the interfin region of the test tube. The actual $p/e$ increases as the fouling deposits continue to fill in the interfin region. When the fouling resistance reaches asymptotic value, the $p/e$ of each helical rib roughness tube will be larger than five. Figure 2(a) in Li et al. [7] shows that the $R_{pp}^f/R_{pp}^e$ in Table 1 is close to asymptotic value. On the other hand, PPF deposits exist on the total heat transfer surfaces of the tubes. The actual $p/e$ values remain closed to the $p/e$ values of clean tubes as the fouling deposits increase, which is confirmed by Fig. 2. Therefore, there are two ranges in Fig. 3 corresponding to $p/e \geq 5.0$ and $p/e < 5.0$, respectively.

Conclusions

The fouling resistances of helical rib roughened tubes were higher than that of plain tube at low $Re$ (16,000) for both practical long-term PPF and accelerated particulate fouling.

With respect to PPF, the $R_{pp}^f/R_{pp}^e$ ratio linearly increases with increasing $\beta - \eta$. There are two linear ranges of fouling characteristics corresponding to $p/e \leq 5.0$ and $p/e \geq 5.0$. With respect to accelerated particulate fouling, the ratio of $R_{pp}^f/R_{pp}^e$ linearly increases with increasing $\eta$. There is one range of fouling characteristics.

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Nomenclature

- $A = \text{nominial internal surface area based on plain tube, m}^2$
- $A_e = \text{cross sectional area, m}^2$
- $A_v = \text{inside wetted surface area, m}^2$
- $e = \text{internal rib height (average value), m}$
- $f = \text{fanning friction factor, dimensionless}$
- $j = \text{Colburn} \ j = \text{-factor (St}^{1/2} \text{), dimensionless}$
- $p = \text{axial element pitch, m}$
- $R_f = \text{fouling resistance, m}^2 \cdot \text{K/W}$
- $R_{pp}^e = \text{asymptotic fouling resistance, m}^2 \cdot \text{K/W}$

Greek Letters

- $\beta = \text{area index,} (A_v/A_v^e)/(A_e/A_e)$, dimensionless
- $\eta = \text{efficiency index,} (j/j_p)/(f/f_p)$, dimensionless

References


Monte Carlo Simulation of Radiative Heat Transfer in Coarse Fibrous Media

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Direct Geometric Monte Carlo modeling of a fibrous medium is undertaken. The medium is represented as a monodisperse array, with known solidity, of randomly oriented cylinders of known index of refraction. This technique has the advantage that further radiative properties of the medium (absorption coefficient, scattering albedo, scattering phase function) are not required, and the drawback that its Snell- and Fresnel-generated data suggest a limitation to large, smooth fibers. It is found that radiative heat flux results are highly dependent on bias in the polar orientation angle (relative to the boundary planes) of the fibers. Randomly oriented fiber results compare well to both the large (specular radiosity method) and small (radiative transfer equation) limits, while the results of previous experiments lie within the range of simulation results generated using varying degrees of orientation bias. [DOI: 10.1115/1.1571092]

**Keywords:** Heat Transfer, Insulation, Monte Carlo, Porous Media, Radiation

**Introduction**

In studies of radiative transfer through fibrous materials, the medium is usually represented as spatially continuous and the Radiative Transfer Equation (RTE) is applied, using radiative properties determined from experiment or by applying electromagnetic theory. In the present note fibrous media are represented as discrete arrays of randomly located and oriented cylindrical surfaces. Bundles of radiation intensity are released from the boundaries of the medium in a Geometric Monte Carlo (GMC) simulation. The bundles interact as surface reflections and absorptions with the fibers in their path. By more detailed representation of the fibrous medium morphology, this method avoids breakdown of radiative continuum near the boundaries of the medium, and allows bias in the fiber orientation direction to be conveniently considered. Snell’s laws are used for fiber-surface reflections directions, and Fresnel’s equations are used to derive surface radiative properties. Hence the method applies only to smooth-surfaced fibers in the geometric limit. However, the method has the advantage that the only radiative property required is the complex index of refraction for the fiber material, and there is no uncertainty over the form of the scattering phase function. The technique is similar to that employed by Nisipeanu and Jones [1] in a study of spherical particle suspensions. This note reports an extension of the technique into the geometry of a dispersion of cylindrical particles, which require a different geometrical development, and admit the complication of non-random fiber orientation.

