Using Stochastic Discounted Cash Flow and Real Option Monte Carlo Simulation to Analyse the Impacts of Contingent Taxes on Mining Projects

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ABSTRACT

The great variability of mineral and energy prices has motivated some governments to incorporate a windfall profits tax into their mineral taxation regimes. Windfall profits taxes are a means for governments to capture additional mineral revenue when mineral prices are considered higher than some historic level. Governments need to be aware of the effects of such contingent taxes on the exploration, development and operating decisions of mining companies. Mining companies need to be able to calculate the full impact of windfall profits taxes on the economics of their projects. This paper examines the valuation of a multi-phase copper-gold project in the presence of a windfall profits tax. Real options Monte Carlo simulation is used to characterise the different exposures of the mine owner and of the government to the risky cash flow streams that they receive from the project. The results highlight that Monte Carlo simulation paired with the real option valuation method is able to account appropriately for the differing risk exposures, while the traditional discounted cash flow valuation model is not. Our key conclusion for governments and mining companies is that the impact of contingent tax and royalty terms can and should be assessed using advanced valuation techniques, and the results used to improve contract designs under the constraint of actual and perceived fiscal system stability.

INTRODUCTION

The current environment of high mineral and energy prices has motivated many governments to review their mining tax laws and in some instances modify existing taxes and royalties or create new ones. These reviews have been conducted throughout the mining world and include such countries as Peru, Venezuela, Mongolia, South Africa and Canada. Their advertised objective is the legitimate desire that the host country receive fair compensation for the extraction of its resources as the domestic and global mining business environment changes through time. However, the impact of these tax changes on cash flow risk and value are often not well understood because the conventional analytical tools used to model them have important limitations. In particular, spreadsheet cash flow models relying on single-point forecasts may produce significant errors in asset value estimates. These valuation errors are especially large for sliding scale royalties or windfall taxes.

The primary limitation of cash flow models relying on single-point forecasts is their inability to compute and value non-linear cash flow pay-offs that accrue to government, smelters and refineries, creditors and equity. Non-linear cash flows have a curved or 'kinked' dependence on the uncertain variables (like prices and grades) that are the inputs into the cash flow model. A prevalent example of this is tax rates that depend on the mineral price, as in sliding scale royalties, or asset-level profits taxes with tax loss carry-forwards or profitability triggers. These contingent tax and royalty cash flows are best estimated using Monte Carlo simulation of the uncertain input variables. Monte Carlo simulation is one of several methods of random sampling. This type of approach is preferred because it is best able to account for high dimensional non-linearities that arise from path- dependence inherent in tax loss carry-forwards and profitability triggers. In this paper, we shall use the term 'Monte Carlo simulation' to refer generically to simulation using any sort of random sampling.

This paper uses Monte Carlo simulation to investigate the impact of a tax regime change on the project economics of a copper-gold project. An analysis of the recent Mongolian windfall tax illustrates how tax changes affect the uncertainty characteristics and value of both government and equity cash flow streams when both discounted cash flow (DCF) and real options (RO) methods are used to calculate net present value (NPV) for the government royalty, the corporate income tax, the windfall tax and the residual equity cash flow. A comparison is made between the DCF and RO risk adjustments applied to each cash flow stream to show how each method deals with the effects on value of the differences in uncertainty of the government and equity cash flows. To keep the analysis uncluttered we abstract from the impacts of the tax on the future management of the project, and simply evaluate whether the project is still economic given the imposition of the tax.

A review of mineral taxation theory

Mineral profits taxes and royalties impose a claim on revenues from extraction. In this sense they are similar to other variable costs of extraction such as wages and energy. They are different, however, in that typically the impact of the tax is explicitly contingent on the income flow. Windfall profits taxes, for example, only tax unusually high income scenarios. Even proportional income taxes are non-linear if there are incomplete tax loss offsets; that is, if, at low or negative taxable incomes, tax loss offsets do not contemporaneously match the full tax burden imposed in high taxable income scenarios. Even where tax loss offsets are complete, depletion allowances can make the effective marginal tax rate non-linear (MacKie-Mason, 1990). This non-linear contingency means that in uncertain environments, where taxable income is stochastic, one cannot correctly estimate the tax burden using one-point forecasts of the input variables and ordinary spreadsheet techniques due to a mathematical result called Jensen's inequality. Simulation techniques must be used, sampling over the entire range of possible taxable income outcomes.

Table 1 provides a simple example of this point. Assume that in year t of production there are three possible revenue scenarios, each with a one third chance of occurrence. Taxes are payable at a flat rate of 40 per cent. In the low case there are tax loss offsets of only US$2 million at time t. In the medium and high cases proportional taxes accrue at a rate of 40 per cent. A simple spreadsheet analysis, which calculates the tax based on the expected taxable income of US$1 million, would estimate a tax

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TABLE 1

A simple non-linear tax pay-off calculation.

Monte Carlo analysis				
Revenue scenario	**Probability**	**Taxable income under scenario (US$ M)**	**Taxes payable under scenario (US$ M)**	**Probability-weighted tax outcome (US$ M)**
Low	0.33	-8.0	-2.0	-0.667
Medium	0.33	1.0	0.4	0.133
High	0.33	10.0	4.0	1.333
Static spreadsheet analysis				
Revenue scenario	**Probability**	**Expected taxable income (US$ M)**	**Expected taxes payable (US$ M)**	**Probability-weighted tax outcome (US$ M)**
N/A	N/A	1.0	0.4	N/A

payment of US$0.400 million in year *t*. A correct analysis of the various outcomes reveals that the expected (probability-weighted) tax is US$0.800 million, twice as high as the estimate from the spreadsheet analysis.

The difference between the tax at the expected income level (US$0.400 million) and the expected tax given all possible income levels (US$0.800 million) arises from Jensen's inequality. This inequality states that an expected value of a strictly concave (convex) function calculated with the expected value of an uncertain variable *X* is greater than (less than) the expected value of a strictly concave (convex) function evaluated at a number of uncertain variable outcomes. This is no mere play on words. A conventional spreadsheet cash flow model calculates equity, government and creditor expected net cash flow as a function of expected (forecast) underlying variables such as gold price. This type of cash flow model samples the net cash flow function at only one point (the expectation of the uncertain variables) and so its estimation of expected net cash flow is not influenced by the curvature of the cash flow equation. A full stochastic analysis of the cash flows recognises the curvature of the net cash flow pay-off because it samples and calculates cash flow across a range of uncertain outcomes.

Another complication is the path dependence of tax flows. Because of depreciation and depletion schemes that depend on cumulative deductions to date, the path of cash flows leading up to any single period's tax analysis matters. For example, the taxes due in year *t* will depend not only on the mineral price in that year, but also on the path that mineral price has taken up to that point. High prior prices may have exhausted certain depreciation shields, yielding a high tax payable, while low prior prices may have left considerable unused depreciation shields, yielding low tax payable. Path-dependence can introduce additional complex non-linearities into the tax structure because the impacts of low price paths and high price paths on current taxes payable are not symmetric. Because of the large number of uncertain variables at play, simulation using random sampling is the best way to calculate the cash flow statistics needed to estimate tax flows.

