GROUND WATER

COMPUTER NOTES

THE POWER OF THE ELECTRONIC WORKSHEET: MODELING WITHOUT SPECIAL PROGRAMS

by T. N. Olsthoorn[a]

Abstract. So-called spreadsheet programs are forerunners of new general software from which the ground-water engineer is likely to benefit—no more special programs each capable of fulfilling just that single task it was explicitly designed for, but programs suitable to do almost any job, where only the fantasy of the user limits the possibilities. Moreover, this new software shows us what user friendliness looks like and proves that it is completely superfluous to be a computer specialist to use it.

With a spreadsheet you can build a simple finite-difference model from scratch within one or two minutes. With some extra time a wide versatility of problems can be solved as the multiaquifer, three-dimensional, phreatic and transient ground-water flow examples prove. This way any hydrologist should be able to solve diverging ground-water models himself without depending on special numerical models. All he needs beyond a microcomputer and a spreadsheet program is a good knowledge of the water balance.

Introduction

The use of special ground-water computer programs, capable of doing just those tasks they were explicitly designed for (many of them only run well when managed by a specialist) will be forced into the background more and more. The reason for this is the tremendous explosion of the capabilities of general purpose software. The software I refer to will be as useful to the local grocer as it is to the nuclear specialist or anyone else who has to deal with numbers. The ground-water specialist may just as well benefit; I will show you how any hydrologist can build his own ground-water models using the same piece of general software he may have been using to do the bookkeeping of his golf club. Moreover, if the model is not too complex, it will run from scratch in just a couple of minutes. There is no need to get either a Master's degree in computer science or numerical analysis to do it; all you need is a good spreadsheet program and a microcomputer, as well as a good sense of water balances.

Spreadsheets, Forerunners of Future Software

A spreadsheet or electronic worksheet is a computer program for general use. With it you can specify any relations you want in a simple and user-friendly way. The large software houses make their popular spreadsheets run on almost any "micro." The most popular ones like "visicalc" and "multiplan" will run on a great variety of personal computers. Their price is normally about $300, for which you receive a thick manual together with the program on a floppy disk that will run right away on your computer. Of the visicalc program alone, over 300,000 copies have been sold. ("Visicalc" and "multiplan" are trademarks of VisiCorp and MicroSoft Corporations, respectively.)

The possibilities of spreadsheets are so extensive and versatile that it is often stated that they alone justify the purchase of a microcomputer. The calculation work for reports which usually takes several days even with the programmable calculator is now fixed in a matter of hours with the spreadsheet.

A spreadsheet gives you a matrix of compartments (cells) on the screen lined up in rows and columns. The total number of cells exceeds many times the number that will fit on your screen at one time. However, with the "arrow-keys" on the keyboard, you can move to any part of the worksheet and with the windowing features you can have different parts of the worksheet on the screen at the same time.

With a spreadsheet you can do the following:

1. Place a piece of text in any cell (for instance, "pi=" in one, "r=" in another one, and "circumference=" in the third cell).

2. Place a number in any cell (for instance "3.1415965" next to the cell holding the text "pi=" and "7" next to "r=").

3. Put an equation in any cell that connects any number of compartments in any way to the cell which keeps the equation. The number that comes out of the equation then becomes visible in the cell you put the equation into. In this example: to obtain the value of the circumference, you will not calculate 2*pi*r on your calculator and fill the result into the cell next to "circumference=," you

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Received July 1984, accepted September 1984.
Discussion open until November 1, 1985.

say: "what I want in this cell is 2 times the value next to 'pi=' times the value in the cell next to 'r='." This "saying" is accomplished in an extremely simple way using the arrow keys on the keyboard and the multiplication key. After this, the result appears. If you change the value next to "r=" the equation is automatically and instantly recalculated, yielding the new circumference in the compartment where the equation has been specified.

In this way any cell may be connected with several other cells in any way. You are not limited to adding and multiplication; there are a great number of special functions directly on hand (ln, sin, square root, etc.). Equations, numbers, and text that you need elsewhere in the sheet are not retyped; they are simply copied into the new fields. In this way you can build your own model on the screen of your personal computer and calculate your way through a hydraulic installation or whatever. The result will always be a neat report you can readily printout and publish and, of course, store on a "floppy" disk to refine your work at a later time. With the more advanced spreadsheets you can also use conditional equations and iterate. At this point it will be clear to any engineer that such a means will extend his personal capabilities, and the purchase of a micro really pays.

A Ground-Water Model Within Two Minutes

Such an electronic worksheet (I use "multiplan"), makes it possible to build and run a ground-water model from scratch within a couple of minutes. A special ground-water program is not necessary. By way of illustration, consider the most simple case: a steady-state two-dimensional one-layer ground-water model having the same transmissivity all over. Adopt a square mesh of points in which to calculate ground-water-head values, since such a mesh fits naturally in the row-and-column pattern of the sheet. These starting points lead to the Laplace equation in two dimensions which after discretization changes to the relationship that the value in any mesh point is one-fourth of the sum of the values in the four surrounding points. (This is the most simple form of the finite-difference method, also called relaxation method; see Verruijt, 1970).

To achieve this you send the cursor, this is the flashing rectangle on the screen showing just where you are, to the cell where you want the top-left corner of the model. This is done using the arrow keys. Here we say that the value in this cell must equal "(his right neighbor + his upper neighbor + his left neighbor + his lower neighbor)/4." With five keystrokes we copy this equation into all of the other cells that are going to be part of the model area, and the model is ready! No more than a minute has passed. Of course, we left an open row on all sides to place the boundary conditions. If this room is not available, simply insert as many rows and columns as you need by just a couple of keystrokes. If the head at the boundary should be zero, you don't have to specify anything as the program will interpret any empty cell as zero. If you need something else, fill it in and copy it into all other cells that should have the same value. The only thing left to be done is to specify internal boundaries like rivers, wells, etc. The simplest way to do this is to fill some fixed head into one or more of the inner points of the model area. Touching the exclamation point of your keyboard suffices to start the iteration process. After this you will notice the numbers start changing on the screen. The program stops automatically when the greatest change in one iteration is less than 0.001; if you want some other criterion, it may be specified easily.