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Analysis

The fibers are randomly oriented, infinitely long, straight cylinders. The fibers are limited in axial extent only by the rectangular boundaries of the medium. Bent fibers could be represented by two or more of these cylinders. Fiber intersection volumes are ignored.

Randomly oriented cylinders are generated in a three-dimensional rectangular control domain of variable size. One of the control domain corners is the origin of the global coordinate system. Randomly oriented position vectors \( \mathbf{v} \) are generated having a length \( S \) (Fig. 1):

\[
S = \lambda x + \mu y + \nu z \tag{1}
\]

is the equation of a plane \( PP \) normal to \( \mathbf{v} \) at a distance \( S \) from the origin, where \( \{x, y, z\} \) is the global coordinate system and \( \{\lambda, \mu, \nu\} \) are the direction cosines of \( \mathbf{v} \). A local coordinate system \( \{x_1, y_1, z_1\} \) is constructed with one axis \( \mathbf{t}_1 \) normal to plane \( PP \), and the other two included in the plane (Fig. 1):

\[
\mathbf{t}_1 = \lambda \mathbf{i} + \mu \mathbf{j} + \nu \mathbf{k}
\]

\[
\mathbf{i}_n = \frac{\mathbf{i} \times \mathbf{t}_n}{|\mathbf{i} \times \mathbf{t}_n|}
\]

\[
\mathbf{i}_2 = \frac{\mathbf{t}_n \times \mathbf{i}_1}{|\mathbf{t}_n \times \mathbf{i}_1|}
\]

\[
\mathbf{t}_2 = \frac{\mathbf{i} \times \mathbf{t}_1}{|\mathbf{i} \times \mathbf{t}_1|}
\]

Randomly oriented cylinders with random offsets from the position vector are generated until the desired volume fraction is obtained. A representation of just a few randomly oriented and positioned fibers is shown in Fig. 2, along with a representative intensity bundle path.

The bottom boundary of the control domain is taken to be hot and black, emitting radiative flux simulated as energy bundles released from random positions and directions. The top boundary is taken to be black, cold in some cases, and cooler than the bottom boundary (though still emitting) in others. The coordinates of an energy bundle after it travels a distance \( P \) are

\[
x = x_{st} + l_1 P
\]

\[
y = y_{st} + l_2 P
\]

\[
z = z_{st} + l_3 P
\]

where \( \{x_{st}, y_{st}, z_{st}\} \) initially represent the starting position of the bundle, \( x_{st} \) and \( y_{st} \) are random, and \( z_{st} \) corresponds to the top or bottom boundary, and \( \{l_1, l_2, l_3\} \) are the direction cosines of the

If

\[
A = \begin{bmatrix}
0 & \frac{-\nu}{\sqrt{\mu^2 + \nu^2}} & \frac{\mu}{\sqrt{\mu^2 + \nu^2}} \\
\frac{\lambda}{\sqrt{\mu^2 + \nu^2}} & \frac{-\lambda \mu}{\sqrt{\mu^2 + \nu^2}} & \frac{-\lambda \nu}{\sqrt{\mu^2 + \nu^2}} \\
\lambda & \mu & \nu
\end{bmatrix}
\]

then

\[
x_j \mathbf{t}_1 + y_j \mathbf{t}_2 + z_j \mathbf{t}_n = x_{st} \mathbf{i} + y_{st} \mathbf{j} + z_{st} \mathbf{k} = A \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]

relates the local to the global coordinate system.