In practice, path dependence and the non-linearity of effective tax rates has been recognised by academia (eg MacKie-Mason, 1990; Lund, 1992; Bradley, 1998; Blake and Roberts, 2006), but it is not usually taken into account by industry or governments. With taxes usually (but not always, eg Mackie-Mason, 1990) being a strictly convex function of a stochastic taxable income stream, one-point forecast spreadsheet analysis usually underestimates the level of the tax burden and overestimate the cash flows attributable to the equity and debt holders of the project, as was the case in the example in Table 1. Yet most mining companies do not use Monte Carlo analysis, or its equivalents, in asset valuations (Blanco and Zanibbi, 1992; Bhappu and Guzman, 1995), and therefore cannot be measuring tax flows correctly because they are ignoring the interacting effects of uncertainty and non-linearity. A recent World Bank study of the impacts of taxes and royalties on mining projects (Otto *et al*, 2006) likewise uses only expected revenues and costs when calculating tax and royalty flows, again ignoring the interacting effects of uncertainty and non-linearity.

A second impact of taxes is neither well recognised nor well measured. Taxes, because they are typically not proportional to the pre-tax cash flow, have a different risk profile from the overall asset and from the after-tax cash flow. If they have a different risk profile, they ought to be valued with a different risk discounting structure (Bradley, 1998). Some taxes are less risky than the asset as a whole, such as ad valorem royalties in situations where the revenues are more risky than the costs. Others are more risky, such as the typical income tax for a corporation in a tax paying position, especially in the presence of an ad valorem royalty that can be deducted for income tax purposes. Windfall profits taxes also tend to be more risky than the asset as a whole if the mined output is risky itself, for reasons we shall make clear below.

The difference between DCF and real options Monte Carlo NPV calculations

The mining industry for the most part uses the one-point forecast single-rate DCF method to estimate asset values. This method discounts the stream of asset net cash flows in the forecast realisation of the future at a constant rate. This constant discount rate is typically set to an estimate of an average discount rate appropriate for valuing the assets of the corporation as a whole. The estimate of asset value is the sum over time of these discounted cash flows.

The RO Monte Carlo method is an approach to valuation that has been introduced to the mining industry as an alternative to the DCF valuation technique. Monte Carlo simulation is used to correctly estimate the cash flow distribution at each future time over all possible realisations of the future taking any non-linearities involved into account. RO uses financial market information and models to determine risk adjustments for each possible realisation of the future. The cash flow in each realisation of the future is multiplied by the corresponding risk adjustment, and the expectation of this product is called the risk adjusted expected cash flow. These risk-adjusted expected cash flows are discounted for time using the term structure of risk-free discount factors. The estimate of asset value is the sum of the time-discounted risk-adjusted expected cash flows.

This difference in risk-adjustment between the DCF and RO valuation methods appears to be nuanced but its consequences are potentially large. This process allows senior management to use financial market information to determine the underlying structure of risk adjustments for the uncertain variables of interest to the corporation. The detailed asset cash-flow dependence on these underlying uncertain variables then determines how these underlying risk adjustments are transformed implicitly into risk discounts for the asset cash flow.

Risk discounting the asset cash flows in this way ground the valuation in the financial markets of relevance to investors. It also tunes the risk discounting in a controllable way to the types and amounts of risk actually in the asset cash flows, as opposed to using some average discounting that it is not likely to be appropriate for the risk involved. Moreover, the cash flows that are claimed by different participants in the project, including governments, are individually discounted for risk according to their possibly different risk profiles. All of this rids the valuation process of some biases that are inherent in the DCF method, which assumes that all assets and claims against those assets have the same structure of risk. In the mining industry, these biases include a general bias against investment to reduce future costs, a bias against mines and mine designs (other than those producing gold exclusively) with long-term production profiles, and, finally, a bias against assets that face taxes, like windfall profits taxes, that are geared toward taking disproportionately more revenue in realisations of the future characterised by high output prices.

A detailed discussion about the difference between RO and DCF risk adjustments and their value effects is found in Jacoby and Laughton (1992), Salahor (1998) and Samis *et al* (2006). Laughton (2007) provides a general review of the evolution of the RO method and the differences between DCF and RO NPV methods.

THE 'GALORE CREEK' PROJECT IN MONGOLIA

Safe harbour disclaimer

The economic analysis in this case study depends on inputs that are subject to a number of known and unknown risks, uncertainties and other factors that may cause actual results to differ materially from those presented here. Factors that could cause such differences include but are not limited to:

- *changes in world commodity markets,*
- *costs and supply of materials relevant to the mining industry,*
- *the extent of resources actually contained in the mineral deposit,*
- *actual recoveries achieved in processing ore,*
- *technological change, and*
- *change in government and changes to regulations affecting the mining industry.*

Forward-looking statements in this analysis include statements regarding:

- *future mining plans,*
- *milling plans,*
- *concentrate production,*
- *tax and royalty terms,*
- *smelter and refinery terms, and*
- *mineral price forecasts.*

The only purpose of the results presented in this paper is to illustrate the effect that windfall taxes and other mining taxes have on a project economics. These results are expressly not an opinion on the economic prospects of the Galore Creek Project or any other mining project. Furthermore, none of the authors have ever discussed the Galore Creek Project or any other project with representatives of NovaGold Resources Incorporated.

Introduction

NovaGold Resources Incorporated's Galore Creek Project is used as a case study in this paper in order to give us a real-world project on which to base our analysis. The Galore Creek Project was the subject of a NI43-101 report (Hatch, 2005) published on 25 October 2005 and only information contained in that report is used to build the cash flow model. We assume that the deposit is located in Mongolia to analyse the effects of the windfall tax on project economics and to demonstrate the merits of stochastic cash flow simulation.

The 'Galore Creek' Project is a gold/copper development project containing 475 million tonnes of run-of-mine (ROM) ore distributed between several adjacent deposits. The average recoverable grades are 0.58 per cent copper and 0.31 g/t gold. Production is expected to last 20 years at a production rate of 23.8 Mtpa. Overall production costs are approximately US$6.40 per ROM tonne and include mining, milling and G and A costs. Concentrate is shipped to a smelter and subject to transport costs, smelter mineral deductions, smelter and refining charges, and copper price participation. The project will take four years to build at an overall capital cost of US$1.1 billion. Sustaining capital over the production life of the project is US$245 million.

Economic environment and mineral price uncertainty

Gold and copper price uncertainties are described over the life of the project by correlated one-factor diffusion processes. An important feature of these processes is the recognition that market participants update their price forecasts as new pricing information arrives. There is a large literature discussing commodity price models. The structure of the model we use is described in Salahor (1998) and Bradley (1998), where it is used for a single oil or natural gas price. The correlated structure we use is described in Laughton (2005), applied to CO_2 emission rights and natural gas.

For simplicity of exposition, we take the costs and production profiles to be known with certainty. Cost and production uncertainty can be included, and the input and output prices could be correlated or related in a functional form possibly with lags, and the qualitative structure of our results would not change as long as the costs remain less risky than the revenues.