The number of nodes that will fit into the 64k memory of my Apple //e computer is approximately 1000 and may be called reasonable. With any one of the 16-bit machines that are becoming less expensive and which can have a megabyte of memory or more, the number of possible nodes will be an order of magnitude higher!

Leaky aquifer conditions, simultaneous flow in a number of aquifers, three-dimensional flow, even transient and purely phreatic flow are just some of the many possibilities that can be directly calculated with a good spreadsheet, without the need of a special ground-water program.

Simple Situation

We start with a simple situation, namely a single-layer aquifer in two dimensions with the same transmissivity throughout, and a network that consists of squares. The equation already given is valid for any point, provided it is fully surrounded by water. This equation and all that follow are easily and straightforwardly dervied from a simple water balance around the point under consideration. For Figure 1A, this is done as follows:

Continuity states that:

$$q_1 + q_2 + q_3 + q_4 = Q$$

in which q_1 $[L^2/t]$ is the specific ground-water flow

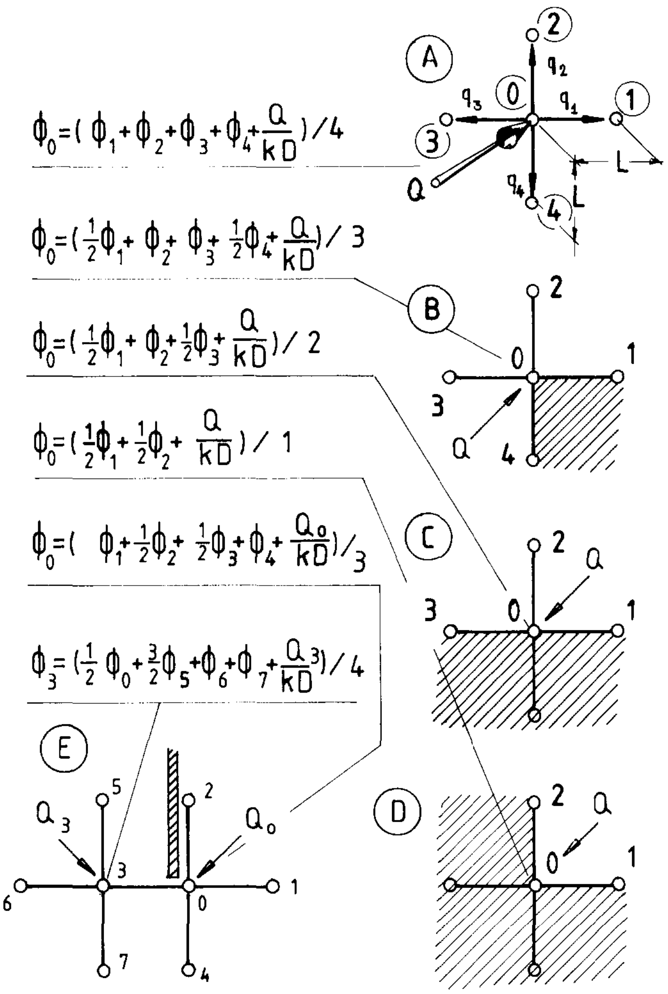


Fig. 1. Single-aquifer model: calculation of the head in a meshpoint from the values in the surrounding points for various situations.

from the node under consideration to the surrounding nodes as given in Figure 1A, and Q $[L^3/t]$ is a flow from the outside world into this node.

Now, applying Darcy's law:

$$q_i = kD\ L\ (\phi_0 - \phi_i)/L$$

in which ϕ_0 is the head in the node under consideration and ϕ_i is the head in one of the surrounding nodes [L], L is the width of one square of the mesh of nodes [L], and kD $[L^2/t]$ is the transmissivity of the aquifer, being the product of its permeability k [L/t] and its thickness D [L].

Combination yields as the end result the relaxation equation already given but now extended with the flow Q:

$$\phi_0 = (\phi_1 + \phi_2 + \phi_3 + \phi_4 + \frac{Q}{kD})/4$$

(note that Figure 1 only shows one single node with its immediate surroundings; it is a so-called calculation molecule).

We also need equations for special points in the model area such as points near impermeable walls, corners and the like. These equations and the cases for which they are valid are given in Figure 1.

It will be clear that these relations permit the use of a given flow as a boundary condition instead of a given head. On the other hand, the flow into or from points with a given head may be calculated directly with these equations. For a node as in Figure 1A, this becomes:

$$Q = kD\ [4\phi_0 - (\phi_1 + \phi_2 + \phi_3 + \phi_4)]$$

which may be placed anywhere in the spreadsheet. It is also possible to incorporate precipitation. Simply extend Q with the precipitation flow into the node:

$$Q \rightarrow Q + nL^2$$

in which n is the precipitation rate [L/t], and L^2 is the area of influence of the node. As will be shown, head-dependent fluxes may just as well be used in this scheme.

If the transmissivity is not a constant, it may be incorporated into the equations (Figure 2A). Very often the transmissivity varies along a line rather than randomly, so that the case of Figure 2B applies, simplifying the equation.

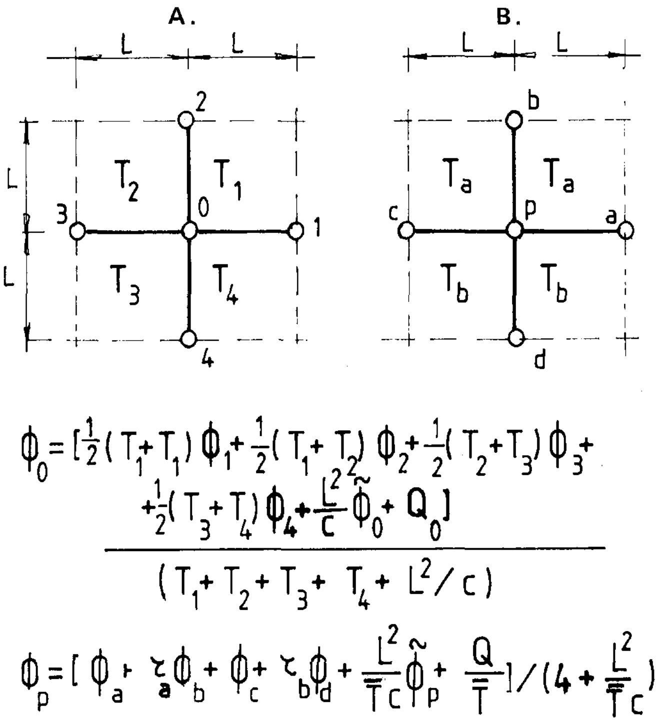


Fig. 2. A: Top view on heterogeneous aquifer. B: A constant transmissivity along one line strongly simplifies the equation. Omit terms containing the aquitard resistivity c for fully artesian aquifers (and in vertical cuts like Figure 3, where permeabilities take the place of transmissivities).