The equations of a cylinder normal to plane \( PP \) in the local coordinate system are

\[
R^2 = (x_j - x_{lo})^2 + (y_j - y_{lo})^2
\]

where \( x_{lo} \) and \( y_{lo} \) are the coordinates of a point arbitrarily chosen on plane \( PP \), and \( R \) is the fiber radius. The local coordinate system is used in order to construct cylinders that are not necessarily centered around the position vector \( \mathbf{v} \). By applying Eq. (4), the infinite cylinder equation in the local coordinate system becomes

\[
R^2 = \frac{[(\mu^2 + \nu^2)(x - x_{lo}) - \lambda \mu(y - y_{lo}) - \lambda \nu(z - z_{lo})]^2 + [\mu(z - z_{lo}) - \nu(y - y_{lo})]^2}{\mu^2 + \nu^2}.
\]
bundle’s random direction. Writing Eqs. (7) into Eqs. (6) yields a relation where \( P \) defines a point of intersection between the bundle and a cylinder. The discriminant of this second order equation in \( P \) is computed. If this discriminant is positive, the equation has real roots and there is an impact at the smaller of the two \([2]\), the other impact point being impossible for opaque fibers. The incident polar and azimuthal angles relative to the fiber surface are computed from the direction of the in-coming ray and the surface normal at the intersection point.

At the point of impact with the fiber’s surface, the energy bundle is absorbed or reflected depending on the absorptivity or reflectivity of the fiber. The surface radiative properties are predicted by Fresnel’s equations, which are functions of the fiber material complex index of refraction \( m = n - ik \). Reflections by individual fibers may be taken as specular without loss of accuracy \([1]\). The direction of a reflected ray is determined using Snell’s law:

\[
\hat{s}_r = \frac{\hat{s}_i + 2\hat{n}_i \cdot \hat{n}}{|\hat{n}_i|} \hat{n}
\]

in which the unit surface normal \( \hat{n}(n_1, n_2, n_3) \) is

\[
\hat{n} = \frac{\nabla g}{|\nabla g|}
\]

and \( g \) is the right-hand side of the surface equation of the cylinder given by Eq. (6). The bundle is followed in its new (reflected) direction, from \((x_i, y_i, z_i)\) at the impact point until it either strikes another fiber, reflects off the (perfectly specular) sides of the medium, or exits the medium through its top or bottom surfaces. For each reflection of each bundle, each fiber is checked for possible impact. If more than one fiber shows a possible impact, then the closest is taken as the true impact.

If the energy bundle is absorbed by an interaction with a point on the fiber surface, and then re-emitted from the interaction point (a consequence of radiative equilibrium), then its direction is taken to satisfy an overall diffuse directional distribution. A local coordinate system is chosen with one unit vector \( \hat{t}_n \) at the impact point on the fiber surface and axially directed, and the other defined by

\[
\hat{i}_t = \hat{t}_n \times \hat{n}
\]

The direction of the emitted ray is given by

\[
\hat{s}_e = \hat{t}_1 + \hat{t}_2 + \hat{t}_3 + \hat{n} = (\lambda l_{12} + l_{31} n_1 - v l_{12} n_2 + \mu l_{12} n_3)\hat{i}_t + (\mu l_{12} + v l_{12} n_1 + \lambda l_{12} n_2 + l_{31})\hat{k}
\]

where the local direction cosines result from a random local azimuth, and a local polar angle whose probability density function corresponds to a diffuse distribution.

If Eqs. (6) and (7) with unknown \( P \) have no real solution, then the energy bundle strikes one of the bounding surfaces. The side walls are considered to be perfect specular reflectors in order to account for contributions to radiative transfer from the regions beyond the computational domain. The energy bundles undergo multiple reflections, absorptions, and/or re-emissions until they eventually get to the top or bottom boundaries. The total number of bundles that pass through the top boundary determines the transmitted flux.

