Chapter 14
Using Real Options to Value and Manage Exploration

GRANT A. DEVIS
Colorado School of Mines, 1500 Illinois St., Golden, CO 80401

AND MICHAEL SAMI
AMEC Americas Ltd., 2020 Wiston Park Drive - Suite 700, Oakville, Ontario, L6H 8T7, Canada

Abstract
This paper considers the two main uncertainties facing an exploration manager, geological uncertainty and price uncertainty, within a real options framework. The paper presents two models of exploration. The first model considers only geological uncertainty and reveals how geological and economic factors influence exploration activity and the value of an exploration property. In particular, the model shows that exploration activity will logically respond to changes in the economic and technical environment. One outcome of this model is that higher geological uncertainty enhances exploration project value where initial resource estimates are small and harms projects that have quality resources. The second model considers both geological uncertainty and price uncertainty, and reveals important differences between exploration for commodities such as gold, which exhibit random walk prices and increasing future price uncertainty, and exploration for base metals, which exhibit reversion to a mean price level and slowly reducing future price uncertainty. Under a value-maximizing policy, gold exploration should, during periods of low and moderate prices, focus on promising greenfields deposits and deposits with previous positive exploration results while defending geologically unpromising greenfields projects until prices improve. This is because given gold’s random walk price characteristic, there is always a chance that higher and higher gold prices could make up for poor geological potential. An extended exploration policy that includes the deferred greenfields projects is warranted when prices are higher. On the other hand, our model suggests that unpromising greenfields copper projects should not be kept in deferred mode during low prices, since future prices are capped by the trend of copper prices to a long-term mean. Greenfields copper projects should therefore either be explored immediately if promising or permanently abandoned if unpromising, regardless of current copper price. Since the geology and economies of the gold and copper examples have been designed to be as comparable as possible, these model differences in exploration behavior are driven entirely by the difference in price behavior of base metals, which exhibits reversion, and gold, which does not. That optimal exploration policy depends on the commodity being mined highlights the importance of modeling both geological and economic (price) uncertainty when valuing and managing exploration projects.

All of this is done within a real options analysis. Traditional discounted cash flow analysis ignores the nonlinearity of the payoffs to exploration and thereby undervalues exploration activity. The real options framework, which captures the nonlinearity of these payoffs, shows that in some cases where the traditional discounted cash flow value of an exploration project is negative, exploration activity can be value creating and should therefore proceed. Real options modeling thus provides economic validation of decisions that exploration managers often take in spite of traditional valuation criteria that recommend against taking such action.

Introduction
The main purpose of exploration is to generate reserves. In this paper we stay from the Joint Ore Reserves Committee (JORC) very specific definitions of resource and reserve classifications and define reserves as the quantity of mineralization in a specific location that is of sufficient geological assurance and economic value to be profitably mined, either now or in the future, under currently forecast market and technical conditions. Thus, in our usage, the reserve quantity is not known with certainty, but is rather the mean of a probability density function representing possible quantities of mineral that might be extracted under the various economic and technical scenarios that could unfold given current information. A resource is the portion of that same mineralization that either has insufficient geological assurance or economic value to be considered a reserve at the time. Our generic definition allows us to discuss reserves and resources in a stochastic and inter-temporal sense, which is vital in a real options perspective.

Real options are options on real assets as opposed to financial assets. Howell et al. (2001) provide a readable introduction to real options. Early real options research specifically focused on the option to develop a mine or explore for oil...
Brennan and Schwartz, 1985; Paddock et al., 1988). The focus was on price and cost uncertainty, in contrast to the decision-analytic models of exploration dating back to the 1960s that considered only geological uncertainty and took the economic variables as known. Only recently have real options models of exploration quantified both price and geological uncertainty (e.g., Cortazar et al., 2003; Martzoukos, 2000, 2002), although one of the main technical artifacts of exploration that appears in the decision-analytic models, imperfect information from sampling, is still omitted. Our paper extends these recent models by introducing imperfect information into a real options model of exploration.

Given the increasingly sophisticated quantitative geological models of exploration there has been a call to add equally quantitative models of the economic parameters affecting exploration decisions (e.g., Lerche, 1997, p. 637). In this paper we seek to do just this. Our hope is that by quantitatively modeling exploration as an economic activity, exploration managers and geologists can make better decisions, decisions that add more value to the firm and the economy and that ultimately result in the more efficient generation of reserves. Our model also allows for the valuations of exploration projects, even those that are highly speculative.

Models are, by design, simplifications of reality. Our model is no exception; we have left out many of the complexities that geologists and exploration managers must wrestle with and that make the job of exploration management and valuation so difficult. That said, we include two of the largest drivers of exploration activity, mineral quantity uncertainty and mineral price uncertainty. The result is a tool for exploration planning and management that appears to explain certain real-world phenomena. Perhaps more importantly, the insights derived through quantification of the exploration process may lead to improvements in exploration decision-making.

Guiding Principles

In guiding our modeling efforts we look to five irrefutable facts that characterize exploration. First, exploration is a lumpy irreversible investment activity; it has an initial irreversible capital outlay over some finite period, followed by sequential options for further irreversible capital outlay over subsequent periods. Modeling exploration activity requires investment theory that characterizes investment as lump, irreversible, and sequential.

Second, dollars are spent on exploration only where there is some expectation of an economic return at the end of the exploration sequence. That expected return is principally the discovery of new reserves. This does not mean that every project needs to lead to the discovery of reserves. Indeed, economic failure, where the discovery is a resource too small or of too low a grade to be extracted at a profit, is the norm (Keating, 1994, p. 56). Exploration has probabilities as low as 1 in 1,000 of moving beyond grassroots stages, and as low as 1 in 10 of finding an orebody in the few properties that warrant major exploration effort (Petrie, 1978). This second irrefutable fact reveals that the appropriate economic model of exploration activity should be grounded in investment theory where expected (or mean) payoffs, rather than most likely payoffs, guide decision making.

Third, the payoffs to exploration are nonlinear—the payoff to success is proportional to the quality and quantity of reserves while the cost of failure is limited via abandonment options that can be exercised at any point in the exploration sequence. This means that our investment theory, in calculating the expected payoff, must adequately take into account the nonlinearity of the payoff function that is generated via the abandonment option.