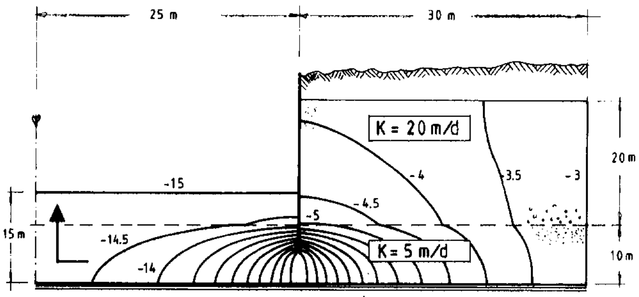


Fig. 3. Flow underneath a retaining wall in vertical cut. Boundary conditions: heads −3 m and −15 m; calculated total flow: 28.2 m²/d.

With the equations derived thus far, a number of interesting ground-water problems may already be solved. Figure 3 gives an example of a vertical cut through the ground-water flow underneath a retaining wall, where the deepest part of the aquifer has a permeability that differs from that of the upper part. The total flow has been calculated as explained above. This water balance has been used here as the criterion to stop the iteration process. (At the moment I broke off this process the total outflow amounted to 28.27 m^2/d and the total inflow 28.08, the difference decreasing further with every iteration step. It is clear that the spreadsheet for such a problem can be stored on a floppy disk so that similar situations arising in the future can be solved by simply changing some of the values on the sheet.

Speed of Convergence

If you apply the equations as given, you will find that sometimes several hundreds of iterations are needed. This may cost some time, a night for instance. This problem occurs mainly if there is only a small number of points with a given head, in general, when the problem is poorly conditioned. The number of iterations needed may, however, be reduced effectively by so-called over-relaxation.

Over-relaxation works as follows: let the difference of the head in a point between two iterations be $\Delta\phi$. In that case, the old value ϕ^- in this point will be incremented by $\Delta\phi$ to obtain the new value ϕ^+. If this process is too slow, don't increment the old value by $\Delta\phi$, but by $\alpha^*\Delta\phi$ instead, in which the over-relaxation coefficient α is greater than 1. A value of 1.5 to 1.7 is often capable of speeding up the iteration process by one order of magnitude.

We can incorporate over-relaxation as follows. The new value ϕ^+ is derived from:

$$\phi_0^+ = \tfrac{1}{4}\,(\phi_1 + \phi_2 + \phi_3 + \phi_4)$$

Its change in one iteration, $\Delta\phi_0$, therefore equals:

$$\Delta\phi_0 = \phi^+ - \phi^- = -\phi_0^- + (\phi_1 + \phi_2 + \phi_3 + \phi_4)/4$$

To obtain over-relaxation, we apply α^* to this change to calculate the new value:

$$\phi^+ = \phi_0^- + \alpha\Delta\phi_0$$

Written out, this gives:

$$\phi_0^+ = (1 - \alpha)\,\phi_0^- + \alpha\,(\phi_1 + \phi_2 + \phi_3 + \phi_4)/4$$

The new value in the cell considered is therefore obtained from the values in the surrounding compartments as well as the over-relaxation coefficient and the value that was already in the cell. α is placed somewhere else in the spreadsheet and is used by all the equations. This technique is applied in the same manner for all of the other equations.

More Complex Situations

If you have several aquifers with simultaneous ground-water flow interconnected via semipermeable aquitards, you have a more complex situation, one which occurs often in practice.

Proceeding in the same manner as before we obtain the following relationship for the situation in Figure 4A:

$$\phi_0 = (\phi_1 + \phi_2 + \phi_3 + \phi_4 + \zeta_1\phi_5 + \zeta_2\phi_6 + \frac{Q}{kD})/(4 + \zeta_1 + \zeta_2)$$

in which $\zeta_1 = L^2/(kDc_1)$, $\zeta_2 = L^2/(kDc_2)$, and where c_1, c_2 [t] are the resistances of the top and bottom aquitards, respectively.

Of course, over-relaxation may be applied in just the same way as before. Also, the relationships for special points along boundaries, etc., may be derived in a straightforward manner. The results are given in Figure 4. Figure 5 gives an example of simultaneous flow in two interconnected aquifers calculated in this way with the spreadsheet.

It is clear that fluxes may be introduced which depend on the head itself. You need to extend the ζ-values in the equations with:

$$\phi_0 = (\ldots + \zeta_i\phi_i + \ldots)/(\ldots + \zeta_i + \ldots)$$

in which $\zeta_i = A_i/(kDc_i)$, and where A/c is the entrance resistance for that point. A [L^2] may be thought of as the area of, for instance, a piece of canal or river that is hung to that particular point, and c [t] as the resistance of the sludge layer at the bottom of this canal. An arbitrary number of different canals, rivers, ditches, etc., each having its own given head, area, and resistance, may in this way be connected to a single mesh point, together with semipermeable layers (being really the same),