The gold price model incorporates a flat time zero gold price forecast of $450/oz. Figure 1 outlines with a jagged black line a simulated price path generated by the gold price model for the first ten years of the project. This is just one path among many generated during the Monte Carlo simulation. The grey lines with geometric shapes are the conditional price forecasts and confidence boundaries associated with the simulated price path for Year 0 (squares), Year 2 (triangles) and Year 6 (crosses). The forecasts change with the updated spot price because the model recognises information updating.

The gold price model has two other important characteristics. First, uncertainty grows at a constant rate so that gold price uncertainty continually increases with time. Second, a price shock of X per cent to the current spot price results in an X per cent revision of price expectations over the life of the project. This is illustrated in Figure 1 when the gold price increases from US$450 per ounce at Year 0 to US$589 per ounce in Year 2. The revised expected price and confidence boundaries due to this price change are delineated by grey lines marked with triangles.

The copper price uncertainty model assumes a long-term equilibrium price of US$1.15 per pound and a Year 0 spot price of US$2.60 per pound. Figure 2 outlines a set of expected price projections and confidence boundaries associated with a price path (the jagged black line) generated by the copper price model over the first ten years of the project. The grey dashed or solid lines with square markers delineate the Year 0 forecast copper prices and confidence boundaries. The copper price forecast is updated to the solid grey line marked with triangles when the spot price falls to US$1.46 per pound in Year 2. Finally, the price

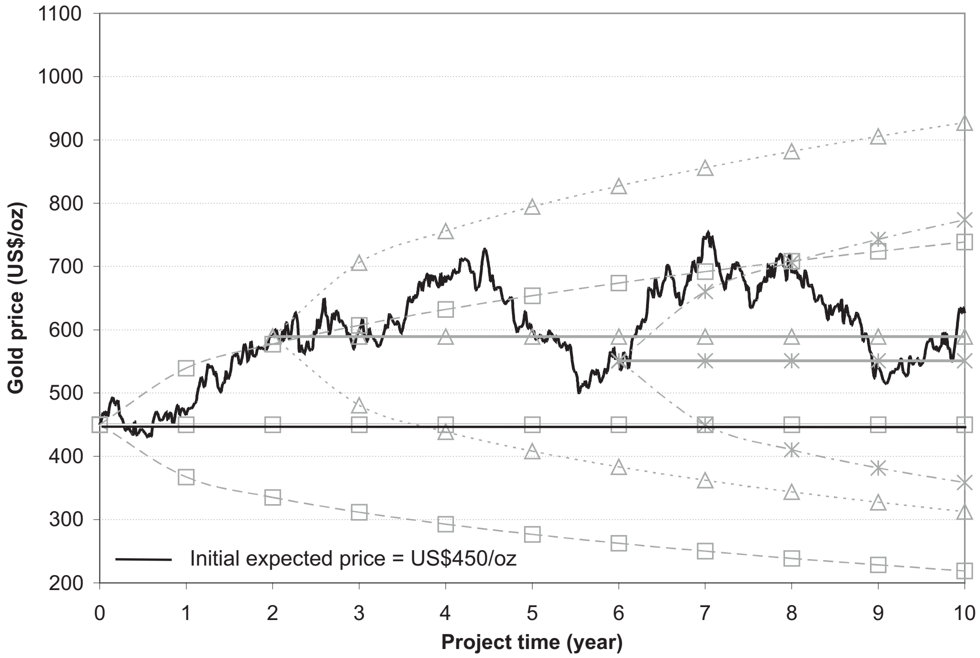

FIG 1 - Gold price expectations and confidence boundaries with forecast updating.

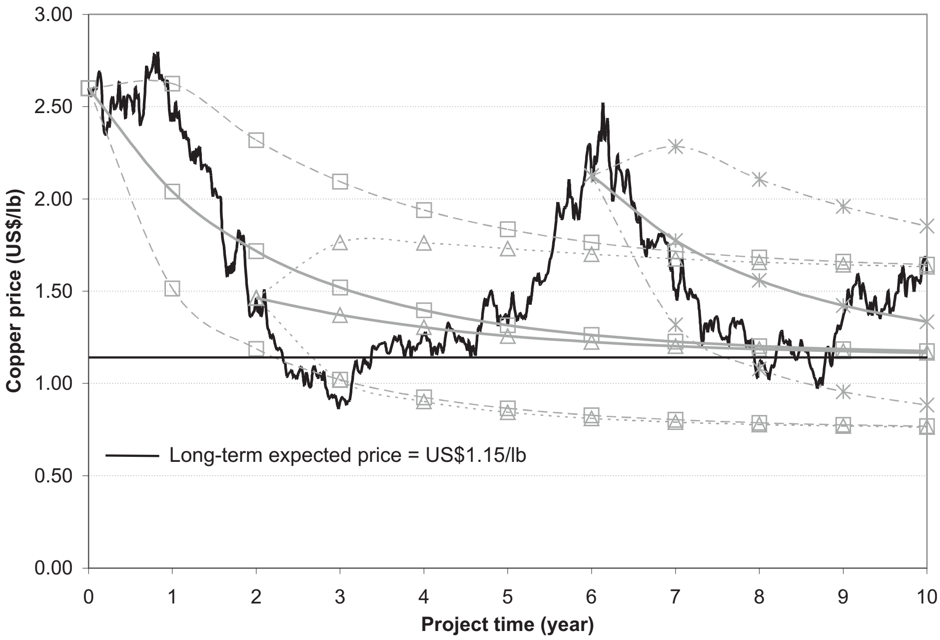

FIG 2 - Copper price expectations and confidence boundaries with forecast updating.

forecast is updated again when the price increases to US$2.13 per pound in Year 6; the revised forecast is demarcated by the solid grey line with crosses.

A key characteristic of the copper price model is price reversion, which is the result of supply and demand forces influencing price behaviour. Reversion limits the growth of price uncertainty over time and impacts price expectations after a price shock. In the presence of reversion, price uncertainty saturates (stops growing) in the long term. The confidence boundaries in Figure 2 exhibit this in that the 80 per cent confidence range for long-term copper prices stabilises with an upper boundary price of US$1.64 per pound and a lower boundary price of US$0.77 per pound. Even after a large price shock, expected long-term prices and their associated confidence boundaries are almost unaffected by the short-term price movement.

Government taxes and mining royalties

Mineral revenues are subject to a five per cent royalty after a deduction for smelter penalties and treatment charges. A windfall minerals tax of 68 per cent is applied against windfall gold and copper revenues when the current copper spot price is greater than US$1.18 per pound and when the current gold spot price is above US$500 per ounce. The costs of smelting and refining concentrate are deductible in the calculation of windfall revenues:

Windfall revenue:

= **maximum** [**maximum** (Au spot price – US$500, 0) × Au production + **maximum** (Cu spot price – US$1.18, 0) × Cu production – smelter deductions, 0] (1)

The interior maximum conditions in Equation 1 ensure that there are no negative windfall revenues for either gold or copper. The exterior maximum condition ensures that a negative windfall tax is not paid by the government when smelter deductions exceed the windfall revenues. The 'maximum' statements introduce non-linearity into the windfall revenue scheme.