Fourth, exploration investments are unlike most other investment activities due to the long time frame between the expenditure of capital and the realization of revenues. An analysis of 54 major base- and precious-metal deposits around the Pacific rim by Stillhöhe (1995) reveals that the time from initial exploration spending to the discovery drill hole averaged 14 years for base metal deposits and 22 years for gold deposits. There is then an average of a further 13.5 years to first production for base metal deposits and 7 years to first production for gold deposits. That is, where exploration is successful there is an average of 27.5 years from initial spending to cash flow generation for base metal deposits. The average at gold deposits is 29 years. We do not know of any other private-sector economic activity with such a long lag between the start of investment spending and the beginning of income receipts. Under traditional discounting this distant income is worth almost nothing in present value terms at the start of an exploration program, causing Newendorp (1975) to recommend an undiscounted analysis of exploration cash flows. This is clearly unacceptable given that there is a cost of capital and investment risk. What we need is an investment model that can adequately discount far-off payments for cost of capital and risk without destroying their present value.

Finally, exploration investment is not investment in tangible capital such as buildings or machines. It is sequential investment in information, initially with the goal of locating a resource, and then with the goal of reducing the uncertainty over the quantity and quality of that resource to bring it into a reserve. Research and development (R&D) investment spending has this same characteristic. Models of R&D recognize that risk is not necessarily compoundable over time, that multiple expansion and abandonment options exist during the development path, and that the main payoff to investment is information (Paxson, 2003). Our model should have these characteristics, too.

The first and second facts of exploration, that it is a lumpý irreversible investment decision guided by expected payoffs, lend themselves to traditional discounted cash flow (DCF) analysis. However, facts three, four, and five—that exploration has nonlinear payoffs and abandonment options, has a long time frame between expenditure and cash flow generation, and has an outcome that is in intermediate stages updated information rather than cash flows—indicate that traditional DCF models are not likely to be suitable. An options framework, on the other hand, is a good candidate, as it deals explicitly with each of these
nuances of the economics of exploration. Instead, explora-
tion is an option on information, and it is difficult to
value options with anything other than option pricing the-
ory. Since at issue here are decisions over real assets, real
option theory is ideal.

Exploration has long been guided by economics (e.g.,
Grayson, 1969; Kuzfnan, 1963; Davis et al., 1975; Peters,
1978; Megfli, 1979). Expenditures in aggregate have been
observed to vary with mineral prices, exploration costs,
exploration success rates and mining firm profitability in a
way that is consistent with exploration activity being under-
taken in response to varying economic factors (Eggert,
Furthermore, considerable evidence exists that exploration
is an economic activity that provides a positive return on
investment (e.g., Mackenzie and Woodall, 1988; M.S.
Enders and R.A. Leveille, unpub. data, 2004; Scholte,
2004). The application of real options represents a natural
progression in the study of exploration and the enhance-
ment of this economic return.

Some Preliminaries

Prior to developing our model we review some prelimi-
naries for readers new to the concept of uncertainty and
real options. Other readers can skip directly to the next sec-
tion, where we present our analysis of the exploration
process.

The nature of uncertainty in exploration

Real options (RO) models always specify quite clearly
the nature of uncertainty to be considered. The two main
sources of uncertainty affecting exploration decision-mak-
ing are current geological uncertainty and future commod-
ity price uncertainty. Geological uncertainty is, in the
absence of exploration effort, constant through time. To
a small extent one can learn more information about a min-
eval lease or property by learning of exploration results
from surrounding leases. But for the most part, geological
information must be purchased, thus making learning
endogenous.

In the limit, it is technologically feasible to resolve most
of the geological uncertainty to this regard; it is possible to drill
the entire lease at 10m spacing, generating, for all intents
and purposes, an exact map of any mineralization within the
lease boundary. A more reasonable sequence of exploration
activities would be as follows: identify an area of likely min-
eralization, either through nature’s signals such as outcrops,
or through some preliminary information gathering such as
soil sampling and trenching; drill in selected areas in the
hope of further reducing rather than eliminating uncer-
tainty; and then make decisions based on this new informa-
tion. Broadly speaking, wherever there is motivation for
planned spending to reduce uncertainty, agents are weigh-
ing up where to spend and when to stop spending. These
decisions are based on the recognition that spending in
some areas is likely to create more valuable information
than spending in other areas, and that at some point an
extra dollar spent drilling does not produce an incremental
dollar’s worth of information. Exploration is, solidly, an eco-
nomic activity that produces information, monetized by cost-
benefit analysis, and that is amenable to analysis using fairly
simple and well-established economic tools.

Economic uncertainty, on the other hand, is partially or
fully resolvable only through waiting. This is called exoge-
nous learning. For example, the London Bullion Market
Association afternoon $US fixing price of gold on January
7, 2010, is uncertain as of writing this paper, but will have
less uncertainty tomorrow, much less uncertainty on Janu-
ary 8, 2010, and will be known with certainty on January 8,
2010. One can spend money now to try and forecast the
price on January 7, 2010, but no amount of money will
resolve the uncertainty. Other uncertainties that affect
the economics of a mine, such as mining cost inflation, envi-
ronmental and social cost requirements, and taxation struc-
tures, also become evident over time. This difference
between the two major areas of uncertainty (geological and
economic) explains why previous models have either dealt
with one or the other but not both.

The value/uncertainty relationship

Exploration creates information, and information has
value whenever it enables better decision-making. For
example, the more information about the reserve location,
grade, and tonnage at mine development, the more likely
that the mine and mill will be optimally sized, designed,
and used (Glavan, 1985; Gocht et al., 1988; Setith, 1997).
The value penalty arising from uncertainty in the geology
of a reserve is evident in the market. In the oil and gas
industry, developed and producing proven reserves have
drawn the value of developed and producing probable
reserves, and there is evidence that similar discounting
factors with mineral reserves (Dasis, 2005). Traditional eco-
nomic views of exploration, which see exploration as only
increasing geological assurance at a technical requirement
prior to extraction (e.g., McKeever, 1974), fail to recognize
this value/uncertainty relationship. In RO models, increas-
ing geological assurance not only moves resources between
the traditional inferred, indicated, and measured cate-
gories, but can turn a resource into what we call a reserve in
this paper simply by reducing geological uncertainty and
thereby assuring profitable extraction by increasing the
likelihood that optimal mine and mill location and decision
will be made.

