Family of stars
Reminder: the stellar magnitude scale

- In the 1900’s, the magnitude scale was defined as follows: “a difference of 5 in magnitude corresponds to a change of a factor 100 in brightness”.

\[ \Delta m = m_2 - m_1 = 5 \Rightarrow \text{flux ratio: } \frac{I_1}{I_2} = 100 \] (remember the Hipparchus convention!)

\[ \frac{I_1}{I_2} = \alpha^{(m_2-m_1)} = 100 \]

with \( m_2 - m_1 = 5 \).

from which we can deduce \( \alpha \approx 2.51 \)

- Using \( \log_{10} \) (exact formula):

\[ m_1 - m_2 = -2.5 \log_{10} \left( \frac{I_1}{I_2} \right) \]
Intrinsic luminosity

- **Apparent magnitude** tells us how bright stars appear to be at Earth. Of more interest is their **intrinsic luminosity** - to calculate this we need to know the star’s distance.

- **Luminosity** \( L \) = total radiation emitted by star in all directions (in [W])

- At a distance \( r \) (e.g. distance to Earth), that radiation flows through the surface of an imaginary sphere, radius \( r \), surface area \( 4\pi r^2 \)

- Hence the intensity at Earth, or brightness in [W.m\(^{-2}\)]:
  \[
  I = \frac{L}{4\pi r^2}
  \]
• Absolute magnitude ($M$ or $M_v$) is defined as the apparent magnitude a star would have if it were at a standard distance of 10 parsecs [pc] (1 pc = 3.26 ly).

• With this standard (but arbitrary) definition, one can use the absolute magnitude as a measurement of the luminosity $L$. It is an intrinsic property of the star, unrelated to its position relative to Earth.

• One can show that: $m - M = -5 + 5\log_{10}(d)$
  where $d$ is the distance to the star in unit of pc.
  and $(m-M)$ is called the distance modulus
Betelgeuse and Rigel have an apparent magnitude of $m_v=0.45$ (average) and $m_v=0.12$ respectively, and are 152 pc and 244 pc away from Earth respectively.

1. How much (intrinsically) brighter is Rigel compared to Betelgeuse?

2. What is the absolute magnitude $M$ of Betelgeuse and Rigel?

3. Does the formula given from the distance modulus check out?
Classification of stars (I)

- Blackbody radiation is a universal way of looking at stars, hence this is used to classify stars in categories. Stars with similar temperature have similar composition on the surface.
A word of caution

This is the Sun full absorption spectrum in the visible
- Absorption lines for H, He, but also C, O, Ne, Fe and many more elements
- The raw analysis of this spectrum lead astronomers to believe that the Sun’s surface composition was very similar to what is observed on Earth.
- In 1925, **Cecilia Payne** suggests in her PhD that the intensity of the lines is not simply proportional to the abundance of those elements.
- The surface temperature also defines which lines are strongly (or weakly) expressed. Once taken into account, Payne found out that the Sun’s surface composition is actually dominated by H and He.
Classification of stars (II)

- Spectral class names are historical
- Each class has 10 subclasses for finer classification, e.g. F0 – F9 as temperature increases

Useful mnemonic (pick your favorite):
- **Oh Be A Fine Girl (Guy), Kiss Me!**
- **Oh Boy, A F Grade Kills Me!**
- **Only Bad Astronomer Forget Generally Known Mnemonics!**
H is present in all stars - 75% of mass of typical star, but presence of H spectral lines depends on T, temperature

- **If T is much hotter than 10,000 K:** high speed collisions between atoms ionize hydrogen. So there are no atomic electrons left. So we see no H absorption lines

- **If T is much cooler than 10,000 K:** collisions are not energetic enough to excite H atoms. The Balmer absorption requires the atom to be in an excited state initially. So again, no H lines

- **Strongest H spectrum** result from stars with surface temps of about 10,000 K
Almost all hydrogen atoms in the ground state (n=1), hence few transitions from n=2 to n=1 (weak Balmer lines).

Note: \( \langle E(10,000\text{K}) \rangle \approx kT \) is only 0.8 eV!
Probability of being in the n=2 state

- From statistical physics, the ratio of the number of hydrogen atoms in the n=2 level and in the n=1 (ground state) level is given by:

\[
\frac{N_2}{N_1} = \frac{g_2 e^{-\frac{E_2}{kT}}}{g_1 e^{-\frac{E_1}{kT}}} = \frac{g_2}{g_1} e^{-\frac{(E_2-E_1)}{kT}}
\]

where \(g_i\) are the orbital degeneracies (\(g_1 = 2\) and \(g_2 = 8\)) and \(E_1\) is the ground state of the H atom (-13.6eV) and \(E_2\) is the first excited state (-3.4eV).