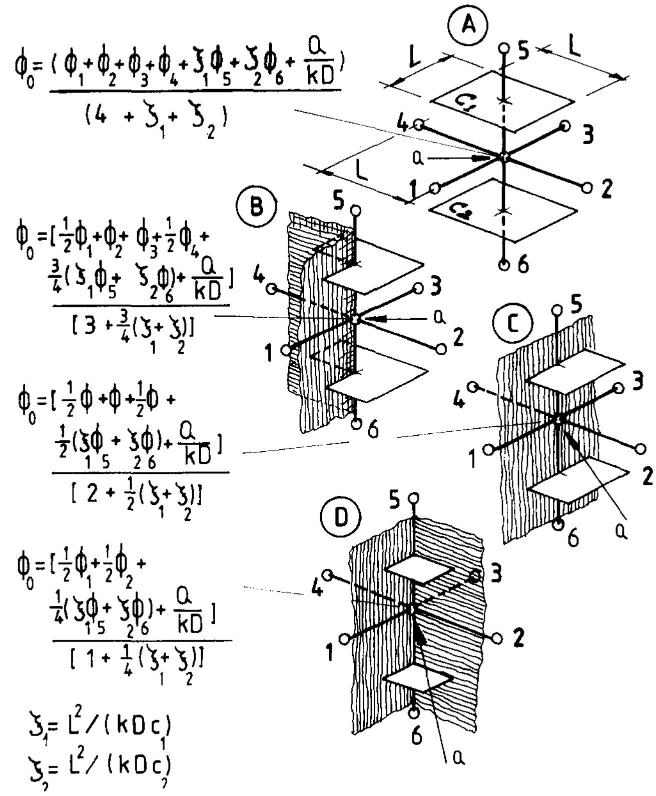


Fig. 4. Multilayer model: calculation of the head in a node from values in the surrounding meshpoints for various situations.

and after some ordering, we get:

$$2\phi_0^2 - 4z\phi_0 + \frac{L^2}{kc}\phi_0 = \tfrac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$$

$$-z(\phi_1+\phi_2+\phi_3+\phi_4) + L^2\phi_6/(kc) + Q/k + nL^2/k$$

Now this equation is linearized in an exact way in order to apply iteration as usual:

$$\phi_0 = \frac{[\tfrac{1}{2}(\phi_2^2 + .. + \phi_4^2) - z(\phi_1 + .. + \phi_4) + \frac{L^2}{kc}\phi_6 + \frac{nL^2}{k} + \frac{Q}{k}]}{(2\phi_0 - 4z + \frac{L^2}{kc})}$$

In other words: the new value ϕ_0 (ϕ_0 at the left side of the "=" sign) is obtained from the values in the surrounding mesh points as well as the value that was already in the cell considered (ϕ_0 in the denominator of the right-hand side of the equation). In this equation we also incorporated flow toward another aquifer via a semipermeable layer as well as precipitation and a flow Q from the outside world into the node. You may, of course, remove a flow Q into the node, and precipitation (the latter being really the same as a flow Q).

Phreatic Water

Phreatic water may seem a problem, since the aquifer thickness itself is now dependent on the ground-water head, introducing nonlinearity between the head and the ground-water flux. The problem, however, proves not very serious when the right "trick" is applied, and convergence occurs readily. By way of introduction we now derive the equation. The only difference from the previous equations is the necessity to connect the aquifer thickness, D [L], to the head [L], and the floor height, z [L], of the phreatic aquifer. To do this we write for the thickness the average of the water depth between two adjacent points, i, j:

$$D_{ij} = \tfrac{1}{2}(\phi_i + \phi_j) - z$$

Thus, the combination of continuity and Darcy's law yields:

$$k(\phi_0-\phi_1)[\tfrac{1}{2}(\phi_0+\phi_1)-z] + k(\phi_0-\phi_2)[\tfrac{1}{2}(\phi_0+\phi_2)-z] +$$

$$k(\phi_0-\phi_3)[\tfrac{1}{2}(\phi_0+\phi_3)-z] + k(\phi_0-\phi_4)[\tfrac{1}{2}(\phi_0+\phi_4)-z] +$$

$$(\phi_0-\phi_6)\,L^2/c = Q + nL^2$$

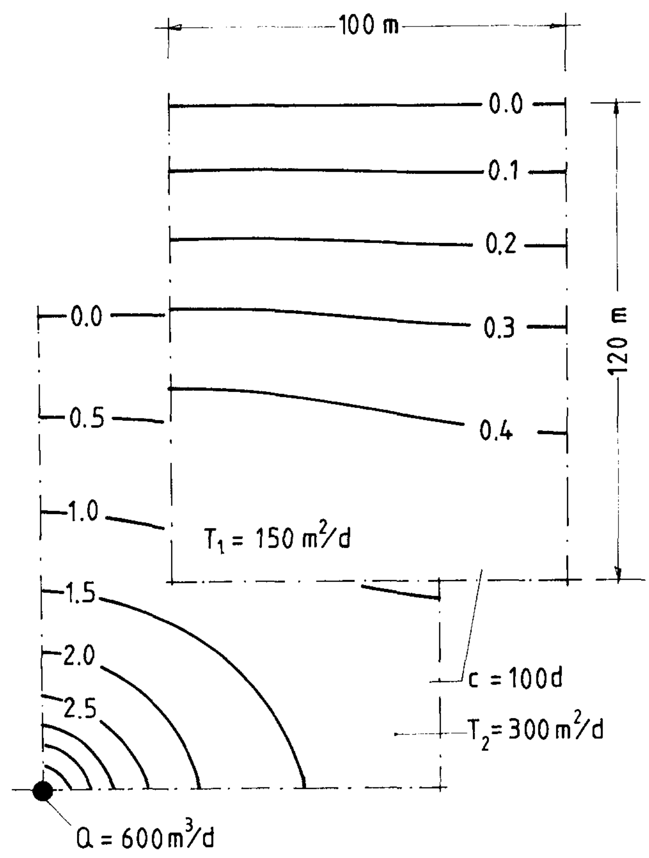


Fig. 5. Simultaneous flow in two aquifers. The 0.0-head boundary is fixed; the other boundaries are impermeable. Withdrawal takes place at the left-hand corner in the lower aquifer. For sake of presentation, both aquifers are shifted with regard to each other.