Valuing future uncertain payoffs

The discounted cash flow (DCF) method is the most com-
mon method of calculating the net present value (NPV) of
the payoffs from investment, be they the value of informa-
tion gained within an exploration program or the payoffs cre-
ated by any other investment. This technique aggregates
time and risk adjustments into a single discount rate. That
discount rate is used to bring the future payoffs to a present
value so that their sum can be weighed against the present
cost of the investment. Unfortunately, the use of DCF dis-
count rates in the resource industries is often inherently
flawed since it is common practice to use a single constant

discount rate across a wide range of projects, with perhaps an arbitrary basis for early stage exploration projects. Most projects have widely varying risk characteristics so that different discount rates are required. Yet, there is no easy way to determine the required project-specific discount rate (Fama, 1977; Myers and Turnbull, 1977). Even within a given project, the use of a constant discount rate over all cash flow periods presumes that cash flow risk is increasing in a specified manner that is unlikely to match the nature of the risk in the project (Robichuel and Myers, 1966; Myers and Turnbull, 1977).

The RO method shares the same theoretical foundation as the DCF method but uses a risk adjustment that is unique to each project and the level of risk inherent in that project (Jacoby and Laughon, 1992; Salabos, 1998; Samis et al., in press). In particular, each source of uncertainty affecting a project net cash flow is identified, the net cash flow risk that this generates is priced using market information or financial models, and then the risk-adjusted net cash flow is adjusted for the time value of money. For example, if a net cash flow is affected by price uncertainty, the RO technique replaces that uncertain price stream with its certainty equivalent, in this case a market-derived forward price. Geologically uncertain and other similar manner. Because of their differing risk factors, copper projects are discounted at a different rate than gold projects, and early stage projects are discounted at a different rate from late stage projects (Laughon and Jacobs, 1993). Discount rates tend to decline for the more distant cash flows, resulting in even the distant positive cash flows of an exploration project having significant positive present value.

Modeling Resource Uncertainty and the Value of Information

In modeling and valuing exploration activity, current geological uncertainty and future economic uncertainty are intertwined. The value of drilling another hole now is affected by the expected price of the mineral several years in the future. This is the price that each unit of mineral found will ultimately fetch upon extraction. And the price of the mineral several years in the future depends on the supply forthcoming at that time, which depends on the amount of exploration activity now. Clearly this is a complex process, and one that will have to be simplified if we are to make any progress. As with earlier works (e.g., Kaufman, 1965; Newendorp, 1975; How, 1979; Ch. 10; Harris, 1990) we begin by taking prices as certain and focus only on initial geological uncertainties. We extend the earlier work by emphasizing the economics of exploration decisions. After developing this economic model of exploration decision-making, we add price uncertainty in the case study in the next section of the paper.

A model of resource uncertainty and the outcome of exploration

In this section of the paper we build a simple model of exploration through which particular management and evaluation insights can be revealed. Our statistical model of the information gained by exploration is based largely on Campbell and Lindner (1983, 1985) and Kaufman (1963). To simplify matters we will consider exploration activity at the level of the project. We assume that exploration is guided by rational decision-making aimed at spending dollars only when there is an acceptable expected return to doing so.

At each point in time the company or government weighs up taking some discrete exploration activity at a given prospect, comparing the costs associated with this activity against the rewards that the activity is likely to produce. Rewards come in the form of new geological information. In all cases information will have value, in the sense that it will create better decisions on subsequent spending on exploration or development than in the absence of that information. In some cases the information will reveal that it is optimal to terminate an exploration program because the information likely to be gained from the next stage of exploration is not worth the cost; the project will either be deferred or abandoned, or it will move into development.

In other cases the information will indicate that additional exploration spending is warranted. The value created by the improved information is discounted for risk and time such that it can be weighed against the exploration costs. Exploration creates wealth when the present value of expected benefits, appropriately discounted in a RO framework, exceeds the present value of expected costs.

To begin, consider a mineral lease that contains $R > 0$ units of economically recoverable mineral, though $R$ may not be large enough to warrant development of a mine and therefore may not be a reserve. The level of $R$ is unknown to the entity that owns the lease. Reconnaissance has revealed some information about the mineralization on the lease and is the basis for what we will call prior beliefs about the level of $R$. There is evidence that within a basin or geological region reserves have a log normal distribution (Kaufman, 1965; De-Goffrey and Wignall, 1985, p. 25-26; Harris, 1990, p. 300-301). A log normal distribution therefore seems to be a reasonable starting point when modeling geologists’ inferences about $R$. Let these prior beliefs therefore be the log normal distribution $\ln(R) \sim \ln(\mu_{R}, \sigma_{R})$, where $\mu_{R}$ is mean of the log of the beliefs and $\sigma_{R}$ is the variance of the log of beliefs. The value of $R$ is a random variable, and the 0 subscript indicates time, with current time being zero. Given the properties of log normal distributions, the expected or mean size of the economic mineralization at time zero is

$$\bar{R}_0 = \exp(\mu_{R} + 0.5\sigma_{R}^2).$$

(1)

The variance surrounding this estimate is

$$\text{Var}(\ln(R)) = \exp(2\mu_{R} + \sigma_{R}^2)[\exp(\sigma_{R}^2) - 1],$$

(2)

which has the desirable property that more promising prospects have more variance surrounding the estimate of economic mineralization; the coefficient of variation is constant for all deposit sizes and is equal to $(\exp(\sigma_{R}^2) - 1)^{1/2}$. Depending on the level of $\bar{R}_0$ and geological assurance sur-
ronding the estimate, $\bar{R}$ may define either a resource or reserve at this point. For conciseness, we assume that it is a resource due to a large Vari($\bar{R}$). The initial beliefs about $\bar{R}$ and Vari($\bar{R}$) contain composite information about the distribution of means of reported field sizes of reserves of this geological nature or area, the variance of possible field sizes associated with each mean, and information from early stage exploration (Kaufman, 1963). The initial resource estimate is almost certain to be wrong, with $\bar{R} \neq R$. Nevertheless, it is the best information available and must be used when making decisions about whether or not to continue exploring the property.

A tractable case treats the next stages of exploration as a series of discrete sampling events, $i = 1, 2, \ldots, n$, ranging from geological mapping $(i = 1)$ to pattern drilling $(i = s)$. For now we assume that all stages of exploration, if undertaken, must be completed sequentially with no option of going directly to development and no option for abandonment at any stage. If exploration is not undertaken the project must be abandoned. Only through exploration can enough information be gained for a warrant proceeding with project development should the resource become a reserve.

