Neutron stars & black holes
A little bit of history

- 1932: Neutrons are discovered in the lab – Chadwick
- 1933: Feasibility of neutron stars suggested by Zwicky – incompressible mass of tightly packed neutrons, at the same density as an atom’s nucleus.
- Zwicky suggested there were two types of stellar corpse - white dwarfs and neutron stars
- The theory is not accepted for some time - too weird!
Let’s consider the original liquid drop model developed for nuclei:

\[ B \left( \frac{AX}{Z} \right) = a_v A - a_s A^{2/3} - a_c Z (Z - 1) A^{-1/3} - a_{sym} (A - 2Z)^2 A^{-1} + \text{pairing terms} \]

with \( a_v = 15.85 \text{MeV} \), \( a_s = 18.34 \text{MeV} \), \( a_c = 0.71 \text{MeV} \), \( a_{sym} = 23.21 \text{MeV} \). We want to extrapolate this model to a supernucleus made almost exclusively of neutrons. For this, we need first to add a gravitational binding energy term to the liquid drop model. This term is written as follows:

\[ B_{grav} \left( \frac{AX}{Z} \right) = \frac{3}{5} \frac{G}{r_0^2} M^2 A^{-1/3} \]

with \( G = 6.7 \times 10^{-11} \text{J.m.kg}^{-2} \), \( r_0 = 1.2 \times 10^{-15} \text{m} \) and \( M = A M_n \) with \( M_n = 1.67 \times 10^{-27} \text{kg} \) (\( M_n \): mass of the neutron).
Exercise

FROM A NUCLEUS TO A NEUTRON STAR!

1. Should the sign of this contribution be positive or negative? Justify.

2. Which terms of the new model can be neglected when considering a neutron star? Justify. [Hint: be blunt, but careful!].

3. A system is bound if its binding energy is positive. Deduce an equation that describes the neutron star at the limit of stability and find the minimum number of neutrons needed for a stable neutron star.

4. Deduce the radius (in km) and mass (in solar mass, $1M_\odot \approx 2 \times 10^{30}$ kg) of the corresponding neutron star.

Weird? Yes. But nuclear physics is on your side!
A supernova explosion of a $M > 8 M_\odot$ star blows away its outer layers.

The central core will collapse into a compact object of ~ a few $M_\odot$.

Pressure becomes so high that electrons and protons combine to form stable neutrons throughout the object.

Typical neutron star
- Size: $R \sim 10$ km
- Mass: $M \sim 1.4$ to $3 M_\odot$
- Density: $\rho \sim 10^{14}$ g/cm$^3$

A piece of neutron star of the size of a sugar cube has a mass of ~100 million tons!!!

Escape velocity? ~ $\frac{1}{2}$ of the speed of light!
The neutron star is supported by the neutron degeneracy pressure, and lead to a similar relationship between mass and volume:

\[ M_{ns}V_{ns} = \text{constant} \]

One can show that:

\[
R_{ns} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M_{ns}^{1/3}} \left( \frac{1}{m_H} \right)^{8/3}
\]

For \( M_{ns} = 1.4M_\odot \), we find \( R_{ns} = 4.4\text{km} \), which is about a factor 3 too small (recall that we observed a similar discrepancy for white dwarf).

And, similarly to the white dwarf model, when the velocity of the confined neutrons approaches the speed of light, the neutron degeneracy pressure becomes smaller than predicted leading to a zero-volume object for a finite mass \( \rightarrow \text{Black hole} \).
Exercise

• Assuming (incorrectly) that one can apply Newtonian mechanics at the surface of a neutron star, calculate the escape velocity for a $1.4M_{\odot}$ neutron star with a radius of about 10km.

• Another way to see that we cannot use Newtonian mechanics to describe what is happening at the neutron star’s surface is to calculate the ratio of the Newtonian gravitational potential energy to the rest energy of an object of mass $m$ at the star’s surface.
The neutron star merger of Aug 17, 2017
Recall: gravitational waves – observed for the first time in 2015

- Release of gravitational energy inducing a deformation of space-time (first predicted by Einstein, who didn’t believe they could ever be detected)

From: PRL 116, 061102 (2016)
August 17, 2017 – Date of observation

- 12:41:04 UTC – Gravitational wave signals detected [GW170817]

- + 1.7s – Short gamma-ray burst detected within the same region of the sky [GRB170817A]