The taxable income for corporate income tax is defined as:

Taxable income:

= **maximum** (Mineral revenue – concentrate and smelter expenses – royalties – windfall tax – operating cost – depreciation – tax-loss carry forwards, 0) (2)

Operating tax losses are applied first against taxable income and can be carried forward a maximum of seven years. Depreciation is added to a capital pool each year and then applied against any taxable operating profits. Any residual operating profits are taxed at a rate of 25 per cent. Once again, the 'maximum' statement introduces a non-linearity into the tax scheme, and the depreciation and tax-loss carry forwards introduce path dependence.

Uncertainty characteristics of equity and government cash flow streams

The uncertainty characteristics of equity, government royalty, corporate income tax and the windfall tax cash flows over the life of the project were modelled with Monte Carlo simulation using the copper and gold price models displayed in Figures 1 and 2. The equity cash flow is net of all government cash flows but does not include sustaining capital expenditures.

A coefficient of variation (CoV) was recorded annually for each cash flow. The coefficient of variation is the standard deviation of an uncertain variable divided by its expected value, providing an indication of cash flow dispersion (the higher the number the greater the uncertainty). It is used to highlight the level of uncertainty in each participant's cash flow stream.

Figure 3 presents the various cash flow stream CoVs during production operations when the windfall tax is in effect. The solid black line delineates the CoVs for the cash flows owned by equity and shows that they gradually increase over the life of the project as grades deteriorate and mining costs increase with pit depth. The spikes in Years 12, 16 and 20 are caused by increased stripping and waste haulage costs associated with accessing an adjacent deposit. The profile of equity cash flow CoVs indicates that equity cash flow uncertainty varies sharply over the life of the project and is sensitive to increasing unit operating costs.

The long dashed grey line with square markers outlines the government royalty CoVs. Uncertainty in the royalty increases gradually during the project. This is due to the influence of increasing gold price uncertainty.

The short dashed grey line with triangular markers provides the CoVs for the corporate income tax stream. There is a large jump in uncertainty during Year 8 associated with the corporate income tax because this is the year that depreciation balance of pre-production capital begins to be exhausted and the tax stream becomes extremely sensitive to if and when this occurs. This can be called depreciation risk. Prior to this time depreciation write-offs are predictable. After this time uncertainty is reduced as pre-production capital is almost surely exhausted and the corporate income tax reflects only contemporaneous price risk.

Finally, the dot-dashed grey line with circle markers delineates the windfall tax CoVs. Uncertainty increases in the windfall tax stream as copper prices tend towards a long-term equilibrium of US$1.15 per pound. During the early stages of operations, there is reasonable probability that copper prices will be high enough that the windfall tax price boundary will be breached. There is a smaller probability that the windfall tax will be paid during the later years of the project once copper price expectations revert to equilibrium levels.

The impact of the windfall tax on the uncertainty levels of the equity and corporate income tax streams is shown in Figure 4. The windfall tax reduces the overall uncertainty of the equity cash flow stream, demonstrated by the equity cash flow CoVs being lower when the windfall tax is in effect. This result may seem counter-intuitive until it is remembered that the windfall tax reduces the number of large cash flow outcomes in each year by reducing the benefit of high mineral price environments – it is a cash flow hedge. The windfall tax also reduces uncertainty in the corporate income tax stream in most years by decreasing the probability of large taxable cash flows. Windfall taxes are a deduction for income tax purposes and so reduce taxable income on which corporate income tax is paid.

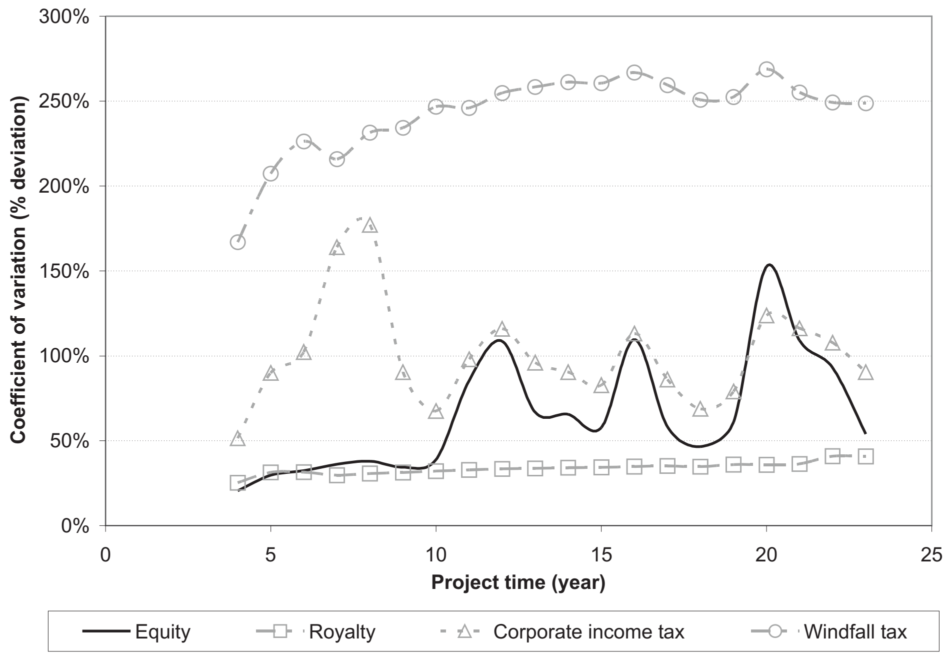

FIG 3 - Coefficients of variation for equity and individual government cash flow streams.

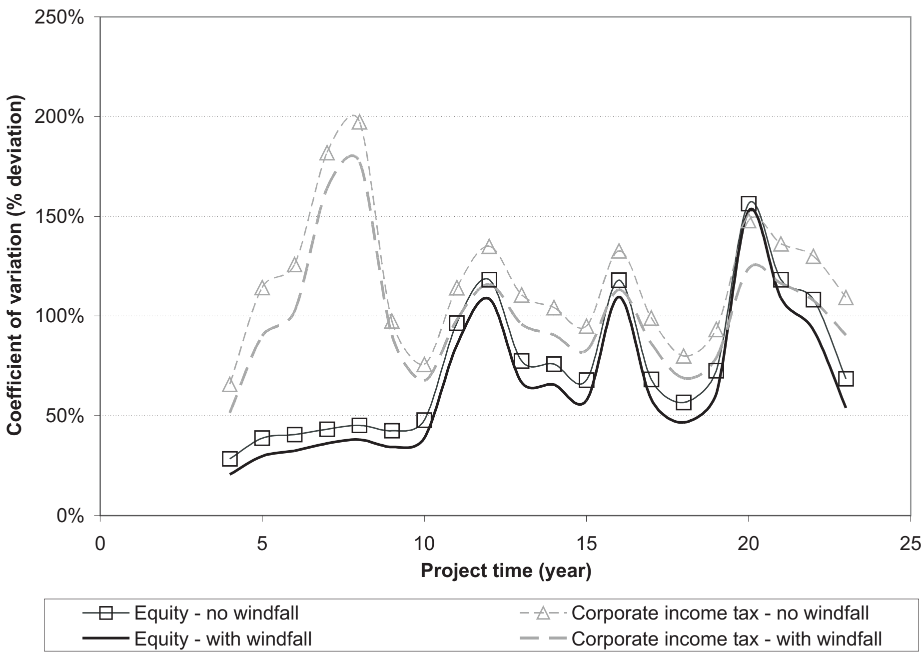

FIG 4 - The windfall tax impact on equity and corporate income tax uncertainty.