In essence, we assume that existing reserve uncertainty is too high to warrant moving directly to development drilling, and that management must decide whether or not to commit now to an all-or-nothing exploration program to prove the resource.

In a departure from most deterministic analytic exploration models, we treat exploration as Bayesian sampling from a continuous distribution rather than sampling from a single binomial or trinomial distribution.1 In this sense our model will take on the flavor of a real options model in which uncertain variables are modeled as continuous. Each stage of exploration generates an estimate $\bar{R}_i$ of the quantity of economic mineralization. These exploration outcomes randomly deviate from $R$ because there is known error or noise $\sigma^2$ associated with sampling, akin to a detection probability of less than 1 (De Geoffrey and Wignall, 1985).2 The level of $\sigma^2$ indicates the degree of sampling error in sampling stage $i$, typified modeled as being a decreasing function of exploration expenditure and exploration efficiency within the stage (De Geoffrey and Wignall, 1985, p. 75–76). Alternatively, stages with less sampling error could be treated as having a sample-equivalent size $s_i > 1$. Although $\sigma^2$ will probably decrease as each sampling stage becomes more targeted, to simplify matters we treat $\sigma^2$ as constant across each stage of exploration.

We noted above that $\bar{R}_i^2$ is the variance of the log of initial beliefs about the resource size. It is useful to think of $\bar{R}_i^2$ as hypothetically being generated by an initial stage of exploration given sampling error $\sigma^2$ in each stage, of $\bar{R}_i^2 = \bar{R}_i^2 + \sigma^2 \pi$.

1 It is common practice to model exploration within a Bayesian framework (see Kaufman, 1960; Peters, 1978, p. 53–549; Geha et al., 2001, 2002; Asmerrin et al., 2005). For example, of an Bayesian updating given discrete sampling outcomes in mining exploration, see Howe (1979, p. 219–239).

2 More sophisticated models treat the initial resource parameters and measurement error $\sigma^2$ as known (Kaufman, 1972).

(Kaufman, 1983, p. 162). That is, the initial resource uncertainty could equally be expressed as the outcome of $n$ previous stages of exploration, each stage identical to the stages of exploration to follow. Then, define

$$\bar{X}_i = \frac{1}{2} \log \bar{X}_0,$$

as the log mean of the succeeding stage subsequent sampling exercise. This new information is combined with the previous information about $\bar{R}$ to create an updated or posterior resource distribution $\bar{R}_i$.

A priori, an unbiased sampling exercise is expected to return the prior resource estimate, with post-exploration uncertainty around this estimate going to zero as the number of stages (exploration effort) goes to infinity. These properties of the sampling program are brought about by the Bayesian assumption that sampling error introduces an independent log normal process with known precision. In other words, sampling will have an independent log normal distribution $f_i(\mu, \sigma^2)$ with process mean $\exp(\mu + 0.5\sigma^2) = R$ and process variance $\exp[\exp(2\mu + \sigma^2) - \exp(\exp(\mu + \sigma^2))] - 1$. Kaufman (1963) shows that given this assumption the post-exploration resource distribution under Bayesian updating is $\bar{R}_i = \bar{R}_0 - \bar{R}_0^2 (\mu_0^2 / \bar{R}_0^2)$, where

$$\mu_0 = \exp(2\mu + \sigma^2) - \exp(\exp(\mu + \sigma^2)),$$

$$\alpha = \frac{n}{\mu^2 + \sigma^2}, \quad 1 - \alpha = \frac{n}{\alpha + n},$$

$$\bar{R}_i = \exp[\mu_0 + 0.5\sigma^2] = \exp(\mu + 0.5\sigma^2/n + \mu_0),$$

and

$$\text{Var}[\bar{R}_i] = \exp[2\mu_0 + \sigma^2] - \exp[\exp(\mu + \sigma^2)] - 1.$$

Equation (1) shows that the new information will be combined with the old via a linear weighing system. Equation (2) describes the weighing system, where more weight is placed on the sampling results when $n$ is larger and $s$ is smaller, perhaps due to a large sampling program or low sampling error. Equation (3) reveals that the new resource variance parameter is less than the initial variance parameter since $\sigma^2_i < \sigma^2_i/(n + n) = \sigma^2_i/n^2$. This is the essence of proving a resource. Equation (4) provides the new resource estimate, which will only change from the original estimate $\bar{R}_0$ if $\bar{R}_0 \neq \mu_0 + 0.5\sigma^2 \pi$. In other words, that is, if the log sample mean is different from the log of the initial resource expectation. Equation (5) is the variance of the revised resource distribution. Some (e.g., Haskett, 2003) argue that exploration should always decrease variance, whereas Galli et al. (2001) suggest that a variance

1 less sampling error will result in a $\sigma^2$,$\pi^2$ being smaller.
increasing with mean resource size is a desirable property of any sampling program. Equation (8) shows that the variance of the resource estimate can increase if the revised mean of the log of resources, μ_x, increases, consistent with Galli et al.

Compare these outcomes to popular decision-analytic models of exploration activity that assume that exploration reveals a binomial positive or negative outcome R_{low} or R_{exp} with totally resolved uncertainty. That is, sampling would predict $X = X_1 + X_2 + \ldots + X_n = R_{low}$ or $X = R_{exp}$ with $\sigma^2 = 0$ (or $\alpha = \pi$, $\tau = 0$, and $\delta^2 = 0$ in this model). Of course, in this case there would be no need for sampling stages $S$ through $n$, as all resource uncertainty would be resolved in stage 1. Another way of saying this is that sample outcomes are perfectly correlated or perfectly dependent. In our model, sampling instead updates the resource estimate $\hat{R}$ and the variance around that estimate without ever completely resolving uncertainty.

