Review the papers I turned back

If I have written "let's discuss" or "let's talk" have a look at the issue and be sure to find time to talk with me about it today.

Remember: If you want to improve your score. Revise the assignment and turn in my mark up and the revision by next class February 2

DUE TODAY Assignment #2 a) Finite Difference Calculation & b) Grid

a) Calculation: For the problem using 5 finite difference grid blocks numbered 1 to 5 left to right and with the following parameters:
confined flow ; y = 3 ft ; b = 3 ft ; K = 0.02 ft/day ; T= 0.06 ft2/day ; s = 0.00033 ft-1 ; S = 0.001
initially, h1=h2=h3=h4=h5=8.2ft ; constant head left h1=8.2ft ; for t>0 constant head h5=3.6ft
as posed on: http://inside.mines.edu/~epoeter/583/06/exercise/finite_diff_exer.htm
using the explicit approach on http://inside.mines.edu/~epoeter/583/06/exercise/explicit_exer.htm
and the implicit approach on http://inside.mines.edu/~epoeter/583/06/exercise/implicit_exer.htm
For both approaches:

1. Calculate h4 @ 0.07 day increments to 0.7 days using the implicit approach
2. Repeat #1 @ 0.14 day increments to 0.7 days using the implicit approach
3. Compare the mass balance at each step for the 0.07 and the 0.14 day time steps.
4. Graph your results as head vs. time
5. Compare your result to those from the explicit method **YOU MAY USE A SPREAD SHEET IF YOU PROVIDE A HAND CALCULATION AS A CHECK FOR EACH TYPE OF CALCULATION**

b) Grid: Layout the finite difference grid over a map of the model area for your problem and in a vertical cross-section. Discuss why you chose to grid the problem as shown. Keep your grid small (e.g. less than 20rows x 20col would be best, but absolutely no larger than 40x40) in order to make the project manageable such that you optimize your learning about modeling. The goal is for you to learn about modeling, not to produce a detailed model of your system. Use at least 2 layers so you can become familiar with issues related to multiple layers. Even if you are only simulating one geologic unit you can break it into an upper and lower portion which will give you a bit of information about vertical gradients in the system. Label the diagrams to describe the initial properties and boundary conditions you will use. These will be adjusted later in the calibration process. Your submission should include:
Drawing of plan view of each layer with properties & boundary conditions labeled
Drawing of cross section view of grid with properties & boundary conditions labeled
submit a description and the drawings as hard copy OR as ASSGN2_LASTNAME.ZIP

DUE TODAY

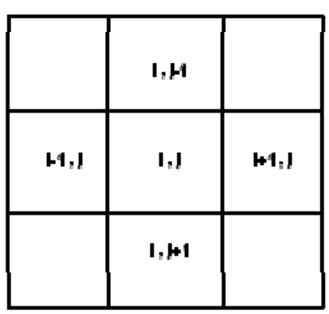
Assignment #3 Analytical Model: Choose an analytical model to represent some aspect of your modeling project and implement it with your model conditions. Describe the problem set-up and solution in a concise and clear manner. If you use a spreadsheet, mathcad, or other code for calculation, provide at least one hand calculation to confirm that your results are correct. Your submission should use illustrations to describe the conceptual model and how it fits your problem. It should include the following items:

Title
Objective
Problem Description
Analytical Model Description
Simplification of System in order to use the analytical model
Parameter values used
Calculations
Results
References

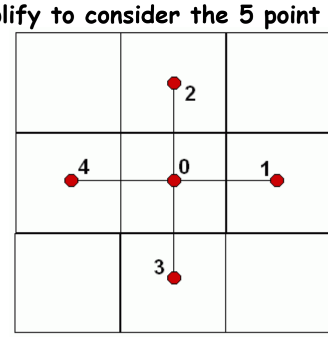
submit the write-up as hard copy and if you have electronic files include it in your zip file labeled: ASSGN3_LASTNAME.ZIP

Simple steady state finite differencing by spreadsheet:

Consider the general steady state case at a node completely surrounded by active nodes:



Simplify to consider the 5 point star:



By continuity flow from node 1 to 0 plus the flow from node 2 to 0 plus the flow from node 3 to 0 plus the flow from node 4 to 0 must equal the net flux at the node:

$$q_1 + q_2 + q_3 + q_4 = Q$$

The net flux may be zero, or it might be equal to a pumping/injection rate, or to a head dependent flux such as flow to a surface water body, or to a recharge rate times the area of the cell, or some combination of these. Let Qwithdrawal represent any pumping or other flux, w be the recharge rate and cells be square with L the cell size, and we can express Q as:

$$Q = Q_{withdrawal} + wL^2$$

By Darcy's law with hydraulic conductivity K and thickness b:

$$q_i = K_i b_i L \frac{(h_o - h_i)}{L}$$

For square cells of constant size:

$$q_i = K_i b_i (h_o - h_i)$$

Given that T=Kb, then:

$$q_i = T_i (h_o - h_i)$$

Substituting for q and Q, then rearranging and simplifying, we arrive at the expression we can use in a spread sheet for 4 surrounding nodes:

Always h_o minus adjacent h produces consistent signs

$$T_1(h_o - h_1) + T_2(h_o - h_2) + T_3(h_o - h_3) + T_4(h_o - h_4) = Q$$

expanding

$$T_1 h_o - T_1 h_1 + T_2 h_o - T_2 h_2 + T_3 h_o - T_3 h_3 + T_4 h_o - T_4 h_4 = Q$$

rearrange order so the same heads are adjacent

$$T_1 h_o + T_2 h_o + T_3 h_o + T_4 h_o - T_1 h_1 - T_2 h_2 - T_3 h_3 - T_4 h_4 = Q$$

simplify

$$h_o(T_1 + T_2 + T_3 + T_4) - T_1 h_1 - T_2 h_2 - T_3 h_3 - T_4 h_4 = Q$$

terms with h_o on left side

$$h_o(T_1 + T_2 + T_3 + T_4) = T_1 h_1 + T_2 h_2 + T_3 h_3 + T_4 h_4 + Q$$

solve for h_o

$$h_o = \frac{T_1 h_1 + T_2 h_2 + T_3 h_3 + T_4 h_4 + Q}{(T_1 + T_2 + T_3 + T_4)}$$

expand definition of Q as presented earlier

$$h_o = \frac{T_1 h_1 + T_2 h_2 + T_3 h_3 + T_4 h_4 + Q_{withdrawal} + wL^2}{(T_1 + T_2 + T_3 + T_4)}$$

Notice the T values are for the cell faces so they are the harmonic mean T for adjacent cells. Given the constant cell size, this simplifies to:

$$T_i = \frac{2}{\frac{1}{K_0 b_0} + \frac{1}{K_i b_i}}$$

For cells near no flow boundaries, the portion of the equation representing flow across that boundary must be eliminated. For example, if the face between cell 0 and the top of the page (cell 2) is a no-flow boundary then the terms related to flow from node 2 will be omitted:

$$h_o = \frac{T_1 h_1 + T_3 h_3 + T_4 h_4 + Q_{withdrawal} + wL^2}{(T_1 + T_3 + T_4)}$$

Let's look at an examples:

Links from today's date on class web page

1 Basic example spreadsheet

2 Heterogeneous spreadsheet

3 No-flow boundary on any side of cells

4 Head dependent flux boundaries (see next few slides)

Note the solution requires an iterative calculation so you must allow for circular references in Excel. Currently this it done via:
Tools > Options > Calculation
Check iteration, then
adjust maximum# & tolerance to get a precise answer

Items to consider:

1 Basic example spreadsheet

Does changing K change the heads? Why, why not?
Does adding Recharge change the heads? Why, why not?
Once recharge has been added then does changing K change heads? Flows? Why, why not?
Can heads within the grid ever be lower than the constant head boundary values? Why, why not?
How does the model change if K and Recharge are changed by the same factor? Why is this?

2 Heterogeneous Sheet

Without adding any fluxes, can we get a head above the highest boundary head, or below the lowest boundary head? Why, why not?
What changes when a new K field is applied? Why?