VALUATION OF THE MONGOLIAN 'GALORE CREEK' PROJECT

The valuation results for the reference project and for the project with taxes are presented in Table 2 when there is no windfall tax and in Table 3 when the windfall tax is present. The reference project is defined as the project where the operator receives the full operating net cash flow (ie the project's free cash flow) because there are no financing, royalty, or government tax obligations to pay. When taxes are imposed on the reference project its value is divided between the various equity, creditor and government participants based on the financing and taxation terms. The following cash flows and net present values were calculated for each project type using both a conventional single-point forecast cash flow model ('Static') and a Monte Carlo simulation model:

1. cumulative expected net cash flow,
2. time-adjusted cumulative expected net cash flow where discounting is performed at the risk-free rate of three per cent,
3. a DCF NPV calculated using an eight per cent discount rate, and
4. a RO NPV calculated with risk-adjusted expected mineral prices and the risk-free rate of three per cent.

The reference project in Table 2 has an expected cumulative cash flow of US$2566.4 million. The Monte Carlo analysis estimates the same number, within numerical approximation error, since the reference project has no non-linearities in the cash flows. Once the government is introduced as a project participant non-linearities are introduced. The expected cumulative income tax cash flow, for example, is estimated at US$456.5 million using a spreadsheet model and at US$467.9 million using Monte Carlo simulation. Note that, as is typical, the Static method underestimates the magnitude of the income tax. The royalty stream is estimated correctly using the Static model since the royalty is in this case a fixed rate. This would not be the case for a sliding scale royalty.

At an eight per cent discount rate, the reference project is valued at US$756 million. This is true whether Static spreadsheet or Monte Carlo analysis is used because there are no non-linearities in the cash flows. Once taxation is introduced the present values of the various cash flow streams differ depending on whether the static expectation of the cash flow is used or the Monte Carlo expectation of the cash flow is used. The US$1762.7 million residual cash flow received by equity has a present value of US$424.6 million using the static spreadsheet analysis and an eight per cent discount rate, and the US$1749.6

TABLE 2

Cumulative net cash flow, time-adjusted net cash flow and net present value for individual project participants when there is no windfall tax.

Model type	Static		Monte Carlo	
Cumulative net cash flows (US$ M)				
Reference project	2566.4		2564.5	
Equity	1762.7		1749.6	
Royalty	347.1		347.0	
Corporate income tax	456.5		467.9	
Windfall tax	N/A		N/A	
Time-adjusted cumulative net cash flow (US$ M)				
Reference project	1601.2		1600.1	
Equity	1054.6		1043.7	
Royalty	238.7		238.7	
Corporate income tax	307.8		317.7	
Windfall tax	N/A		N/A	
Net present value (US$ M)				
Risk discounting approach	**Discounted cash flow**		**Real options**	
Model type	*Static*	*Monte Carlo*	*Static*	*Monte Carlo*
Reference project	756.6	756.0	916.4	915.3
Equity	424.6	415.8	555.0	521.2
Royalty	147.6	147.6	209.3	209.3
Corporate income tax	184.4	192.7	152.0	184.8
Windfall tax	N/A	N/A	N/A	N/A

million equity cash flow estimated using Monte Carlo analysis has a value of US$415.8 million when discounted at an eight per cent discount rate. Corporate income tax has a present value of US$184.4 million using the standard static spreadsheet analysis and an eight per cent discount rate, and US$192.7 million if the tax cash flow is estimated using Monte Carlo and the result discounted at eight per cent.

None of these values is likely to be correct, however. It is well-recognised that cash flows that are more sensitive to price variability should be discounted at a higher rate. Looking back to Figure 2, the variability of the income tax stream is higher than the equity stream. It is therefore unreasonable to discount both types of cash flow stream at the same discount rate. It is also unreasonable to discount the equity cash flow in Year 12 at eight per cent, and then discount the equity cash flow in Year 13, which has much lower variability, at that same eight per cent.

Real option valuation calculations take this varying cash flow uncertainty across time and across cash flow type into account. Under the RO approach, which uses market information and formal models of risk discounting to estimate appropriate cash flow risk discounting, the reference project has a value of US$916 million, indicating that the DCF discount rate of eight per cent was too high for this project. Once government taxes are added, the equity portion of the project is worth US$521.2 million and the corporate income tax portion is worth US$184.8 million. Note that this corporate tax valuation implies an effective discount rate for these income tax cash flows of greater than eight per cent, since the Monte Carlo real options value is lower than that calculated under the Monte Carlo DCF approach. Note also that the equity, royalty and income tax values add up to the reference project value: all taxes do is divide the 'project value pie' amongst the various participants.

Table 3 shows that with the introduction of the windfall profits tax the value of the equity portion of the project is unchanged at US$555 million in the static spreadsheet analysis, while it falls from US$521.2 million to US$439.7 million under the real options Monte Carlo analysis. The static spreadsheet analysis determines that the windfall tax has no impact on the project because the spreadsheet forecast prices are never high enough to generate a taxable windfall profit net of smelting and refining charges. This is clearly an underestimate of the windfall profits tax; the correct analysis, taking into account all possible price paths and the appropriate risk and time discounting of likely windfall taxes shows that it lowers the project value to the equity owner by about US$81 million. The total government interest in the project correspondingly increases by US$81 million. The government interest does not rise by the full value of the windfall profits tax due to the fact that this tax is deducible from the corporate income tax. The royalty owner's interest is in this example unaffected by the windfall profits tax. The individual cash flows streams do not add up to the reverence project value under the Monte Carlo method due to slight numerical approximation error associated with Monte Carlo. This problem would disappear when increased simulations are performed.

TABLE 3

Cumulative net cash flow, time-adjusted net cash flow and net present value for individual project participants when there is a windfall tax.

Model type	Static		Monte Carlo	
Cumulative net cash flows (US$ M)				
Reference project	2566.4		2563.6	
Equity	1762.7		1493.7	
Royalty	347.1		347.0	
Corporate income tax	456.5		382.6	
Windfall tax	0.0		340.4	
Time-adjusted cumulative net cash flow (US$ M)				
Reference project	1601.2		1600.1	
Equity	1054.6		866.5	
Royalty	238.7		238.7	
Corporate income tax	307.8		258.6	
Windfall tax	0.0		235.8	
Net present value (US$ M)				
Risk discounting approach	**Discounted cash flow**		**Real option**	
Model type	*Static*	*Monte Carlo*	*Static*	*Monte Carlo*
Reference project	756.6	756.0	916.4	915.3
Equity	424.6	304.2	555.0	439.7
Royalty	147.6	147.6	209.3	209.3
Corporate income tax	184.4	155.5	152.0	157.7
Windfall tax	0.0	148.5	0.0	108.2

Cash flow estimation differences using DCF and Monte Carlo calculations

In Tables 2 and 3 any difference between the cumulative cash flow estimates using single point forecasts and cash flow estimates using Monte Carlo simulation is due to Jensen's inequality. The conventional cash flow model using single point forecasts is shown in all cases to underestimate corporate income tax and windfall tax payments while overestimating the equity cash flow stream. The equity cash flow is overestimated by a conventional spreadsheet model because its cash flow equation has a concave pay-off shape. This shape arises from equity cash flows being reduced in advantageous price environments by various taxes, while its cash flows are fully exposed to the negative impact of adverse price situations. Conversely, the government tax cash flows are underestimated by a conventional cash flow model because these cash flow equations have a convex pay-off shape. The government receives a portion of project cash flows as taxes when price levels are high enough to generate a taxable cash flow and receives nothing when price levels are so low as to cause an effective operating loss. Both of these outcomes are consistent with Jensen's inequality.