Exploration decision must be made prior to having the information resulting from the exploration program, and so for our analysis we must treat the exploration outcome $X_n$ as the random variable $\hat{X}_n$ with an anticipated mean $X_n$ (a mean of possible means). Given the prior resource distribution $f_n$ sampling (with error) is a priori assumed to be from a log normal process $f_n(R_n, \sigma^2)$. Since expected exploration payoffs are based on the mean or expected resource, it is the preposterior distribution of the possible posterior resource means $\hat{R}_n$ (see eq. 7) that is of interest. Let the distribution of these possible updated resource means be $f_n,R_n$. Then incorporate prior information $\hat{R}$ as well as the anticipated sampling outcome $X_n$. Kaufman (1983) shows that given $n$ stages of sampling, the preposterior mean resource quantity has a log normal distribution

$$\hat{R}_n = f_n(\mu_n + 0.5\sigma_n^2, m+n, \delta_n^2),$$

(9)

where

$$\delta_n^2 = \delta_1^2 + \delta_n^2; \quad \delta_n^2 = \frac{\sigma^2}{m} + \frac{\sigma^2}{n} + \frac{\delta_0^2}{(m+n)}.$$  

(10)

Equation (9) indicates that there are many possible resource quantities after exploration, only one of which will be realized. The distribution of these possible resource quantities is log normal. Equation (10) gives the variance parameter reflecting the range of possible resource quantities, with

$$\text{Var}(\hat{R}_n) = \exp(2\mu_0 + \delta^2)[1 - \exp(-\delta_0^2)] - 1.$$  

Comparing equation (11) to equation (2), we can see that exploration is expected to reduce the variance around the resource estimate since via equation (10) $\delta_n^2 < \delta_0^2$

Kaufman also shows that the anticipated mean of the updated resource estimate is

$$\hat{R}_n = \exp(\mu_0 + 0.5\sigma^2) \cdot \exp[m \mu_n + 0.5\delta_n^2].$$

(12)

Exploration should, if it is unbiased, not be expected to change the initial resource estimate. From equations (12) and (1), as desired. Endogenous or controlled learning that is not expected to change the prior belief about the mean of a random variable, as is the case here, is called pure learning (Munzuurk, 2003), and is often used to model the R&D process within firms. We mentioned the similarity between exploration and R&D above.

Calculating the value of exploration information

This statistical model of geological uncertainty resolution, which would seem to model real-world attributes of exploration and information updating quite well, is at the heart of the exploration program. Preposterior beliefs are described in equations (9) through (13) can now be used to generate expected benefits of the exploration sequence, which in turn are weighed against the costs of the sequence. If the anticipated benefits are greater than the anticipated costs, all in present value terms, the exploration program creates wealth and will be undertaken. If the anticipated costs outweigh the anticipated benefits, all in present value terms, the property will be abandoned. We will now show that although exploration is not expected to increase resources (eq. 13) it can nevertheless be valuable, even where the initially estimated resource $\hat{R}_0$ is too small to warrant mine development should more resource not be found.

Adding the economics to the model, let the present value of the expected cost of the n-stage exploration program be $C_n(R_n, \sigma^2)$, where cost is increasing with the length of the exploration program for and the initial resource estimate $\hat{R}_0$ and decreasing in the most associated with each sampling stage $\sigma^2$ (more detailed exploration is more expensive). Upon completion of the program the property may be developed if the resource becomes a reserve or abandoned if the resource is uneconomic. Given the prevailing mineral price, let the time $t$ NPV of this option to develop the property after exploration be $NPV(R_n, \delta_n^2) \geq 0$, where the option value is increasing in posterior resource size $\hat{R}_n$ and decreasing in posterior resource uncertainty $\delta_n^2$. The value discounted associated with $\delta_n^2$ is not only a result of risk aversion as in Campbell and Linden (1985), but also due to the fact that ex post facto suboptimally sized mines and mills are inefficient and can only be made more efficient through expensive expansion or contraction expenditures. Again, information has value.

Define resource quantity $y$ as the minimum that will lead to exercise of the mine development option upon completion of exploration. For a guideline on representative values of $y$ over various mineral types, see Goeh et al. (1988, p. 112), although $y$ will vary with resource quality and commodity price. The gross payoffs of the exploration program are now as shown in Table 1. If after exploration the updated resource size is $\hat{R}_n \geq y$, the company or government abandons the property and $NPV(R_n, \delta_n^2) = 0$. If, on the other hand, after exploration the updated resource is $\hat{R}_n < y$, $\hat{R}_n$ becomes a reserve and the firm or government will exercise the option to develop the property, gaining a
TABLE I. Gross Payoffs from Exploration

<table>
<thead>
<tr>
<th>Posterior resource size</th>
<th>Gross payoffs for exploration</th>
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</thead>
<tbody>
<tr>
<td>$R &gt; \delta$</td>
<td>$\text{NPV}(\delta, \delta R)$</td>
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project value of $\text{NPV}(\delta R, \delta R) > 0$. The gross project payoff is thus

$$\max(\text{NPV}(\delta R, \delta R), 0) > 0,$$  \hspace{1cm} (14)

which is nonlinear (convex) in $\delta R$. The present value of the net exploration payoff can be represented as

$$\mathcal{P}(\theta) \max(\text{NPV}(\delta R, \delta R), 0) - C(\theta, \delta R, \sigma^2),$$  \hspace{1cm} (15)

where $\mathcal{P}(\theta)$ is a discount factor $e^{-\alpha t}$ that takes into account only the time value of money. We only discount for the time value of money because at the point of making the development decision resource risk has been reduced to $\delta R$, and the value effects of this remaining uncertainty have already been taken into account in the calculation of $\text{NPV}(\delta R, \delta R)$. The time discount factor $\mathcal{P}(\theta)$ is increasing in the number of stages of exploration since more exploration takes more time; discounting increases. However, even if the exploration program will take decades, the discount factor $\mathcal{P}(\theta)$ does not reduce the present value of the ultimate payoff to almost zero, as it would if it included a risk premium, since it only adjusts for the cost of money.

An example will illustrate the model. Consider a small, late-stage gold project with an anticipated $50$ M development cost after a stages of exploration based on the initial resource estimate $\bar{R}_0$. Assume a five-stage exploration program ($n = 5$) that will take five years to complete and a continuously compounded time discount factor of $r = 5$ percent per year. With these parameters, $\mathcal{P}(\theta) = 0.78$. Assume that the present value of exploration costs is $C = \$10$ M, probably reasonable for a property at this stage of development given that Sphorle (2001) calculates a gold finding cost of $\$30$/oz through all stages of exploration. If $\delta R = 75$ oz and $\sigma^2 = 1.5$, then $\mathcal{P}(\theta) = 0.78$.

This reveals that there will be remaining resource uncertainty after exploration, although it is less than initial resource uncertainty. Assume that this remnant uncertainty is low enough that any economic mineralization can be mined, with a post-exploration resource value of $\$50$/oz given price and extraction cost expectations.

1 A traditional risk-adjusted discount rate of $15\%$ would reduce $\mathcal{P}(\theta) = 0.67$, only reducing payoffs by $9\%$ if exploration were to take 20 years, $\mathcal{P}(\theta) = 0.78$ discounting only for time, compared with a $\theta$ of $0.05$ in a 15\% discount rate.