More items to consider:

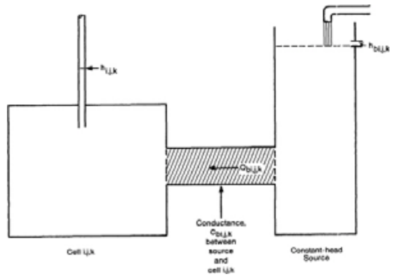
3 No-flow boundary on any side of cells

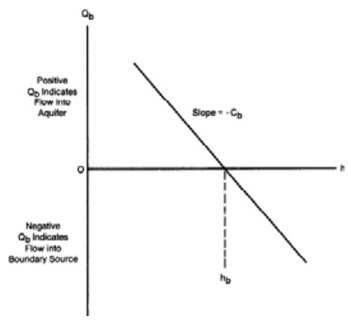
Look closely to see how the boundaries work.
Let me know if you find an error
How might you test that the equations are correct?
How might you be able to use this sheet to make creating your sheet easier?

4 Head dependent flux boundaries

How is this implemented?
How will it change for other situations like those shown in next few slides?
How might you be able to use this sheet to make creating your sheet easier? What will you need to look out for?

Head-dependent Flux: General Head Boundary Q + or -





Conductance (is all of Darcy's Law except the head difference)

Q = KA dh/dl

Conductance = KA/thickness
Q = Conductance dh

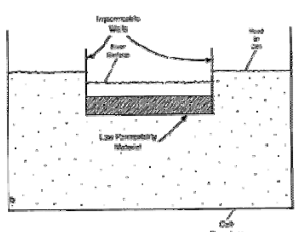
Conductance of the ghb is calculated as:
K * Area / thickness

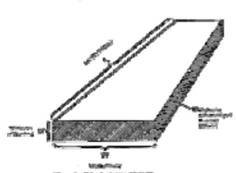
This is implemented in the HDF sheet

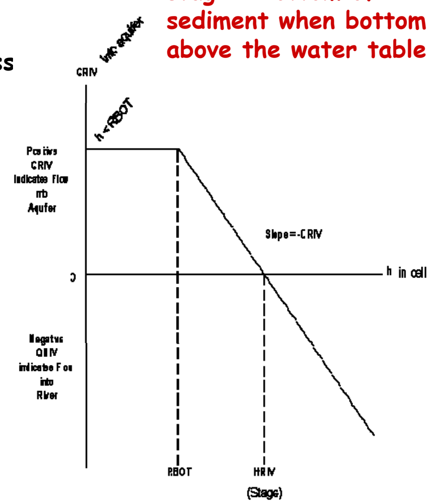
Head-dependent Flux RIVER

Using River Stage and River Bed Conductance
Q = KA dh/dl
Conductance = CRIV = KA/thickness
Q = Conductance dh
Conductance = Kv * L * W / thickness

dh is limited to stage – bottom of sediment when bottom is above the water table

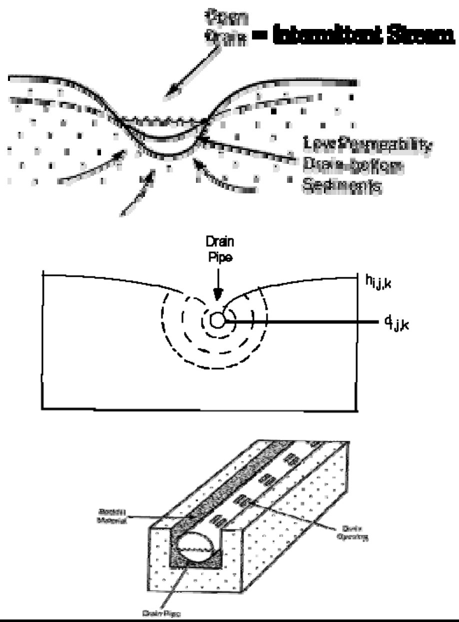






What would you need to add to implement this?