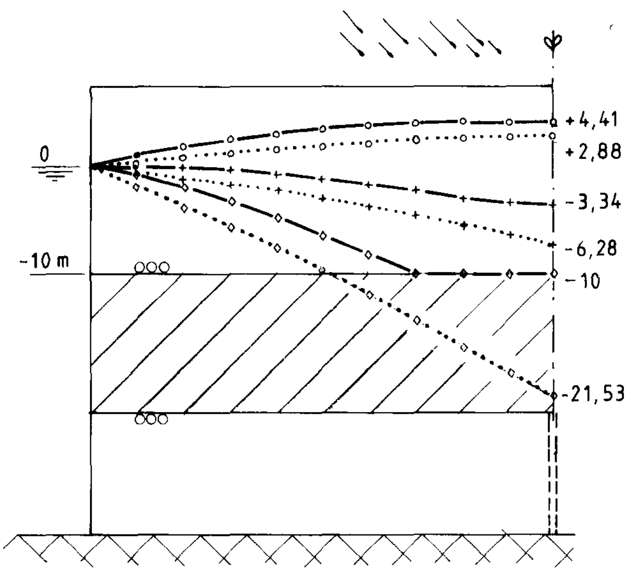


Fig. 6. One-dimensional simultaneous flow in two aquifers, of which the upper one is truly phreatic. Precipitation: 0.001 m/d. Withdrawal from the lower aquifer is as shown. Upper pair of head curves for zero withdrawal, middle pair for 1200 m²/d withdrawal, lower pair for 2400 m²/d withdrawal. In the latter case, the phreatic aquifer dries up partially and the flow towards the lower aquifer is switched off automatically over this area. (The permeability of phreatic aquifer = 10 m/d; the resistance between the aquifers = 2000 d; the transmissivity of the lower aquifer = 500 m²/d).

from the equation those parts you don't need or extend it in the same way as before. Also, the equations for special points are derived in a straightforward way, whereas over-relaxation may be applied as usual; the equations will then become rather complex. Figure 6 gives an example (that was kept one-dimensional for reasons of easy presentation only) of a phreatic aquifer on top with precipitation and a semiconfined aquifer with a withdrawal at the line of symmetry (closed boundary).

It is interesting that the relation holding for the phreatic aquifer allows that aquifer to fall completely dry over some area, when the withdrawal from the lower aquifer is increasing. Numerically no direct problems arise, since the denominator of the equation does not normally become zero when the head of the phreatic aquifer equals its floor height, =z, that is, falls dry. You can simulate this phenomenon by using conditional equations, an option that is standardly available in the spreadsheet I use ("multiplan"). Its form is IF(A,B,C), meaning: If A is true, then we do B, else we do C. In this case we take:

$$\text{IF}(\phi_0 < z + \delta, z, \text{equation})$$

The relationship before the first comma is true when the aquifer falls dry (head under the aquifer floor z; δ is an arbitrary, small value that avoids numerical problems. Take δ, for instance, one inch). z between the commas tells that in that case the head will be fixed at the floor height of the phreatic aquifer. After the second comma we place the equation for phreatic flow as given, which is then evaluated if the phreatic aquifer is not dry.

Doing this, we introduce a hydrological problem with the lower aquifer, since keeping the head of the phreatic aquifer fixed means too great an inflow into the lower aquifer. This problem is solved by applying conditional equations in the lower aquifer as well. These have the form:

$$\text{IF}(\phi_5 < z + \delta, \text{equation 1, equation 2})$$

Thus, when the head ϕ_5 in the point above the considered node in the lower aquifer equals the floor height of the phreatic aquifer, which is therefore dry, equation 1 is evaluated and equation 2 is not calculated. Now equation 1 is chosen to be the relationship holding for completely confined flow, that is without vertical seepage to or from adjacent aquifers, and equation 2 is the one holding for semiconfined flow, therefore receiving water from or losing water to adjacent aquifers. This conditional relationship therefore switches off the seepage from the phreatic aquifer as soon as it falls dry over the point considered. This solves the dry-falling problem of the phreatic aquifer. Figure 6 shows what is obtained when the withdrawal in the lower aquifer is increased from zero via 1200 to 2400 m²/d.

Three-Dimensional Flow

Three-dimensional flow fits directly into the method we have been following. Consider a point in a network in space, consisting of equal cubes. Figure 7A shows a single node of this network together with the nodes directly surrounding it. Continuity states:

$$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = Q$$

while Darcy gives:

$$q_{0\to i} = kL^2 (\phi_0 - \phi_i)/L$$

so that in total, we have:

$$\phi_0 = (\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 + \frac{Q}{kL})/6$$

Figure 7 gives the situation and the relations for other nodes, i.e. nodes which are not entirely surrounded by water.

In our spreadsheet we carry out the calculation of the three-dimensional network in the same way as we did for problems with more than one aquifer. That is, for every layer of nodes a field of cells is reserved in the sheet, and these fields are connected with each other. A regular spacing of these fields allows you to copy entire fields, their connections inclusive, to obtain as many as you want, with only a couple of keystrokes. Figure 8 gives an example of a three-dimensional problem that was calculated in this way.

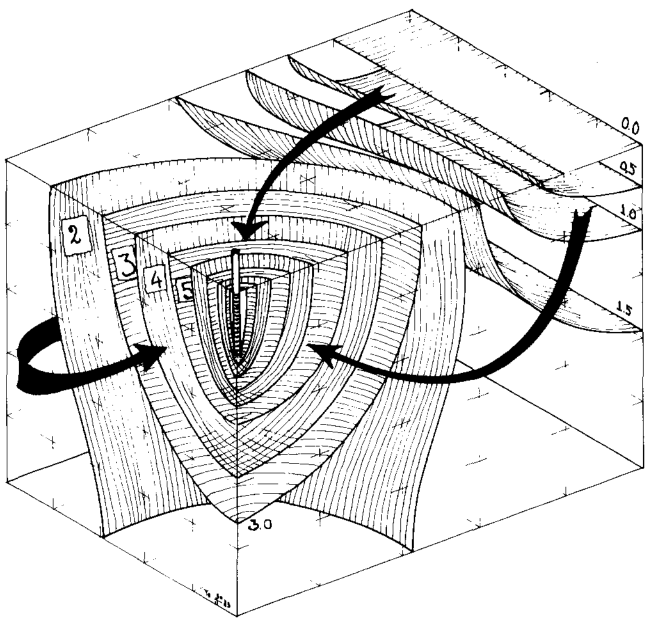


Fig. 8. Calculated three-dimensional problem (6 x 6 x 6 nodes, 10 m mesh) in a piece of aquifer (50 m x 50 m x 50 m, permeability 10 m/d) that is closed at all sides. Shown are the three-dimensional potential surfaces caused by the flow between the ditch (given head = 0) and the well (two nodes with given head of 10 m). The calculated total flow equals 920 m³/d.