2 These statements produce an initial resource coefficient of variation of 1.057, after a deposit with a price expected resource of $\$50$ M has a 0.5\% chance of being greater than $\$50$ M, a 30\% chance of being between $\$50$ M and $\$50$ M, a 63\% chance of being less than $\$50$ M. The posterior resource estimate will have a coefficient of variation of 0.815 due to the reduced uncertainty from sampling.

The task is now to manage and value the option to invest in exploration, as at stage zero. Traditional BCA analysis would estimate the anticipated resource quantity after exploration $\bar{R}_0$ which, from equation (15), is the same as the prior resource estimate $\bar{R}_0$. Calculate the anticipated gross exploration payoff after stage $\theta$ is

$$\max(\text{NPV}(\delta R, \delta R), 0) - C(\theta, \delta R, \sigma^2),$$

discount that anticipated payoff for time and resource estimation uncertainty $\varphi(\delta R)$, and deduct the present value of exploration costs. If we follow the standard finance assumption that resource uncertainty is unsystematic and can be diversified away and that the firm or government is risk neutral, the decision-maker can make a decision based on pure discounted expectations:

$$\text{DCF} = \mathcal{P}(\theta) \max(\text{NPV}(\delta R, \delta R), 0) - C(\theta, \delta R, \sigma^2).$$  \hspace{1cm} (16)
Under the traditional DCF framework, exploration will proceed when the ordered pair \((R_0, \delta_0)\) produces a positive DCF. Restating the numerical example, DCF = 0.78 max(550, R_0 - 800, 0) - 1000R_0. Line (3) in Figure 1 shows DCF values for prior resource estimates \(R_0\) of between 50,000 oz and 2 Moz. Exploration will proceed for any initial resource estimate of greater than 1.54 Moz since DCF > 0 for these resource quantities. For initial resource estimates of less than 1.34 Moz, the DCF of proceeding with exploration is negative, so the property will be abandoned.

In this DCF decision framework, projects where the probability of success is in 1 or 10 or less, as is the case when initial resources are estimated at 500,000 oz or less, are far from satisfying the hurdle to go forward with exploration. This proves problematic with intuition and evidence that exploration is often warranted when the chance of success is in that order. The essential problem with this DCF calculation approach is that the nonlinear (convex) payoff structure is ignored. This is remedied by the more comprehensive real options approach. Let \(h(R_0, \delta_0)\) represent the postproject log normal distribution of expected resource quantities upon completion of exploration given the prior resource estimate parameters \(\mu_0\) and \(\delta_0\). where

\[
P(R_0 | \delta_0, \mu_0 > \gamma) = \int_0^{\gamma} p(R_0 | \mu_0, \delta_0) dR_0
\]

and

\[
P(R_0 | \mu_0, \delta_0 > \gamma) = \int_{\gamma}^{\infty} p(R_0 | \mu_0, \delta_0) dR_0
\]

This last expression represents the probability that the resource will become a reserve after \(n\) stages of exploration conditional on \(\mu_0\) and \(\delta_0\). The preproject real option value of expected benefits from exploration is

\[
\beta(\delta_0) \max(NPV(R_0 | \delta_0, \mu_0), 0) h(R_0 | \mu_0, \delta_0) dR_0
\]

and the stage zero real options expected exploration property value is

\[
RO_0 = \beta(\delta_0) \max(NPV(R_0 | \delta_0, \mu_0), 0) h(R_0 | \mu_0, \delta_0) dR_0
\]

Exploration will go ahead whenever \(RO_0 > 0\) conditional on the ordered pair \((\mu_0, \delta_0)\). When max(NPV(R_0 | \delta_0, \mu_0), 0) is convex in \(R_0\), which is the case whenever \(\gamma > 0\), the value of equation (18) will be greater than the value in equation (16) via Jensen’s inequality, and the resource size cutoff for continued exploration will be smaller than under the DCF approach, possibly even smaller than \(\gamma\). This is a key insight from real options analysis.

For the numerical example above the real option exploitation value, \(RO_0\), is depicted in Figure 1 as line (4). While the log normal distribution of preproject resource estimates lends itself to the potential for closed-form solutions for \(RO_0\), we estimate \(RO_0\) using a Monte Carlo simulation over 50,000 trials. All positive values on line (4) reveal the value generated by exploration, for without exploration all of this value would be lost (the project would be abandoned at stage 0). Now the exploration program will go ahead for any initial resource estimate greater than 500,000 oz, rather than just \(\gamma = 1.34\) Moz cutoff under traditional DCF analysis. If the current resource estimate is less than 500,000 oz the project will be abandoned, since at these levels going ahead with exploration costs more than the present value of any revenues that are likely from mining ultimate reserves. A 1.5 Moz resource has a current value of \(\$10.9\) Moz, more than double the estimate using traditional DCF analysis. A 950,000 oz resource is now worth \$0.6 Moz, compared with being worthless via the traditional DCF approach. Since we have used the same discount factor \(\beta(\delta_0)\) in both the DCF and real options calculations and valued stage \(n\) mineralization in both cases at \(\$0.6\), the difference between the line (4) and line (3) valuations in Figure 1 is solely due to the real options recognition that decisions about whether or not to develop the project are made conditional on the outcome of the exploration program, an option to develop, whereas the traditional DCF approach assumes that the decision is made at time \(t\), based on \(R_0\).

Of note, carrying out exploration at a \(R_0 = 910,000\) oz deposit has a 10% chance of exploration success, success defined as proving an expected 1 Moz or more. Exploration is nevertheless economics for properties with anything more than 900,000 oz of resources at stage 0 because of the high payoffs of an unexpected but large find, with the downside limited to exploration costs. The real options model is showing this and allowing for speculative exploitation, whereas the traditional DCF model does not.

A Generalized Model of Exploration

The above analysis, while depicted in Figure 1, for the specific parameter values given in the example, is completely general. Given any gross payoff function, line (1), that is convex in current resource quantity, Jensen’s inequality will cause the real option resource value line (4) to be a smooth and strictly convex continuous function everywhere above line (3), the DCF resource valuation line.

The real option value line is anchored at the origin and asymptotically approaches line (5) as initial resources become infinite. The exploration cutoff resource quantity will always be lower in the real options analysis than in the traditional DCF analysis, and the exploration project value will always be higher. Most significantly, exploration can be justified where initial resource estimates are less than required to support a mine, \(R_0 < \gamma\).

