A Journal Review:

Introduction
Granular materials, like sand or powders, exposed to an external force will undergo statistical-like motion due to the random nature of the size and shape of the grains and their contacts. This observation inspired the author to use thermodynamical concepts to describe the motion of granular particles in a similar way as one describes the motion of molecules in a gas at a given temperature.

Contrast to molecular thermodynamics
As opposed to the typical thermodynamics of molecular gases the elementary unit of grains is on the mesoscopic scale, which is, consisting of many atoms \((10^{15} - 10^{25})\) per grain. On a microscopic scale the surface of the grains is rough. Solid friction is the immediate consequence - which as a result on the granular level, energy is not conserved but rather dissipated. Plastic deformation of grains during collision also produces energy dissipation.

Constant motion of the grains can only be produced when there is a steady state energy flux. The presence of the energy flux and the fact that on the granular level energy is dissipated forces the author, however, to introduce concepts beyond that of typical equilibrium thermodynamics.

Development of an analog to thermodynamics
A reasonable starting point was to consider energy conservation as the first “thermodynamical principle”:

\[
\Delta I = \Delta E_{\text{int}} + \Delta D \quad \text{ ... (1)}
\]

The internal energy \(E_{\text{int}}\) can be likened to the thermodynamics kinetic and potential energy of all the degrees of freedom of the grains as elastic bodies. \(\Delta I\) is the energy introduced into the system while \(\Delta D\) energy was dissipated in order to maintain steady state. Allowing for changes in the system’s volume, equation (1) will become

\[
\Delta I = \Delta E_{\text{int}} + \Delta D + \Delta W, \quad \text{where } \Delta W \text{ is the work done to change the volume.}
\]

The author then dealt with the excess dissipated energy \(\Delta D = \Delta D - \Delta I\) (nick-named “dissipate) in a similar way as the thermodynamic concept of heat. Like heat, dissipate is not a potential. The dissipate, however, does not stem from the kinetic energy of the particles but is due to particular collisions.

The dissipate is thus proportional to the sum of normal forces \(f_{n}^{i}\) that push the particles together during collision \(i\).

\[
\delta D = p \delta C \quad \text{ ... (2)}
\]

\(p\) is an internal pressure acting at collision, which the author called “compression”. It was defined as \(p = \rho \left( \frac{f_{n}^{i}}{A_{i}} \right)\) where \(A_{i}\) is the area of contact of collision \(i\) and \(\rho\) is the number of collisions per unit volume per unit time. The term \(C\) called “contactopy” is defined as a potential for a dissipative systems and it is the conjugate variable to \(p\).
The contactopy has contributions due to plasticity and due to dynamic and static friction. But although it resembles the internal energy in having kinetic and potential contributions, it is analogous, in the author’s thermodynamic formalism, to the entropy. Geometrically, it can be related to the (average) particle overlap volume $V_{ov}$ that one has (for technical reasons) in molecular dynamic simulations.

$$C = \frac{\gamma}{\rho'} V_{ov} \quad \cdots (3)$$

Division by the number $\rho'$ of collisions per unit volume assures that $C$ is an extensive quantity. $\gamma$ is a material dependent constant.

“Equilibrium” was defined as the ensemble minimizing, at fixed $E_{\text{int}}$, the contactopy. Herrmann then postulated an analog to the second law of thermodynamics:

$$\Delta C \leq 0 \quad \cdots (4)$$

Various ensembles were also characterized. Fixing the compression $p$ led to the “granular ensemble” and fixing the contactopy $C$ resulted in the “atomistic ensemble”. The later case is difficult to implement in practice, so more consideration was given to the granular ensemble.

A granular potential $G_r$ was defined as

$$G_r = E_{\text{int}} + pC \quad \cdots (5)$$

An immediate consequence of (4) is that at constant $p$ the equilibrium is given by the minimum $G_r$. An analog to the specific heat called “dissipativity” $\kappa$ was defined as

$$\kappa = \partial D / \partial p = p \left( \partial C / \partial p \right) \quad \cdots (6)$$

This new quantity describes how much more energy can be dissipated if the compression is increased.

In the granular ensemble a “dissipate” bath (instead of a heat bath) is coupled to the system and consequently $E_{\text{int}}$ (and by extension $G_r$) of the granular material is a fluctuation quantity. A definition for a “granular temperature” $T_g$ is given as:

$$T_g = \left( \frac{\partial G_r}{\partial S} \right)_p$$

$T_g$ is the variable that controls the “granular canonical” ensemble with the granular free energy $F_g$ as the potential, defined as the Legendre transform of the granular potential: $F_g = G_r - T_gS$. One can liberate the number of grains and introduce a “granular grandcanonical” potential controlled by a chemical potential.

**Conclusions**

Herrmann was able to describe within a thermodynamic formalism the fluctuations resulting from the constant flux and dissipation of energy that drives granular material behaviour. A “granular ensemble” was defined and coupled to a “dissipate bath”. A dissipative potential called the “contactopy” was introduced. Fluctuating internal energy was replaced by a granular potential controlled by an intensive variable called the “compression”, which is conjugate to the contactopy. In a “granular canonical” ensemble, a “granular temperature” was defined.