Cash flow non-linearity provides a strong justification to use Monte Carlo simulation when valuing a project. In this case study, a conventional cash flow model can not effectively value the windfall tax because expected windfall profits are negative for the entire life of the project. The windfall tax can only be effectively valued with Monte Carlo simulation since this technique recognises that the asymmetry of windfall tax cash flows in high and low mineral price environments do not add up to zero likely windfall taxes.

Risk discounting differences using DCF and real option NPV calculations

Calculating values for the various cash flow streams in a manner consistent with the principle of investor risk aversion is difficult because the uncertainty characteristics of each stream vary markedly. This task is further complicated because cash flow uncertainty for an individual project participant may vary by year as ore grade, production rates, costs structures and capital depreciation balances fluctuate. This was shown in Figures 3 and 4.

The difference between DCF and RO methods of adjusting for cash flow risk when calculating a cash flow NPV can be illustrated using the concept of Net Cash Flow Risk Discount Factors (NCFRDFs). A NCFRDF calculates the magnitude of risk adjustment that is applied to each dollar of a cash flow in a particular year. It indicates the amount an investor will pay for a dollar of cash flow from a particular cash flow stream on a risk adjusted (but not time adjusted) basis. A NCFRDF is defined as the present value of a cash flow divided by the expected cash flow after offsetting the adjustment for the time value of money:

$$NCFRDF_t = CFPV_t \,/\, (ECF_t * TDF_t) \qquad (3)$$

where:

CFPV = the cash flow present value at project time 't'

ECF = expected cash flow at project time 't'

TDF = time discount factor at project time 't' = $1/(1 + \text{riskfree rate})^t$

RO and DCF have different NCFRDF equations because their respective approaches to risk discounting differ. An *equity* DCF NCFRDF is calculated for time 't' as:

$$NCFRDF_{t,\,DCF,\,Equity}$$
$$= (AuProd_t * AuPrice_t + CuProd_t * CuPrice_t - OpCost_t) * RDF_{t,DCF} * TDF_t \,/ (AuProd_t * AuPrice_t + CuProd_t * CuPrice_t - OpCost_t) * TDF_t = RDF_{t,DCF} \qquad (4)$$

where:

AuProd = gold production

AuPrice = expected gold price

CuProd = copper production

CuPrice = expected copper price

OpCost = expected operating costs including tax and royalty payments but not development capital expenditure

RDF_{DCF} = DCF risk discount factor = $1/(1+\text{risk premium})^t$

The numerator is the present value of the cash flow as calculated using the DCF method. The product RDF_{DCF} * TDF is an approximate representation of the conventional discrete DCF discounting formula that combines the risk and time adjustment into one discount rate. The TDF in the denominator offsets the effects of time discounting so that we can examine only the discounting for risk.

The *equity* RO NCFRDF for time 't' is calculated as:

$$NCFRDF_{t,\,RO,\,Equity}:$$
$$= ((AuProd_t * AuPrice_t * RDF_{t,AU} + CuProd_t * CuPrice_t * RDF_{t,CU}) * TDF_t - PVOpCost_t) / (AuProd_t * AuPrice_t + CuProd_t * CuPrice_t - OpCost_t) * TDF_t = (AuProd_t * AuPrice_t * RDF_{t,AU} + CuProd_t * CuPrice_t * RDF_{t,CU} - PVOpCost_t / TDF_t) / (AuProd_t * AuPrice_t + CuProd_t * CuPrice_t - OpCost_t) \qquad (5)$$

where:

PVOpCost = PV of expected operating costs plus the RO present value of royalties and taxes

RDF_{AU} = a market-based risk adjustment for pure gold price uncertainty

RDF_{CU} = a market-based risk adjustment for pure copper price uncertainty

In the first line of this derivation the numerator is the present value of the cash flow calculated using the real option method. The copper and gold risk adjustments in the RO method are calculated using formulas that are consistent with the Capital Asset Pricing Model and are able to recognise copper price reversion. Salahor (1998) provides details of these risk-adjustment formulas.

The RO and DCF NCFRDF equations for the government royalty, corporate income tax and windfall tax have similar structure but are not included here for brevity.

Equation 4 shows that project structure does not affect DCF risk discounting since the cash flow equations in the numerator and denominator cancel each other. However, project structure is very much part of the RO risk discounting mechanics. Equation 5 shows that the cash flow equations can not be factored out of the value calculation because RO applies a market-based risk adjustment to the source of uncertainty and then filters its effect through to the net cash flow stream. The application of a risk-adjustment to the source of uncertainty results in the effective net cash flow risk adjustment generated by RO recognising changes in cash flow uncertainty as operating leverage varies.

Figures 5 to 8 compare the DCF and RO risk adjustments applied to the various project cash flow streams and links these adjustments to levels of cash flow uncertainty during each year. In each of these graphs the vertical axis on the left side represents the level of cash flow uncertainty. These are plotted with a solid black line, taken from Figures 3 and 4. The vertical axis on the right side represents the NCFRDFs. The DCF and RO NCFRDFs are plotted with grey lines; the dashed line with no data markers delineates the DCF NCFRDF and the lines with data markers outline the RO NCFRDFs. The difference between 1.00 and the NCFRDF is the amount of compensation an investor requires for exposure to uncertainty in a particular cash flow stream.

The equity cash flow uncertainty and risk discounting is provided in Figure 5. Cash flow uncertainty increases with time as ore grades decrease and operating leverage increases. There are jumps in Years 12, 16 and 21 when additional stripping and waste handling costs are incurred opening new resources. The RO NCFRDFs show that the effective equity cash flow risk discounting reflects the cash flow uncertainty. The risk discount increases to \$0.60 or even \$0.80 per dollar of cash flow (ie \$1 cash flow is worth \$0.40 or \$0.20 on a risk-adjusted but not time discounted basis) when additional stripping costs lead to a sudden increase in cash flow uncertainty. The RO method is also able to recognise the influence of the windfall tax on equity cash flow uncertainty, showing how the addition of a windfall tax lowers the risk discounting of the equity cash flows. Conversely, the DCF NCFRDF reflects a risk adjustment that is increasing over time. This highlights a DCF assumption that cash flow uncertainty is growing over the life of the project. The equity CoVs show that equity cash flow uncertainty in fact rises and falls throughout the project horizon.