Transient Ground-Water Flow

Transient, unstationary flow can be tackled with the spreadsheet as well. This proves hardly more complex than the flow in semiconfined aquifers.

Consider the timespan or timestep t_1, t_2 of length Δt. Denote the heads at the beginning of this timespan with ϕ^- and those at the end of this timestep with ϕ^+. Let ϕ (without a "+" or a "-" mark) be the head at some intermediate time within the step for which the total flow at that moment equals the average flow over the entire timestep. From the mid-value proposition from calculus, it follows that such a time exists.

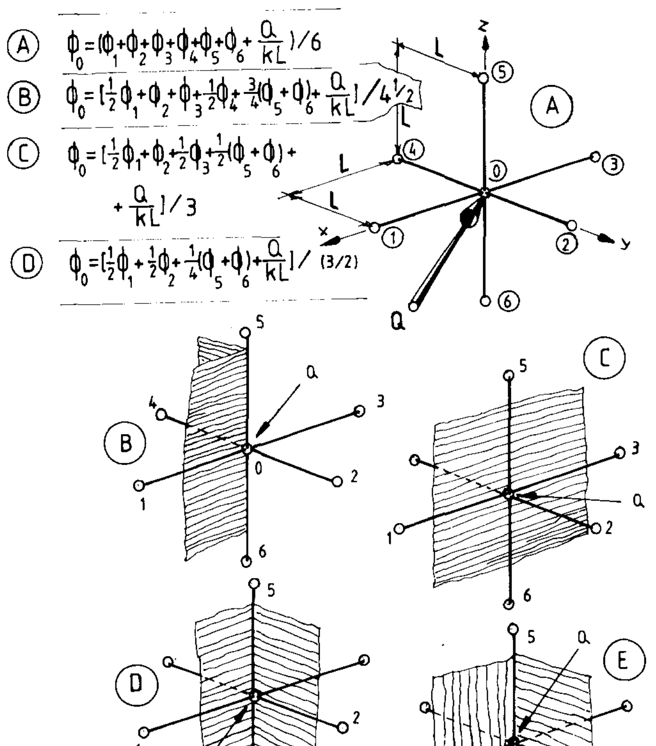


Fig. 7. Three-dimensional model: calculation of the head in a node from values in the surrounding nodes for various situations.

With these definitions, you obtain the following water balance for the node under consideration:

$$(q_1 + q_2 + q_3 + q_4)\,\Delta t + (\phi^+ - \phi^-)\,L^2\mu + (\phi_0 - \phi_5)\frac{L^2}{c}\Delta t + \ldots = Q\,\Delta t + nL^2\,\Delta t$$

After applying Darcy's law and dividing the equation by the transmissivity, kD, and the timestep, Δt, we have:

$$4\phi_0 - (\phi_1 + \phi_2 + \phi_3 + \phi_4) + (\phi^+ - \phi^-)\,\gamma + (\phi_0 - \phi_5)\frac{L^2}{c} + \ldots = \frac{Q}{kD} + \frac{nL^2}{kD}$$

in which:

$$\gamma = L^2\mu/(\Delta t\, kD)$$

and μ is the storage coefficient (specific yield).

ϕ can be expressed in the head at the beginning of the timestep and the head at its end. Let θ be some value between 0 and 1, then:

$$\phi = \phi^- + \theta\,(\phi^+ - \phi^-)$$

so that

$$\phi - \phi^{-} = (\phi - \phi^{-})/\theta$$

Filled in, this gives:

$$\phi_0 = \frac{(\phi_1 + .. + \phi_4 + \frac{\gamma}{\theta}\phi_0^{-} + \frac{L^2}{kDc}\phi_5 + .. \frac{nL^2}{kD} + \frac{Q}{kD})}{(4 + \frac{\gamma}{\theta} + \frac{L^2}{kDc})}$$

With this relationship and given value of θ, the head in the intermediate point within the timestep can be calculated iteratively from the heads at the beginning of the timestep and the heads of the surrounding nodes. (You may have noticed that the time-dependency enters this equation in exactly the same way as the flow to adjacent aquifers.) The heads at the end of the timestep follow directly from those at the start of the timestep and the calculated values at the intermediate time:

$$\phi^{+} = \phi^{-} + (\phi - \phi^{-})/\theta$$

(This method was published by Verruijt in 1972.)

By building up a field with the equations for one timestep, and then copying the entire field to other locations in the spreadsheet, the heads can be calculated for a great number of timepoints in one single run. (There are more ways in which this effect can be achieved, for instance by switching back and forth between fields and so having time incremented automatically until some given point in time is reached).

The value of θ plays an important role. This coefficient determines the amount of explicitness of the calculation. $\theta = 0$ gives a fully explicit calculation; $\theta = 1$ is fully implicit, while $\theta = 0.5$ yields the central difference method of Crank-Nicholson, normally giving the highest accuracy. $\theta > 0.5$ guarantees stability of the solution, while $\theta < 0.5$ yields oscillating solutions if you have a poor ratio between mesh width, transmissivity, storage coefficient, and timestep; $\Delta t\, kD/(\mu L^2)$ should then be smaller than 0.5. A value for θ of 2/3 is practical; it coincides with the finite-element approximation of the time derivative according to Galerkin. Figure 9 gives an example calculated in this way.