In addition, through some comparative static exercises the real options model provides rich insights into factors influencing exploration decisions:

---

Footnotes:

1. As an initial resource estimate of 560,000 oz, there is a 0.00972 chance that the posterior resource will be greater than 1 Moz given our model parameters.

2. Compared to \(\gamma\), the DCF benefit estimate of \(\beta(\delta_0)\max(NPV(R_0 | \delta_0, \mu_0), 0) h(R_0 | \mu_0, \delta_0) dR_0\).

3. By practice, the value calculated using the traditional DCF approach would probably be even lower than \(\gamma\) since this approach typically does not consider remaining stage \(n\) resource risk more heavily than in our treatment.
1. Exploration project value is not monotone in initial resource quantity. Lines (3) and (4) are anchored at the origin and slope downwards and then upwards as initial resources increase (Fig. 1). Traditional and post-discovery valuation shows the worst, or most unfavorable, project, i.e., the lowest point on line (4), to be that with an initial resource estimate of more than \( y \). Real options show the most uncertain project, the lowest point on line (4), to be that with an initial resource estimate of less than \( y \). Real options are thus more optimistic in terms of small-scale resources than line (4) shows real option exploration project value to be less sensitive to resource size than traditional DCF exploration project value shown as line (3).

2. Additional stages of exploration always provide gross value through the value of information effect. Equation (6) shows that additional stages of exploration lower \( D_f \) which we have assumed will increase NPV (\( R_B, D_f \)) for any \( R_B \). This causes the shift left and slope of the upward-sloping part of line (3) to increase. In addition, equation (10), more sampling stages increase the variance of the possible posterior resource quantities. This is a result that is not picked up in the traditional DCF analysis. Both effects will cause the real option value of the exploration property to increase: the first effect causes line (4) to rotate upwards, while for second effect causes the rotated line (4) to be less convex. Exploration creates value even though it is not expected to increase the quantity of the resource.

3. There is an optimal amount of exploration to commit to at stage zero. An increase in exploration effort increases COs (\( R_B, \sigma \)), causing a downward rotation in lines (3) and (4) that offsets the gains outlined in the preceding paragraph. As a result, there will be a value-maximizing exploration effort \( \sigma \) that will cause the optimal exploration effort, and the payoff value function NPV (\( R_B, D_f \)), and initial resource beliefs.

4. Higher sampling precision, if \( \sigma > 0 \), will increase exploration activity and the value of exploration. Higher sampling precision lowers \( \sigma \), and, via the specification \( D_f = \frac{1}{\sigma^2} \), lowers \( k \). From equations (10) and (11) this causes \( \Pi = \frac{1}{k} \) to increase, which line (4) being less convex. The amount of and value of exploration projects will increase. The effects here are very similar to an increase in the number of sampling stages. Indeed, increased sampling precision and increased sampling size are imperfect substitutes in exploration value generation. Traditional DCF analysis would not pick up this effect.

5. Lower exploration costs increase exploration activity and the value of exploration. Lower exploration costs will rotate lines (3) and (4) upwards. The amount of and value of exploration projects will increase.

6. Higher margins upon extraction increase exploration activity and the value of exploration. Higher margins shift \( y \) to the left and make the upward-sloping portion of line (3) steeper, increasing the amount and value of exploration projects. The impact of higher margins on exploration is not identical to a decrease in exploration costs, but has the same broad effects. The impacts of mineral price on exploration activity have been noted (R.C. Schodde, unpublished data, 2003, Natural Resources Canada, 2004).

7. Higher interest rates will reduce exploration activity and lower the value of exploration. Higher interest rates will increase the \( k \) of discount factor, rotating the upward-sloping portion of line (3) clockwise. The resulting line (4) clockwise. Exploration activity will decrease, and the value of any given exploration project will also decrease. This mechanism may not be affected by the change in interest rates. If they are, the impacts of interest rates will be combined with the previously noted impacts of prices (paragraph above) and costs (paragraph 5 above) on exploration.

8. Higher initial resource uncertainty is not necessarily beneficial to exploration project value. Initial resource uncertainty is parameterized as \( D_f \). In examining the effects of an increase in \( D_f \) we wish to preserve the initial resource estimate \( k \) which means that via equation (1) an increase in \( D_f \) must be offset by a decrease in \( k \). That is, \( k \) in equation (5) must be held constant. Equations (10) and (11) reveal that such a compensated increase in \( D_f \) results in an increase in \( \Pi \), with positive valuation impacts as in paragraph (4) above. But, equation (6) reveals that an increase in \( \Pi \) (a decrease in \( k \)) will increase \( D_f \), remaining resource uncertainty after exploration. This will lower NPV (\( R_B, D_f \)). The ultimate impact on exploration property value is ambiguous.

To show this using the numerical example, assume that \( D_f \) the variance parameter on the initial resource estimate, rises from 0.75 to 1.50, with \( k \) consequently falling from 2 to 1. This increases the coefficient of variation on initial resources to 1.866 from 1.057. From equation (6) \( \Pi \) rises from 0.3857 to 0.214. We assume this increase in \( \Pi \) lowers developed resource value from \$50 to \$40 due to the increase in post-tax opportunity cost of the likely design project with an increased \( D_f \). Exploration cost \( C \) remains unchanged. Figure 2 plots the original (lines 3 and 4) and the one with increased \( D_f \) (2) traditional DCF and real option valuations. Increasing initial resource uncertainty has increased exploration project real option value for projects of above \$45,000, while decreasing exploration real option project value for projects of greater than about \$1.5 Moz. A 900,000 oz project has increased from a real options exploration value of \$45,000 to a value of \$145,000, while a 1.1 Moz project has decreased in value from \$5 to \$2.4 M. The observation that increasing uncertainty does not always benefit option holders has been pointed out by Davis (2002). In that paper, as in this model, increasing uncertainty is most likely to be beneficial for out-of-the-money options, or, in Figure 2, initial resource estimates to the left of \( \Pi \). Of course, in traditional DCF analysis, increasing uncertainty always reduces asset value (line 1 in Fig. 2 is everywhere on or below line 3). Yet an exploration property with 900,000 oz of certain resource is clearly less valuable in our example than a resource that has a mean of 900,000 oz and some variance around that mean that allows for reserves to be discovered. It is only through uncertainty, the possibility of ultimate resources being greater than 3 Moz in this case, that exploration becomes valuable at sites with small resource estimates.
Fig. 2. Exploration projects with initial resource uncertainty is high. Higher resource uncertainty causes to make post-exploration resource less valuable, increasing the post-exploration resource value necessary for economic extraction from $5 per oz. Line (1) represents the no discounted pay-off from exploration under higher initial resource uncertainty. Line (5) represents the original discounted pay-off from exploration, which is the same as line (3) in Figure 1. Line (2) represents the new real option value of the exploration project, and line (4) represents the original real option value of the exploration project, which is the same as line (3) in Figure 1. Increased initial resource uncertainty has lowered the traditional exploration property value, since line (1) is everywhere below line (3). The real option value is higher under increased uncertainty when initial resources are above about 1 Moz, and lower when initial resources are above about 1 Moz.