Energy bundles are released until the transmittance of the medium becomes stationary to within at most 3%, which requires as many as 10⁶ bundles for higher optical thicknesses (representing over 200 hours of run time on a 200 MHz UltraSPARC processor). For lower optical thickness, stationarity within 0.1% is achieved with a few as 10⁴ bundles (20 to 30 hours). No results are reported that use fewer than 10⁴ bundles.

In the discontinuous limit, GMC results were checked against the cases of a single fiber, placed both horizontally and vertically in the medium. These cases are readily solved using a radiosity method with specular view factors. GMC results agreed to within 3% with the radiosity method for a horizontal cylinder and 10⁴ bundles. GMC results are essentially identical to radiosity results for a vertical cylinder and 10⁴ bundles.

In the continuous limit, GMC results were checked against Monte Carlo simulations of a radiative continuum (Continuous Monte Carlo, CMC, as derived from Modest, \([3]\), by Nisipeanu, \([4]\)). The CMC simulations use both Linearly Anisotropic Scattering (LAS, with specified angular distribution coefficient) as well as an embedded Scattering-on-Cone (SC) method \([4]\). In the later, the scattering direction compliments an incoming ray’s polar angle from the axis of a randomly oriented scattering fiber, but it is assumed that in the small fiber limit, the azimuthal scattering angle about the fiber axis is uniformly distributed. Comparison of GMC results to CMC is described in the Results section. CMC with LAS was itself verified against an exact integral solution of the RTE \([3]\), agreeing within 1% for 10⁴ bundles.

**Results**

Test case parameters are chosen in order to allow qualitative comparison to the experimental results of Tong et al. \([5]\), concerning fiberglass insulation material 2.54 × 10⁻² m thick with a solubility of 0.0033 and a mean (though not uniform) fiber diameter of 6.9 μm. The directional bias of the fibers, if any, is unknown. These parameters represent a practical engineering example of the application of disperse fibrous media, and are considered here even though the ratio of fiber diameter to wavelength (at low ends of the diameter distribution and for longer radiatively significant wavelengths) is very near the accepted limit of applicability of geometric scattering. Tong et al. measured the total heat flux under vacuum conditions (radiation plus a small amount of fiber-to-fiber conduction) between black boundary plates at varying hot boundary temperatures and a fixed cold boundary temperature (308 K); results are reproduced in Fig. 3.

Figure 3 shows CMC radiative heat flux results of both LAS and SC for the physical conditions of Tong et al.’s experiment. Tong et al. derived theoretical radiative properties for these conditions using a random fiber orientation, yielding a wavelength-averaged extinction coefficient (825 m⁻¹), scattering albedo (0.605), and angular distribution coefficient for LAS (0.804). These properties were used in the gray CMC simulations. The
LAS and SC results agree well, indicating that use of SC might allow CMC simulation without explicit knowledge of the angular distribution coefficient.

Figure 3 also shows GMC results for the boundary conditions and solidity of Tong et al.’s experiment, modeling a monodisperse medium with a fiber diameter of 6.9 μm and a wavelength-averaged index of refraction for glass of 1.49+i1×10^{-4} (derived from the data of Hsieh and Su, [6]). Agreement between GMC and CMC results is quite good, though this should be regarded as somewhat fortuitous since the GMC results did not consider fiber diameter variations, while the radiative properties derived by Tong et al. are integrated over a particular fiber radius distribution. Also, the index of refraction itself was averaged for the GMC simulations, instead of more properly averaging the heat flux results over a range of spectral simulations. This was done because of the extreme run times required by present GMC simulations, and because the intent of the present results is to serve as a general illustration, as follows.