Head-dependent Flux DRAIN (outflow only)



Q = KA dh/dl

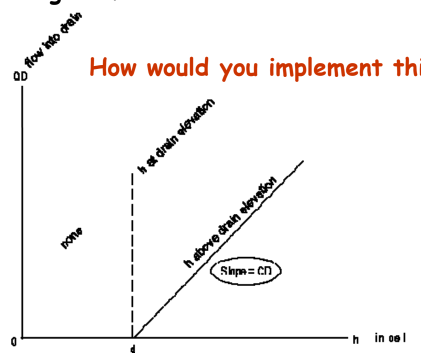
Q = Conductance dh

Conductance of the drain is calculated as:

$K_{\text{of material over which gradient is calculated}}$ * Area/thickness

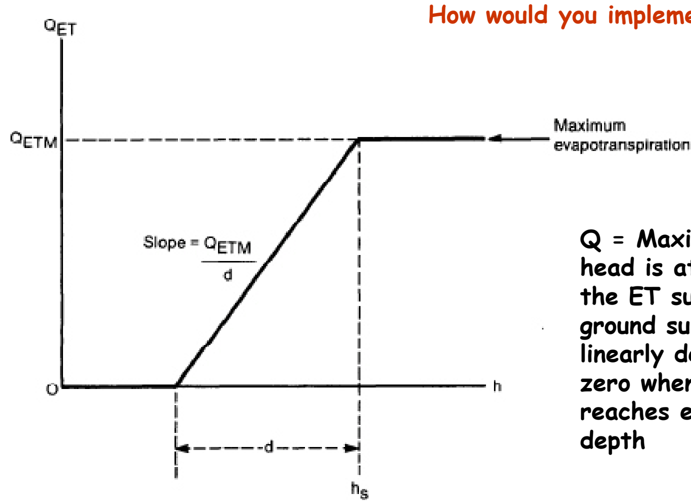
Area may be the cylindrical area midway between where the heads used for the gradient are located* length of the drain

How would you implement this?



Head-dependent Flux: ET only outflow

How would you implement this?



Q = Maximum when head is at or above the ET surface (usually ground surface) and linearly declines to zero when head reaches extinction depth

Recall the solution requires an iterative calculation so you must allow for circular references in Excel. Currently this it done via:
Tools > Options > Calculation
Check iteration, adjust maximum# & tolerance to get a precise answer

Setting up the sheet requires careful input.

Start with all parameter and boundary arrays full of values and a number for head in every cell of your FD array.

Save under a new name often.

If after pasting an equation you obtain a divide by zero or reference error, note what caused the trouble, go back to the previous working copy and proceed from there using the correct approach you figured out by noting where the error occurred.

If you wish to take advantage of cut and paste from my sheet you will need to limit the size of your grid to mine and put everything in the same grid cells so when my equations reference J3 you will get your cell dimension.

ONCE YOUR EQUATIONS ARE SET

Calculate inflows & outflows by Darcy's law for every active face of the model
Always use the constant head cell first for the head difference so a negative flow indicates flow out of the groundwater system

Do not calculate flow between constant head cells SUCH FLOW is irrelvant

Sum up Q from wells, W*Area for active recharge cells, & HDF flows or any other flux you have added

Do not sum recharge on noflow or constant head cells SUCH FLOW is irrelvant

Calculate the MASS BALANCE (%error) as:
(sum of all flows) / (sum of absolute value of all flows)/2 * 100

Compare heads, inflows and outflows for your unstressed and stressed situation

Let's talk about how we could add the following to a spreadsheet:

multiple layers
unconfined flow
transient flow

For more on spreadsheet finite difference models, see: Olsthoorn, 1985, The power of the electronic spreadsheet: Modeling without special programs, Ground Water, Vol 23, no. 3.

inside.mines.edu/~epoeter/583/06/spreadsheet/OlsthoornSpreadsheetModeling1985.pdf

And a response to comment:

inside.mines.edu/~epoeter/583/06/spreadsheet/OlsthoornSpreadsheetModeling1985ResponseToComment.pdf

DUE NEXT WEEK

Assignment #4 Finite Differencing by spreadsheet: Create a simplified 2D steady finite difference spreadsheet model of your problem, explain what it does.

Your submission should include:
Title
Objective
Problem Description
Spreadsheet setup Description
Simplification of System in order to use the spreadsheet model
Be sure to include at least one head-dependent flux boundary
Explanation of spreadsheet calculations
Explanation of Results (if appropriate comparison to analytical solution)

Submit the write-up as hard copy and include it in your zip file with the spreadsheet label the zip file: ASSGN4_LASTNAME.ZIP

Grading considers degree of difficulty as well as correctness