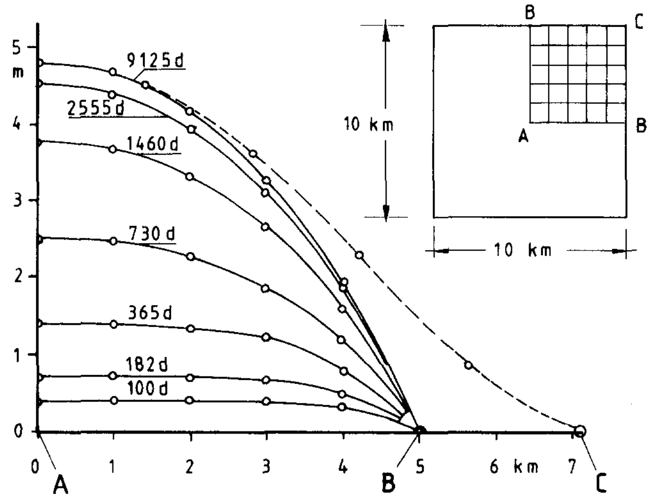
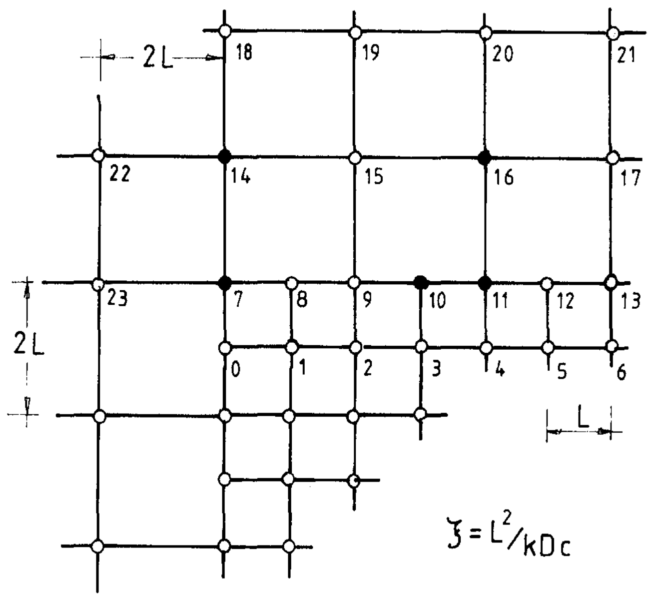


Fig. 9. Calculated head at various points in time in a 10 km x 10 km square area with fixed boundaries (head = 0), which had a horizontal ground-water level equal to 0 at t = 0. (kD = 1500 m²/d, storage coefficient = 0.25, precipitation = 0.001 m/d.)



$$\phi_{11} = [\tfrac{3}{2}\phi_{12} + \tfrac{1}{2}\phi_{16} + \tfrac{3}{2}\phi_{10} + \phi_4 + \tfrac{3}{2}\zeta\tilde{\phi}_{11} + Q/kD]/(\tfrac{9}{2} + \tfrac{3}{2}\zeta)$$

$$\phi_{10} = [\tfrac{3}{2}\phi_{11} + \tfrac{1}{2}(\tfrac{1}{2}(\phi_{16} + \phi_{15})) + \tfrac{3}{2}\phi_9 + \phi_3 + \zeta\tilde{\phi} + \frac{Q}{kD}]/(\tfrac{9}{2} + \tfrac{3}{2}\zeta)$$

$$\phi_{16} = [\phi_{17} + \phi_{20} + \phi_{15} + \tfrac{1}{4}\phi_{10} + \tfrac{1}{2}\phi_{11} + \tfrac{1}{4}\phi_{12} + 4\zeta\tilde{\phi} + \frac{Q}{kD}]/(4 + 4\zeta)$$

$$\phi_7 = [\tfrac{3}{2}\phi_8 + \tfrac{3}{4}\phi_{14} + \tfrac{3}{4}\phi_{23} + \tfrac{3}{2}\phi_0 + \tfrac{9}{4}\zeta\tilde{\phi}_7 + Q/kD]/(\tfrac{9}{2} + \tfrac{9}{4}\zeta)$$

$$\phi_{14} = [\phi_{15} + \phi_{18} + \phi_{22} + \tfrac{3}{4}\phi_7 + \tfrac{1}{4}\phi_8 + \zeta\tilde{\phi}_{14} + \frac{Q}{kD}]/(4 + 4\zeta)$$

Fig. 10. Calculation of head in marked (black) meshpoints from values in surrounding nodes at the boundary of a fine to coarse part of the network.

Arbitrary Refinement of the Network

Arbitrary refinement of the network is something many a ground-water model user has been longing for whenever he found himself bending