Primarily, Fig. 3 shows GMC results under varying conditions of fiber orientation bias. Practical fibrous media might not be accurately represented by randomly oriented fibers, especially in mat applications. Figure 3 shows GMC results in which the fiber orientation azimuthal angle is random, but the polar angle (relative to the boundary planes) is fixed at 0, 45 deg, and 90 deg. Figure 3 shows a significant dependence of the GMC results on fiber orientation. Radiation is mostly back-scattered by horizontal fibers, and the best transmittance is for vertical fibers. Results both from randomly oriented fibers and from Tong et al.’s experiment lie within the range of the orientation bias results. Clearly, quantitative comparison between GMC and experimental results will require knowledge of the fiber orientation bias in the experimental test article.

The issue of the applicability of the radiative continuum assumption to discontinuous media is addressed for spherical particles and randomly oriented cylindrical fibers. The results are very similar, suggesting that as long as the particle orientation is random, the particle shape (varying from spherical to cylindrical) may be relatively unimportant. This suggestion holds only for randomly oriented scatterers.

Confidence in the applicability of GMC to fibrous media would certainly benefit from future research in such areas: sensitivity to index of refraction variations; non-smooth (non-specular) fiber surfaces; and less idealized descriptions of the fibrous medium geometry. The present computational implementation of GMC is rather cumbersome, though future improvements should make a wider variety of results more readily available.

Conclusions

A Geometric Monte Carlo (GMC) model is developed for radiation in discrete (non-continuum) fibrous media. Unlike continuous medium (RTE) models, GMC requires only volume fraction, fiber diameter, fiber orientation, and fiber material complex index of refraction as parameters; independent computation of an extinction coefficient, scattering albedo, and scattering phase function are not necessary. However, applicability of GMC to fibrous media in small diameter, long wavelength situations remains to be demonstrated. GMC results for fibrous media suggest that:

1. The distribution of polar orientation direction of the fibers is very important to radiative flux; comparison between the GMC results and experimental results cannot be meaningful without knowledge of the orientation distribution in the experiment.

The geometric limit, the shape of randomly-oriented scattering particles in a discontinuous medium or solid matrix may be relatively less important, as indicated by the similarity of fibrous medium and spherical suspension results for equal volume fraction and fineness parameter.

Nomenclature

\( A \) = transformation matrix: global to local coordinates

\( f_s \) = volume fraction (solidity)

\( g \) = left-hand side for the surface equation of a cylinder

\( H \) = plane-parallel medium depth, m

\( \hat{i}, \hat{j}, \hat{k} \) = unit vectors in \( x, y, z \) directions

\( l_1, l_2, l_3 \) = direction cosines

\( \hat{n} \) = unit surface normal

\( P \) = dimensionless pathlength

\( PP \) = plane perpendicular on the position vector

\( R \) = fiber radius, m

\( \mathcal{D} \) = real numbers domain

\( S \) = distance from origin to plane \( PP \)

\( \mathbf{s}, \mathbf{s}' \) = original and scattered ray direction vectors

\( \mathbf{s}_i, \mathbf{s}_e \) = incident and reflected ray direction vectors

\( \mathbf{i}_1, \mathbf{i}_2 \) = unit vectors included in plane \( PP \)

\( \mathbf{i}_p \) = unit vector parallel to direction of cylinder

\( T \) = boundary temperature, K

\( \mathbf{\hat{e}} \) = position vector

\( x, y, z \) = position coordinates

\( \lambda, \mu, \nu \) = direction cosines of cylinders

Subscripts

1.2 = hot, cold boundary

1,2,3 = perpendicular directions in local coordinate system
Unified Wilson Plot Method for Determining Heat Transfer Correlations for Heat Exchangers

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The paper deals with the unified Wilson plot method used for obtaining heat transfer correlations for finned heat exchangers. In this approach, the direct nonlinear regression is implemented. The numerical example with uncertainty analysis is included as well. [DOI: 10.1115/1.1576810]

Keywords: Convection, Data Analysis, Energy, Finned Surfaces, Heat Transfer, Heat Exchangers, Parameter Estimation