Figure 5 also provides an explanation for RO Monte Carlo calculating a higher equity NPV than the DCF Monte Carlo method in Tables 2 and 3. On average, the RO method applies a smaller risk adjustment to each dollar of equity cash flow than the DCF method because it is picking up copper price reversion and the low correlation between gold price and financial market uncertainty in its risk adjustments. The constant discount rate of eight per cent is too high for this project.

The royalty cash flow CoVs and RO and DCF NCFRDFs are displayed in Figure 6. Uncertainty in the royalty cash flow grows at a gradual rate during the project. The RO NCFDFs reflect this pattern of uncertainty with RO discount factors that grow at a gradual rate during the project. The DCF NCFDFs, however, reflect a pattern of cash flow uncertainty that grows at higher rate – the same rate as in Figure 5. This is the reason that the DCF Monte Carlo royalty NPVs are lower than the RO Monte Carlo royalty NPVs in Tables 2 and 3 – they are discounted at too high a rate.

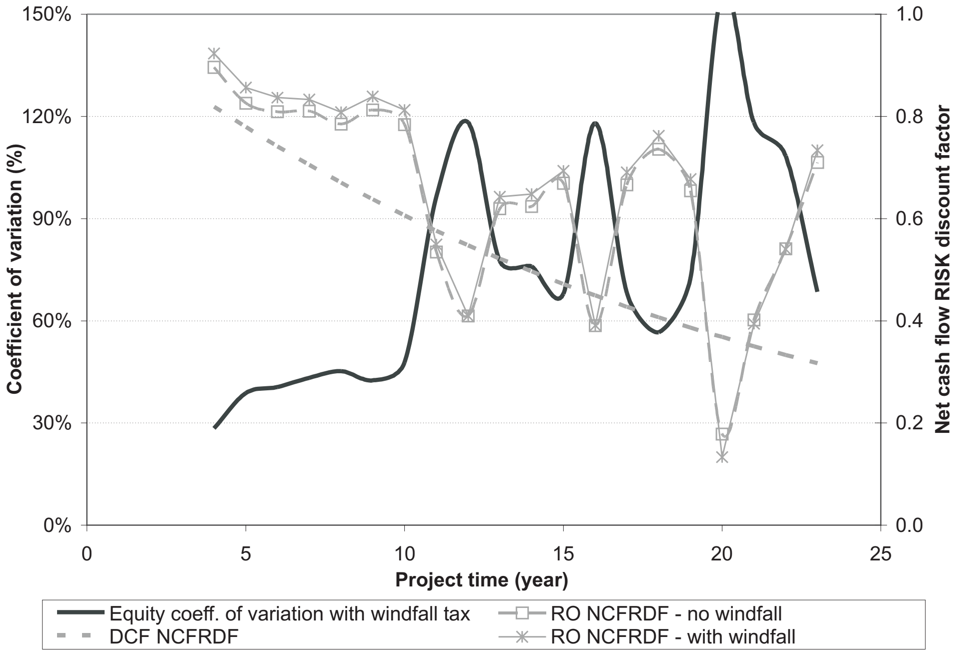

FIG 5 - Equity cash flow DCF and real option NCFRDFs.

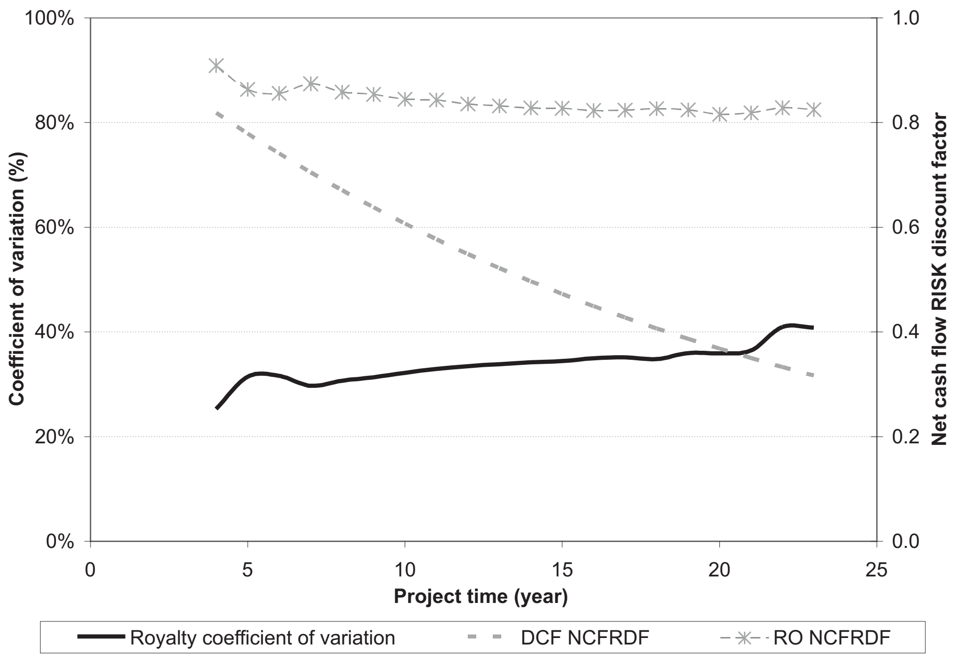

FIG 6 - Royalty cash flow DCF and real option NCFRDFs.

Figure 7 delineates the corporate income tax RO and DCF NCFRDFs and CoVs. There are high levels of tax cash flow uncertainty before Year 10 as depreciation balances are run down. This is reflected in the risk discounting profile of the RO approach, where risk adjustments are higher in the initial stage of the project than in the later stages where corporate income tax is paid on a more regular basis. The figure shows that the RO discounting of the income taxes is lower in the presence of the windfall tax, as is appropriate given that the uncertainty of the income tax stream is reduced by the windfall tax. The DCF method does not recognise this pattern of cash flow uncertainty and discounts all cash flows at the same eight per cent discount rate. This results in an increasing DCF NCFRDF that is not dependent on the variability of the tax cash flow stream and is not affected by the presence of the windfall profits tax.

The CoVs and RO and DCF NCFRDFs for the windfall tax cash flow are shown in Figure 8. Uncertainty in the windfall tax increases with time as copper price reverts to its long-term equilibrium price of US$1.15 per pound and there is less certainty that the windfall profits tax will be paid in any given year. The pattern of RO risk adjustments mirrors the pattern of windfall tax uncertainty except in the project's final year. Windfall tax uncertainty decreases slightly from the previous year but the RO risk adjustment decreases far more abruptly. This change in risk adjustment may seem unwarranted until the change in copper and gold grades is examined. Gold grades double (from a low level) in the final year while the copper grade continues to fall, which increases the gold proportion of the total revenue stream. The RO method picks up the increase in gold revenue and its associated smaller risk adjustment due to low