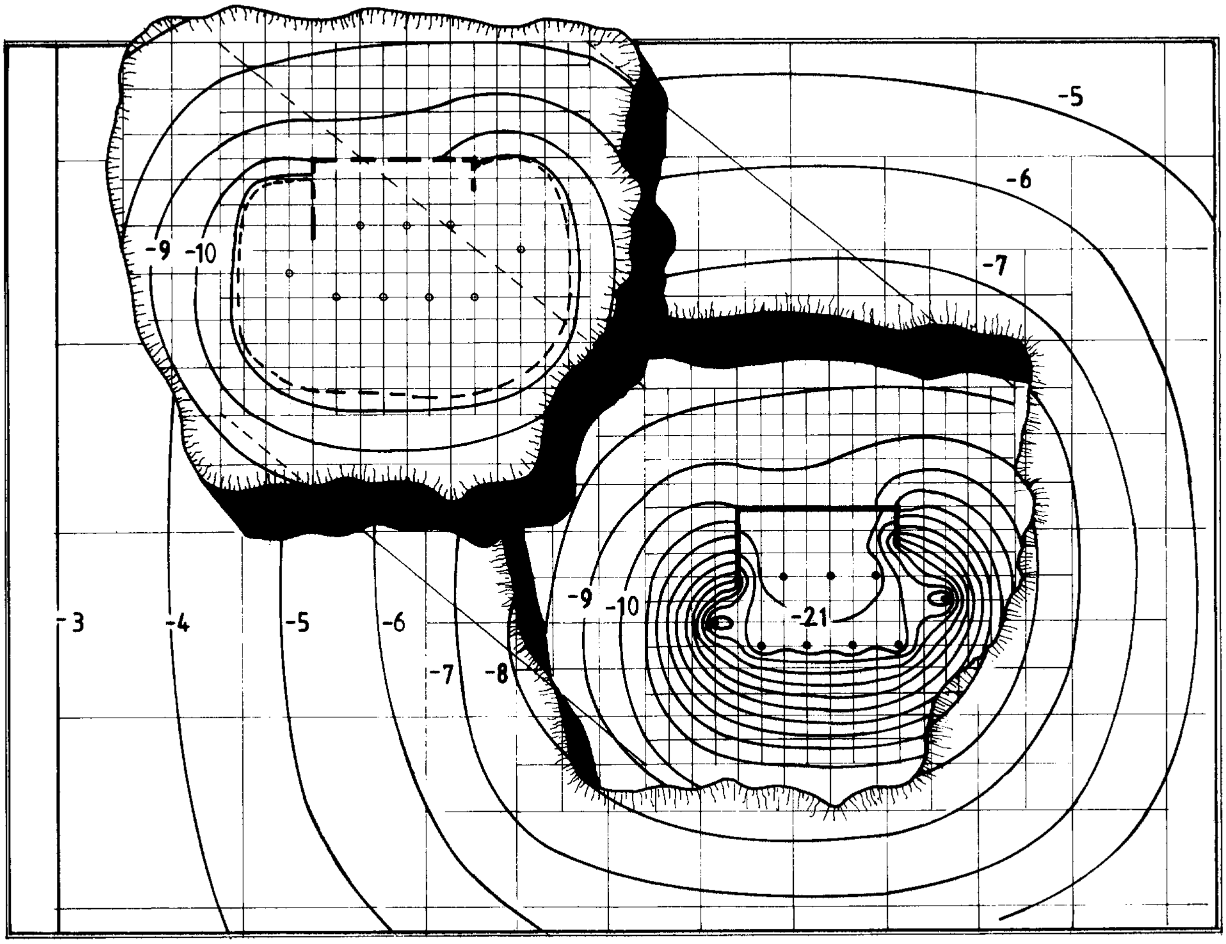


Fig. 11. Example of network refinement and the effect of a retaining wall. The finer meshed inner part of the network (mesh = 12.5 m) contains two aquifers that are connected together to one total aquifer at the boundary with the first network coarsening. The upper aquifer is truly phreatic and falls dry within the dashed line (−11.25 m), because of the ground-water withdrawal from the lower aquifer only. The total withdrawal calculated is 880 m^3/d; the permeability of the top layer = 5 m/d; the resistance between the top aquifer and the lower one = 25 d; the transmissivity of the lower aquifer = 20 m^2/d; the total transmissivity in the outer part of the network equals 50 m^2/d. The coarsening of the network is the same towards all sides and is partly visible in the lower left-hand side of the picture where −3 m marks the given head at the outer boundary of the network.

over one of the many completely unsystematic-looking triangular mesh patterns.

Now, consider a fine mesh of squares connected to a mesh of squares with double side length. (A mesh of squares and doubling the side length makes life easy, but this is not a necessary restriction). Figure 10 gives the situation. We may now develop the water balance for the transition from the fine to the coarse network, carefully considering the width of flow toward any of the nodes. Only one of each of the nodes needing a different equation has been made black in Figure 10. It suffices to consider only these nodes as the equations in similar nodes will be the same and are obtained by copying. The relationships you get are given in Figure 10.

If the inner part of a network is to be refined in this way, there will be too few free cells within the coarser outer part. This can be solved by placing this inner part elsewhere on the spreadsheet and connecting it from there with the coarser part of the network. You can repeat this as many times as you want (as far as your computer memory will allow you) in order to achieve a really impressive refinement. For the problem in Figure 11, I worked this way. You can, however, also start the other way around, with the finest part of the network, and work outwards. Doing this you will be able to get an impressive model area in a very condensed form onto your spreadsheet. Except for the results themselves, Figure 11 shows part of the network used. Figure 11 has some other specialties; the fine inner area consists of two aquifers connected to each other via a semipermeable layer. These two aquifers are combined to just one total aquifer at the boundary of the fine inner part of the network. Within the inner part of the network the top layer contains truly phreatic water as described; moreover, the withdrawals (from the lower aquifer only) made the phreatic aquifer fall dry within the dashed line (head = −11.25 m). Furthermore, the lower aquifer contains a retaining wall around which the ground water has to flow toward the wells. This wall is made up from the special equations given in Figures 1 and 4. The relationships for this wall have been chosen so that it does not have a thickness equal to the square size of the local network, but zero thickness instead. Figure 1E shows how such relations can be derived in the usual way. In fact, by adjusting the given equations appropriately, you are able to have an impermeable or a semipermeable wall run between the nodes of your network in an arbitrary way, without the need to move meshpoints to the wall or vice versa, and still obtain an exact representation of the wall itself.

Summary and Conclusions

This paper shows some of the possibilities the electronic worksheet may offer to the engineer. Electronic spreadsheets form the precursors of a new way of working and a new generation of software, not of specialists' software capable of doing only that specific job it was designed to do, but software that can be used for a large variety of problems, and that excels because of its user-friendliness (i.e., behaves as you expect it to intuitively), and for which computer knowledge is completely superfluous. The creator of this software could never have foreseen all the fields his product will be used for. Its application is only limited by the fantasy of the user.

A modest show of the possibilities has been given with various ground-water problems. It was easy to obtain a simple finite-difference model, consisting of several hundreds of nodes, from scratch, within just a couple of minutes and have it run. In any case, with the means given, any hydrologist should be able to solve a broad variety of problems without a lengthy study to do it analytically (only possible in a limited number of cases, anyway), and without depending on some mystic ground-water models he might have been forced to use until now. Notwithstanding the fact that a good finite-element or finite-difference program stays useful for extensive problems with a lot of heterogeneity, the method presented here will undoubtedly achieve its own place in daily engineering practice. A good feeling of the water balance is again becoming many times more important than computer knowledge or programing ability . . . and so it should be.

Acknowledgment

This article is a personal initiative throughout.

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