- + 10h52m – New bright source of optical light in galaxy NGC4993 (Hydra constellation) [AT2017gfo – optical transient]

- + 11h36m – Infrared emission observed

- + 15h – Bright ultraviolet emission detected

- + 9d – X-ray emission detected

- +16d – Radio emission detected
GW170817 signal

- Signal in VIRGO first, then 22 ms later in LIGO – Livingstone (LA) and 3 ms later in LIGO – Hanford (WA)
- Signal weak in VIRGO, but enough to provide triangulation information.
- Binary neutron star inspiral observed for about 100 s [longest GW observation]
- Total mass of the system: 2.73 to 3.29 $M_\odot$. Component masses between 0.86 and 2.26 $M_\odot$.

From: *PRL 119 (2017) 161101*
Localization of GW170817

GW150914
LIGO detection only

GW170817

• Triangulation using the timing, amplitude and phase of the signals detected by the three observatories.

From: PRL 116 (2016) 241102

Credit: LIGO/Virgo/NASA/Leo Singer (Milky Way image: Axel Mellinger)
Precise localization of the event


NGC4993 – 40 Mpc (130 Mly)
Timing of GW170817 / GRB170817A

Fermi

Gamma rays, 50 to 300 keV

Counts per second

GRB 170817A

LIGO

Gravitational-wave strain

GW170817

Frequency (Hz)

Time from merger (seconds)
GW-GRB timing correlation $\Rightarrow$ at least some short GRBs come from neutron star mergers!

- Gravitational waves arrived first, then $\gamma$-rays after 1.7s. Gravitational waves travel at the speed of light

- Why a 1.7s delay for $\gamma$-ray photons? Characteristic time to form the source? Emission process mechanism? Time for $\gamma$-rays to break out from the source?...

- Light curve properties of the GRB

- Lots of (astro)physics to be learned about neutron stars, neutron star merger mechanism, GRB production…

Gamma-ray bursts (GRBs)

Short (~ a few s), very bright bursts of gamma-rays from very distant locations

- Later discovered with X-ray and optical afterglows lasting several hours to a few days.
- Many have now been associated with host galaxies at large (cosmological) distances.
A candidate for Gamma-ray bursts (GRBs)

- Currently favored model for gamma-ray bursts: **HYPERNOVA**!

- Probably related to the supernova-like deaths of very massive (> 25 M$_\odot$) stars

- The iron core collapses to form a black hole instead of a neutron star (as in a “normal” supernova)

- Considerably larger release of energy because a large fraction of the star mass is converted into energy
GRB170817A – what did we see?

- Dim sGRB (dimmer by 2 to 6 orders of magnitude)
- Low energy $\gamma$-rays, but **NO** high energy $\gamma$-rays (HESS / HAWC...)

We were probably not aligned with the jets

![Relativistic jets](image-url)
GRB170817A – what did we see?

Three possible scenarii

- By the way, we don’t know what was formed after the merger (Black hole, neutron star...)

Death by Gamma Rays!

Yet another way earth could be annihilated
A neutron star merger is predicted to produce a transient optical-near-infrared source called macronova or kilonova. The kilonova is believed to be powered by the nucleosynthesis of heavy elements.

Optical transient observation (the “afterglow”) following GW170817 and GRB170817A gives by far the best precision on the location of the event.

Observed light spectrum in part driven by the existence of absorption lines of the heavy elements produced.
Heavy nuclei nucleosynthesis

- From the data collected, it is estimated that 0.03 to 0.05 $M_\odot$ of heavy elements were produced in the kilonova, including 3-13 Earth mass worth of Gold alone ($1M_\odot=333,000$ Earth mass).
Stellar nucleosynthesis

• Landmark “B²FH” paper from 1957

• Paper suggests (correctly) that the stars are responsible for the nucleosynthesis of most chemical elements beyond Boron (Z=5) or so.

Produced in stars

Production of heavier elements?

Measured abundance of the chemical elements on Earth
Explosive Nucleosynthesis

\( \tau(p/n) \ll \tau(\beta^{+/-}) \)

Rapid proton capture:

rp-process: Novae

Rapid neutron capture:

r-process: BNS merger, ...