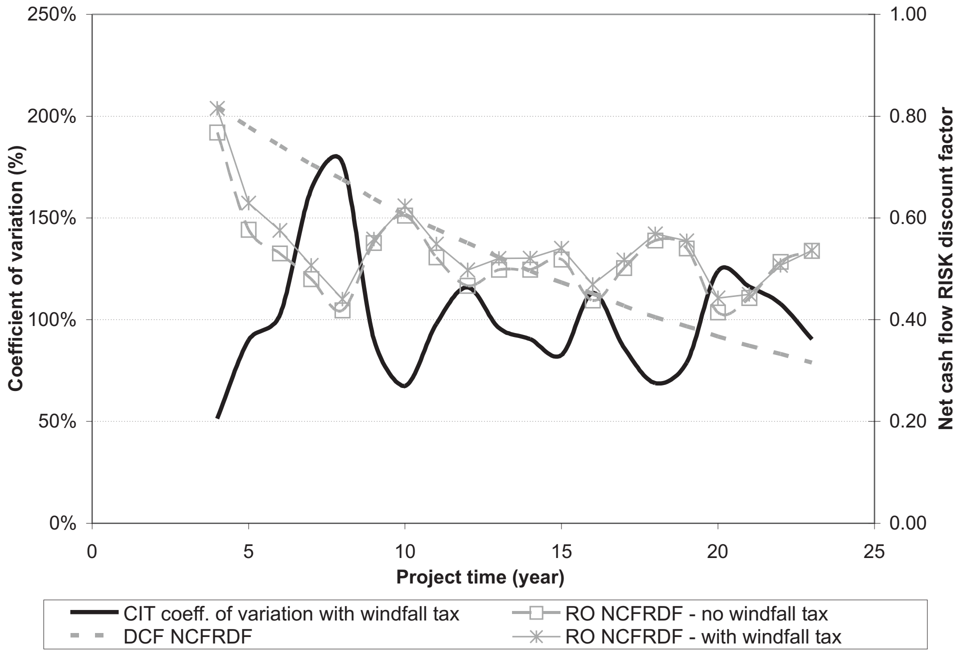

FIG 7 - Corporate income tax cash flow DCF and real option NCFRDFs.

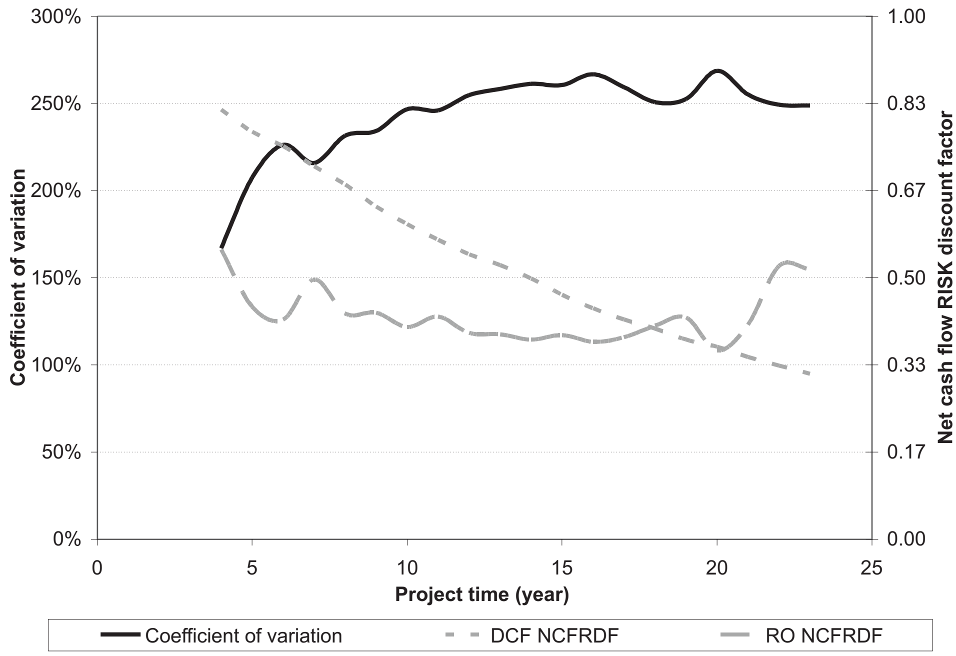

FIG 8 - Windfall tax cash flow DCF and real option NCFRDFs.

financial market correlation and adjusts the effective windfall tax risk adjustment accordingly. The DCF method does not pick up the change in the relative proportion of gold and copper revenues in its risk adjustment, and tends to under-adjust the windfall cash flows for risk. This is why the windfall tax has a higher value with the DCF Monte Carlo method than with the RO Monte Carlo method.

The heavier discounting of the windfall tax cash flows in the RO method has important implications for viewing the impact of the tax on the project. In the RO method, even though the windfall tax and the royalty have roughly the same expected cumulative cash flows (see Monte Carlo in Table 3), the windfall tax has a value that is half that of the royalty stream. In other words, the high variability of the windfall tax stream means that it is heavily discounted, whereas the more certain royalty stream is discounted at a lower rate. The net effect is that the windfall tax does not have as large an impact on the equity position of the mine as DCF analysis indicates. An analysis that ignores these discounting differences will tend to overestimate the burden of the windfall tax on the economics of the project.

CONCLUSION

The recent changes to mineral tax policy in many jurisdictions have caused the economic reassessment of many natural resource projects. Some changes, such as increased royalty rates, appear to be easily recognised and studied within conventional spreadsheet valuation models, while others, such as the imposition of a windfall profit tax, have been much harder to assess because of their contingent nature. The primary valuation challenge confronting mineral economists is to recognise the contingent nature of these new taxes and the impact these new taxes have on the cash flow uncertainty characteristics of existing project participants.

This paper investigated the impact of a windfall profits tax on the cash flows of a multi-staged copper-gold project using Monte Carlo simulation. The results show that a conventional cash flow model using single-point input values may contain important cash flow estimation errors. In the example we developed a static spreadsheet analysis that underestimated the cumulative windfall tax cash flows by $340 million. Monte Carlo simulation corrected these errors by recognising cash flow non-linearities in the cash flow stream.

Taxes not only introduce complicated non-linearities into the valuation process, but they also alter the riskiness of the cash flows received by the project's equity owner. DCF analysis discounts all cash flow streams at the same rate. Windfall profits taxes, being a highly variable cash flow, require higher risk adjustments than the less-risky equity cash flow stream. The real options valuation approach brings this out, with logically higher discounting of riskier cash flow streams and lower discounting of less risky cash flow streams. In the example we examine, based on the Galore Creek project, a Mongolian-type windfall tax, with expected cash flows over the life of the project of $340 million, would lower the equity value of the project by $81.5 million. The static DCF approach indicated that the windfall tax would have no impact on the project economics.

We cannot conclude from this one example that the traditional DCF approach will always tend to underestimate the impacts of taxes on project value. We can say that any time taxes are path-dependent or contingent on the current economic environment faced by the project, non-linearities are introduced that will cause singe-point forecasts of tax cash flows to be incorrect. These errors are corrected in a Monte Carlo analysis, and it is thus imperative to use Monte Carlo analysis when such non-linearities exist. We can also say that taxes alter the risk profile of the cash flows accruing to equity participants in a project, and that these risk adjustments are very difficult, if not impossible, to interpret and account for in a traditional DCF analysis. They should not be ignored, and real options Monte Carlo analysis lends itself perfectly to the correct valuation of taxes and their impact on the value of a project.

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