1 Introduction

For design and analysis of heat exchangers, it is necessary to evaluate the average heat transfer coefficients, if not available, for one or both fluid side surfaces. If they are to be determined for both fluid sides of a heat exchanger or for the case when the thermal resistances on both fluid sides are of the same order of magnitude and we want to determine an accurate heat transfer correlation on the unknown fluid side, the Wilson method appears to be very useful. The main idea of Wilson technique is to split the overall thermal resistance, previously determined, into individual thermal resistances. In order to obtain the overall thermal resistance, inlet and outlet flow rates and temperatures on both sides of the heat exchanger and heat transfer rate have to be measured. Afterwards, we need to make the precise energy balance of the heat exchanger, and to apply a suitable statistical procedure of data analysis. When the modified Wilson plot techniques are applied, the linear regression is used as a statistical procedure. Due to the fact that only linear regression is used to estimate the unknowns, the number of unknowns cannot be greater than 2 in the modified Wilson method. One of the modifications of Wilson method, which allows us to estimate 3 unknown parameters is based on the double use of the linear regression scheme connected with an iterative procedure [1–4]. If the number of unknowns is greater than 3, the required evaluations are more complicated and the estimation results are less accurate. Moreover, when using linearization by means of taking the natural logarithm, heat transfer correlations may have the form of monomials only, which additionally limits possible applications of the modified method. Furthermore, it changes the original equation for the overall heat transfer coefficient and can lead to unsatisfactory estimates. It is worth mentioning that the fin efficiency can be taken into account only by an additional, internal iterative scheme. In the modified techniques, the issue of experimental uncertainty is not addressed at all. For these reasons the unified method has been suggested.

2 The Unified Wilson Plot Method

For any two-fluid heat exchanger, the overall heat transfer coefficient is given by:

\[
\frac{1}{U} = \frac{1}{h_i} + \sum_i R \frac{1}{h_o A_o} \quad l = 1, 2, \ldots, q.
\]  (1)

If, additionally, the finned cooler is considered, the above equation takes the form:

\[
\frac{1}{U} = \frac{1}{h_i} + \sum_i R \frac{A_l}{\chi h_o (\eta_f A_f + A_{fin})},
\]  (2)

where \(\chi\) is a coefficient connected with condensation of air moisture, which is equal:

\[
\chi = \frac{\Delta x m_{d} + \Delta \dot{Q}}{\dot{Q}}
\]  (3)

and \(\eta_f\) is the fin efficiency for the straight fins, defined by:

\[
\eta_f = \frac{\tanh(y h_o)}{y h_o}.
\]  (4)

The value \(\gamma\) in formula (4) is the fin efficiency parameter given by:

\[
\gamma = \sqrt{\frac{2\chi h_o}{k_f \delta_f}}.
\]  (5)

The unified Wilson plot technique allows us to consider different flow regimes, but requires all test data on one side of the exchanger to be in on flow regime and no phase changes in the fluid streams flowing through the exchanger. Other limitations given by Shah [1] are relaxed.

Let us assume the following heat transfer correlations for the outside and inside of the exchanger (respectively) [2]:

\[
\text{Nu}_o = \frac{h_o d}{k_o} = C_o Re_o^{m_c} Pr_c^{1/3},
\]  (6)

\[
\text{Nu}_i = \frac{h_i d}{k_i} = C_i Re_i^{m_i} Pr_i^{1/4}.
\]  (7)

The finned exchanger belongs to the group of highly effective heat exchangers with high values of overall heat transfer coefficients. In that case the left-hand side of Eq. (2) has very small values. It may cause some numerical problems, that is why Eq. (2) is being inversed in the adjustment procedure. Assume four unknowns: \(C_o, p, C_i, m\) in Eqs. (6) and (7), and take into account the following notations for indirectly measured quantities:

\[
x_1 = \frac{d}{k_i Pr_i^{1/4}}, \quad x_2 = Re_i,
\]  (8)

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