Fred Sarazin (fsarazin@mines.edu)
Physics Department, Colorado School of Mines

PHGN324: Neutron stars and black holes
Nucleosynthesis in the r-process

JINA
Joint Institute for Nuclear Astrophysics 2002

Movie: H. Schatz, National Superconducting Cyclotron Laboratory
Calculation: K. Vaughan, J.L. Galache, and A. Aprahamian, University of Notre Dame
Model: B. Meyer, Clemson University and R. Surman, North Carolina State

Temperature: 1.50 GK
Time: 2.7e-14 s
Timeline (cont’d)

• + 9d – X-ray emission detected
• +16d – Radio emission detected

→ Delayed X-ray and radio afterglow emissions are related to the interactions with the Inter Stellar Medium (ISM) and to the environment around the binary system prior to the merger.
• 1967: Jocelyn Bell, a postdoctoral student at Cambridge, observed pulsed radio signals from a particular direction in the sky

• No obvious source. Radio pulses had very precise period (i.e. time between pulses) = 1.33730119 sec

• 1974: Jocelyn Bell’s boss Antony Hewish wins the Nobel Prize

Pulsars are evidence of the existence of neutron stars
Discovery of pulsars

[Graph showing pulsar CP 1919 with reference to interference and dates: 6 Aug 1967]
Discovery of pulsars

- Alien theory quickly discounted. Several other sources were soon found with similar pulsed signals.

- Bell found periods from 1/4 sec (fastest) to 1 1/2 sec (slowest) → PULSARS

- 1968: Discovery of very fast pulsar (period = 0.033 sec) in the center of the Crab Nebula - long known as a SN remnant (Chinese 1054) → Pulsars must be associated with exploding stars
• Normal stars spin (e.g. our Sun has a period of close to 1 month). Therefore, stars have angular momentum.

• As the core of the star becomes a neutron star, conservation of the angular momentum implies that it must spin a lot faster because it is a lot smaller.

• Rapidly pulsed (optical and radio) emission from some objects interpreted as spin period of neutron stars.
Exercise

- Suppose that the Sun ($T_\odot=26$ days, $R_\odot=6.96\times10^5$km) were to collapse down to the size of a neutron star ($R_N=10$km) with no mass loss. Find the rotation period of the neutron star. For simplicity, assume that both the Sun and the resulting neutron star can be modelled as homogeneous spheres.
The Crab pulsar

Remnant of a supernova observed in A.D. 1054

- Theory up until 1968 had said that all SN led to white dwarfs (neutron star theory not taken seriously)

- However, a white dwarf couldn’t pulse 30 times per second (like the Crab object). It cannot rotate or vibrate that fast, or it would fly apart.

- One needs a much smaller object → the neutron star theory is revived.
Light curves of the Crab pulsar

Variation in light intensity

- Main pulse
- Secondary pulse
- Pulsar blinks twice each cycle.

Variation in X-ray intensity

- Pulsar blinking at X-ray wavelengths

Time (milliseconds)
Like on Earth, rotation axis may not be aligned with magnetic field axis.

Rapid radiation + intense dipolar magnetic field makes a huge electric generator. Magnetic fields are amplified up to $B \sim 10^9-10^{15}$ G.

Intense electric field at pulsar surface accelerates protons and electrons to high speeds.

Note: electrons and protons exist freely on surface - not fused into neutrons.
The lighthouse model of pulsars

- Spinning star acts like a lighthouse. Earth may be in path of one or both beams

- Synchrotron process can produce
  - radio waves (what was observed by Bell)
  - visible light
  - UV light
  - X-rays

  But only for “young” pulsars like Crab or Vela (11,000 yr)

- Pulsars slow down with age, eventually only energetic enough to emit in the radio region
Images of pulsar and other neutron stars
• Pulsars blow off a constant stream (wind) of high-energy particles → pulsar winds
• Protons, electrons shoot out from magnetic pole regions, spiral around mag. field lines, and radiate energy - called the synchrotron radiation process
Some neutron stars are moving rapidly through interstellar space. This might be a result of anisotropies during the supernova explosion, forming the neutron star.
Some pulsars form binaries with other neutron stars (or black holes).

Radial velocities resulting from the orbital motion lengthen the pulsar period when the pulsar is moving away from Earth (red arrow)… and shorten the pulsar period when it is approaching Earth (blue arrow).

This is due to the modulation of the pulse arrival time arising from the radial velocity.
Hercules X-1 (1st X-ray source discovered in Hercules constellation) is a pulsar emitting X-rays with period of 1.24 secs.