The higher the temperature, the more H atoms in n=2 state.
So why a peak around 10,000K?
Ionization state (HI: neutron, HII: ionized)

- According to the previous slide: the higher the temperature, the more H atoms in n=2 state.
- But that’s not all, the higher the temperature, the higher the probability of the atoms to be ionized

**Saha equation** (for HI and HII):
\[
\frac{N_{\text{ionized}}}{N_{\text{neutral}}} = \frac{kT}{P_e} \left(\frac{2\pi m_e kT}{\hbar^2}\right)^{3/2} e^{-\chi_i/kT}
\]

- \(m_e\): electron mass
- \(\chi_i\): ionization energy (13.6 eV)
- \(P_e\): free electron pressure

- \(T=8300K, 5\%\) of the atoms have become ionized
- \(T=9600K, 50\%\)
- \(T=11300K, 95\%\)
The Balmer thermometer

- Combination of the two effects:

\[
\frac{N(n=2)}{N_{total}} = \frac{N(n=2)}{N(n=1) + N(n=2)} \frac{N(n=1) + N(n=2)}{N_{neutral} + N_{ionized}} \approx \frac{N(n=2)}{N(n=1) + N(n=2)} \frac{N_{neutral}}{N_{neutral} + N_{ionized}}
\]

- **Note:** only a small fraction of H participate to the Balmer absorption line. As Cecilia Payne found, the strength of the spectral line is not an indication of composition on the star surface.
Stellar spectra

- From the relative strength of absorption lines (carefully accounting for their temperature dependence), one can infer the composition of stars.

- One cannot only rely on H spectral lines.
Another application of blackbody: determination of stellar radii

- A star is too distant to measure its diameter directly using angular size and distance. However, it is possible to measure the radius of a star if one knows its color and intrinsic brightness (luminosity) assuming the star behaves like a perfect blackbody.

- Recall Stefan-Bolzmann law: \( E = \sigma T^4 \) and the fact that the (blackbody) radiation is emitted from the surface of the star (hence \( E \) is in [W.m\(^{-2}\)]), we can deduce the luminosity: \( L = (4\pi R^2) \times (\sigma T^4) \) in [W].

- From the color of the star (wavelength \( \lambda_{\text{max}} \) of maximum emission), we can get its temperature using Wien’s law: \( \lambda_{\text{max}} T = 2.898 \times 10^{-3} \) in [m.K].

- We get: \[ R = \sqrt{\frac{L}{4\pi} \times \frac{1}{\sigma T^4}} \]

**Note:** there is an additional step! Absolute magnitude needs to be converted to [W] (using the Sun as reference for example) – see exercise on the next slide.
A good summary exercise

- What is the (approximate) radius of Sirius A (in [km])?
  
  Sirius A data:
  - apparent magnitude: \( m_v = -1.47 \)
  - parallax: 0.377"
  - spectral type: A1

  Earth / Sun data:
  - Sun absolute magnitude \( M_\odot = 4.83 \)
  - solar constant: 1367.7 W/m\(^2\)
  - distance Earth – Sun: 1 AU

But... but...

why?
The Hertzsprung-Russell diagram

- 1905: Ejnar Hertzsprung (Denmark) discovered regular pattern when he plotted the absolute magnitude of stars against their color.

- 1915: H.N. Russell (USA) independently discovered the same pattern when he used spectral types (OBAFGKM) instead of color.

- The Hertzsprung-Russell (or HR) diagram can therefore be described as (absolute magnitude OR luminosity) VS (spectral class OR color index OR temperature).
Stellar evolution in the H-R diagram

- The HR diagram shows striking clumping of stars into groups. Those groups are related to \textit{stellar evolution}, but H\&R were unaware of this at the time.

- \textbf{Main Sequence (MS):} diagonal band across diagram from hot, bluish, bright stars (top left) to cool, dim, reddish stars (bottom right)
  - Our Sun is a typical MS star: spectral class G2, absolute magnitude +4.83

- Outside of MS: stages in the stellar evolution of the MS stars.
• **Red Giants** are both cool and bright.

• Red? Related to surface temp. ✔

• Giant? Recall Stefan-Boltzmann law: \( R(T) = \sigma T^4 \). A cool object radiate much less energy per unit of surface area than a hot object. For the luminosity to be high, the object needs to be HUGE \( \Rightarrow E(T) = (\sigma T^4)(4\pi R^2) \) ✔

• Red Giants have diameters 10-100 times larger than the Sun.
Radii of stars in the H-R diagram

**Super Giants:** rare, bigger and brighter than Red Giants (ex: Betelgeuse)

1000 $R_\odot$

100 $R_\odot$

10 $R_\odot$

1 $R_\odot$ (Sun radius)

0.1 $R_\odot$
• **White dwarfs** are hot and dim.

• Back to Stefan-Boltzmann law: the white dwarfs have to be **very small** (size of the Earth! – only observable with telescopes).
Classification of very cool stars

- **L** – Very cool, dark red (radiation mostly in infrared)
- **T** – Coolest, infrared

- The major differences between M stars and L/T stars are in the IR part of the spectrum.
Luminosity effects on the width of the spectral lines

Lower gravity near the surfaces of giants

⇒ Smaller pressure

⇒ Smaller effect of pressure broadening

⇒ Narrower lines
• **Which class of star is most numerous?** Astronomers have surveyed the volume of space within a distance of 62 pc from the Sun - this is a sphere of volume $10^6$ pc$^3$!

• Dim, red main sequence stars called **red dwarfs** are the most common stars.

• White and red dwarfs are very common, but **not a single one is visible to the unaided eye**.
What you see vs what is nearby

What you see when you look up

What is nearby (and that you cannot see!)

The brightest stars in the sky tend to be highly luminous stars — upper-main-sequence stars, giants, or supergiants. They look bright because they are luminous, not because they are nearby.

The nearest stars in space tend to be very faint stars — lower-main-sequence red dwarfs or white dwarfs. Nearly all of these stars are faint in the sky even though they are nearby. Only a few are visible to the unaided eye.