BUT every 1.7 days the pulsar appears to turn off for a few hours. The reason is that it is actually eclipsed by a binary companion star.

Note: short orbital periods mean that stars are in close proximity. Since the companion star is usually a MS star, the neutron star (with strong gravitational pull) can capture gases escaping from its companion.
Compact objects with accretion disks

- Black holes and neutron stars can be part of a binary system.

- Matter gets pulled off from the companion star, forming an accretion disk, and heats up to a few million K.

→ Strong x-ray source!
Neutron stars in binary system – X-ray binaries

**X-RAY EMISSION**

1. The material from the companion star is funnelled towards the magnetic poles of neutron star

2. The gas is accelerated to \( \frac{1}{2} \) the speed of light and hits surface of neutron star, heating it to \( 10^8 \) K.

3. Heating leads to emission of X-rays with typical luminosities of \( 10^{31} \) Watts (10,000 times brighter than the Sun over all wavelengths!)

Accretion disk material heats to several million K → X-ray emission
Several bursting X-ray sources have been observed

Rapid outburst followed by gradual decay
- Some pulsars have planets orbiting around them.

- Just like in binary pulsars, this can be discovered through variations of the pulsar period.

- As the planets orbit around the pulsar, they cause it to wobble around, resulting in slight changes of the observed pulsar period.
Magnetars and soft gamma-ray repeaters

- Some neutron stars have extremely large magnetic fields (~1000 times stronger than a normal neutron star): Magnetars

- **Crust quakes** in magnetars might produce X-ray and gamma-ray flashes similar to gamma-ray bursts: Soft gamma-ray repeaters
Black holes

• Just like white dwarfs (recall the Chandrasekhar limit: $1.4 \, M_\odot$), there is a mass limit for neutron stars. The core cannot be supported by neutron degeneracy pressure.

• We know of no mechanism to halt the collapse of a compact object with $> 3 \, M_\odot$.

• The object will collapse into a single point (zero volume) of infinite density – a singularity: a black hole!
• Escape velocity - speed required to escape gravitational pull of an object. For Earth, escape velocity at its surface is 11 km s\(^{-1}\) (38,000 km h\(^{-1}\))

From classical mechanics: \(v_{\text{esc}} = \left(\frac{2GM}{R}\right)^{1/2}\)

• If object is massive enough and/or packed into a small enough volume, it’s possible for the escape velocity to exceed the speed of light (Rev. John Mitchell, 1783) – a black hole.
The Schwarzschild radius and the event horizon

- There is a limiting radius where the escape velocity reaches the speed of light $c$.

- This radius is known as the **Schwarzschild radius (1916)**:

$$R_s = \frac{2GM}{c^2}$$

- Nothing (not even light) can escape from inside the Schwarzschild radius and we have no way of finding out what is happening inside $R_s$ (or inside / beyond the so-called **event horizon**).

**Note: it is incorrect to think that you can derive $R_s$ from the classical escape velocity.**

<table>
<thead>
<tr>
<th>Object</th>
<th>Mass ($M_\odot$)</th>
<th>$R_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star</td>
<td>10</td>
<td>30 km</td>
</tr>
<tr>
<td>Star</td>
<td>3</td>
<td>9 km</td>
</tr>
<tr>
<td>Star</td>
<td>2</td>
<td>6 km</td>
</tr>
<tr>
<td>Sun</td>
<td>1</td>
<td>3 km</td>
</tr>
<tr>
<td>Earth</td>
<td>0.000003</td>
<td>0.9 cm</td>
</tr>
</tbody>
</table>
We assume that core collapse continues inside the event horizon producing region of infinite density and zero volume - called a singularity.

A very strange place – space and time jumbled, laws of physics apparently aren’t obeyed. However, we are shielded from these irrational events by the event horizon and their effects cannot escape past it.

In 1969 Penrose (Oxford) recognising that things are sensible outside the event horizon proposed his “Law of Cosmic Censorship”: “Thou shalt not have naked singularities”
The gravitational potential (and gravitational attraction force) at the Schwarzschild radius of a black hole becomes infinite.

However, at large distances, it is not different from the gravitational potential of a normal star.

If you replaced the Sun with a black hole of the same mass, the orbits of the planets would not change!
General relativity effects near black holes

- **Remember Spaghettification?** An astronaut descending down towards the event horizon of the black hole will be stretched vertically (tidal effects) and squeezed laterally.

- **Time dilation:** clocks started at noon. After 3 hours, this is what the clocks read depending on their position relative to the event horizon for an observer far away from the black hole.
General relativity effects near black holes

- Gravitational redshift: All wavelengths of emissions from near the event horizon are stretched (red shifted), i.e. the wavelength of the escaping light is “stretched” similarly to our astronaut in the previous slide.

\[ 1 + z(r) = \frac{1}{\sqrt{1 - \frac{R_s}{r}}} \]

where \( R_s \) is the Schwarzschild radius

\[ R_s = \frac{2GM}{c^2} \]

Recall that the redshift is defined as:

\[ z = \frac{\lambda - \lambda_0}{\lambda_0} \]

- Note that the gravitational redshift formula cannot be used for \( r \leq R_s \)!
Exercise

- At which wavelength would you observe X-ray photons ($E_0=5\text{keV}$) emitted 1km away from a black hole with a Schwarzschild radius of 1000km.

- What is the mass of the blackhole (in $M_\odot$)? [Data: $1M_\odot = 1.99\times10^{30} \text{kg}$]
Evidence for black holes - indirect

- No light can escape a black hole → no black holes can be observed directly
- If an invisible compact object is part of a binary, we can estimate its mass from the orbital period and radial velocity.
- If the mass of the invisible object is $> 3 \, M_\odot$, it is a black hole

![Table 11-1: Seven Black Hole Binaries](image)

<table>
<thead>
<tr>
<th>Object</th>
<th>Star</th>
<th>Orbital Period</th>
<th>Mass of Black Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cygnus X-1</td>
<td>B0I</td>
<td>5.6 days</td>
<td>$10 , M_\odot$</td>
</tr>
<tr>
<td>LMC X-3</td>
<td>B3V</td>
<td>1.7 days</td>
<td>$&gt;8 , M_\odot$</td>
</tr>
<tr>
<td>A0620-00</td>
<td>KV</td>
<td>7.75 hours</td>
<td>$11 \pm 1.9 , M_\odot$</td>
</tr>
<tr>
<td>V404 Cygni</td>
<td>G-KV</td>
<td>6.47 days</td>
<td>$12 \pm 3 , M_\odot$</td>
</tr>
<tr>
<td>GR0 J1655-40</td>
<td>F5V</td>
<td>2.61 days</td>
<td>$6.9 \pm 1 , M_\odot$</td>
</tr>
<tr>
<td>QZ Vul</td>
<td>KV</td>
<td>8 hours</td>
<td>$10 \pm 4 , M_\odot$</td>
</tr>
<tr>
<td>4U 1543-47</td>
<td>AV</td>
<td>1.123 days</td>
<td>2.7–7.5 $, M_\odot$</td>
</tr>
</tbody>
</table>

Compact object with $> 3 \, M_\odot$ must be a black hole!
If compact object is a neutron star, matter falling onto it, produces an X-ray outburst when it impacts on the surface.

This cannot happen if the compact object is a black hole. Matter would just fall through the horizon and disappear without a trace! Note: even if there is no nova, there can still be some X-ray emission from the matter being accelerated outside of the event horizon.
New indirect evidence – gravitational waves!

- Release of gravitational energy inducing a deformation of space-time (first predicted by Einstein)
Both signals are interpreted as mergers of a pair of black holes. Big surprise for many, neutron star mergers were expected first – maybe because of sensitivity of the interferometers?

These observations have absolutely confirmed the existence of black holes of “stellar” mass i.e. $\sim 30 M_\odot$. 
More interferometers = more constraints on arrival directions!
Currently operating: LIGO (Hanford, WA / Livingston, LA), VIRGO (Italy)
Future interferometer in Australia and in India?

FIG. 4. An orthographic projection of the PDF for the sky location of GW150914 showing contours of the 50% and 90% credible regions plotted over a colour-coded PDF. The sky localization forms part of an annulus, set by the time delay of $6.9^{+0.5}_{-0.4}$ ms between the Livingston and Hanford detectors.
More interferometers = more constraints on arrival directions!
Currently operating: LIGO (Hanford, WA / Livingston, LA), VIRGO (Italy)
Under construction: KAGRA (Japan)
Future LIGO interferometer: India