While our model of exploration and learning is very simple, all of these insights are in accordance with what we would expect. Our main result is that exploration can have value even when the traditional DCF value of the investment is negative, but only for resource quantities above some lower bound; exploration of all projects regardless of initial resource expectations is not warranted. The framework also reveals that over time, as exploration costs rise, exploration will become less valuable and more likely to be curtailed on properties where resources are low. On the other hand, mineral price rises will tend to offset the effects of the higher exploration costs. We have modeled the real-world realization that increasing exploration precision and effort provides value that can offset its cost, and that exploration effort will respond to changes in macroeconomic variables such as exploration costs, mineral prices, and interest rates. Finally, those holding leases with low initial resource estimates would want higher initial resource uncertainty, whereas those holding high quality leases would not.

The model has an additional use. Having built a model of late-stage exploration project value, reconsideration optimization now becomes possible given payoff line (4) (Fig. 1). To date such payoffs to reconsiderance have been presented qualitatively (Hrprovsky, unpub. data, 2004; R.C. Schodde, written comments, 2004) with a focus on estimating a minimum and maximum target size for reconsiderance. We now have a way to model the payoffs quantitatively.

Managing the Exploration Program earlier

Price and Resource Uncertainty

While informative, the analysis to date has ignored economic (price) uncertainty and an important management alternative, the option to defer late-stage or even initial exploration in low-price environments. Project deferral can be a valuable alternative to project abandonment.

As noted earlier, price and reserve uncertainty are rarely modeled together, and exploration deferral is even more rarely modeled. To our knowledge the only other paper to model both reserve and price uncertainty and the option to defer exploration is Cortazar et al. (2003). In that model, there are 11 possible resource outcomes, $R$. Initially, there is a price expectation over these resource outcomes, $R_e$. Exploration over four stages lasting a total of eight years serves to resolve all uncertainty over $R$. A single poor exploration outcome results in project abandonment, whether this is opti- mal or not. Exploration can be costly and perpetually deferred at any stage pending improvement in the economic environment. Under these assumptions Cortazar et al. find considerable value to deferring exploration in low price environments. In their worked example, a $1,000,000 post-development prospect with a current NPV of $-629,000 M has a real option value of $33 M dollars, with the option to defer exploration contributing $1.5 M to the $33 M.

In this section we augment our model via a real-world case study that considers reserve uncertainty and price uncertainty. Our example and treatment is similar to that of Cortazar et al., only we have five possible discrete levels of $R$, assume that there is still uncertainty over $R$ after exploration is completed, allow for repeated exploration upon a negative sampling outcome (since exploration information is imperfect), explicitly model resource updating in a Bayesian framework given exploration measurement errors, impose costs while deferring exploration, and impose finite time periods over which exploration can be deferred. We use a discrete resource distribution because this is how the participants in the case study thought about possible exploration outcomes. While the original case study involved gold, we also fabricate a hypothetical copper exploration example because copper prices exhibit revision to a long-term equilibrium price, whereas gold prices do not (Schwartz, 1997). This difference leads to markedly different exploration decisions in the various price environments.

Exploration base background information

A company wishes to value and manage a geological anomaly that has been identified by an aerial geophysical survey during a greenfields exploration program. The geographical region in which the anomaly is located is large and has seen some previous exploration activity. A government geological database is publicly available with over 700 entries identifying various regional geological anomalies, the extent of exploration, and the results of exploration to date at each anomaly. Company geologists also have long exploration experience in the region. Based on this information, the senior geologist within the company grouped the possible deposit outcomes for the anomaly into five orebody size classifications: (1) a world-class (WC) deposit $R_m$ with a probability of 0.15 percent; (2) a large-scale (LS) deposit $R_l$ with a probability of 0.35 percent; (3) a mid-sized (MS) deposit $R_m$ with a probability of 2.0 percent;
(4) a small-sized (SS) deposit in the case of a probability of $5.0$ percent; and (5) a waste deposit (reserves of less than $7$) at a minimum size needed to mine (the resource) with a probability of $92.5$ percent. Tables 2a and 2b list the geological and economic characteristics of each deposit size for each metal. The probabilities of each deposit size reflect the priors that will inform the analysts, a discrete version of the continuous distribution $\pi (r, c, d)$. The price presented above is the given absolute probability and assuming no economic data and in the case of the small deposit, the gold lease has a time zero. Resource $\ell_1$ of 100,000 oz and the copper lease has a resource size $\ell_2$ of 0.051 billion lb. While these estimates are by nature subjective, they are held to represent an accurate representation of knowledge about the orebody given the information at hand. Exploration sampling will then serve to update these priors as in equations (4) through (8), with more weight being placed on the exploration outcome: (i) the less the exploration measurement error, and (ii) the less the certainty over the original priors. At this stage, for instance, we would expect considerable weight to be given to the exploration outcomes. The NVP values presented in the last column of Tables 2a and 2b are calculated using the traditional DCF cash flow approach that is common in the mining industry. The gold project NVP values are calculated with a forecast gold price of $400/oz and a risk-adjusted discount rate of 8 percent/year continuously compounded. The copper project NVP values are calculated with a forecast price of $100/ton and a discount rate of 12 percent/year continuously compounded. These NVPs assume that the stages of exploration are complete and that a particular deposit type with reserves $\ell_1$ denoted in columns 4 of Table 2a and 2b has been discovered. The expected gross exploration payoff, NVP $(\ell_2, \ell_3)$, is roughly $1$ million for the gold case and $6$ million for the copper case. Prior to development the project is also subject to a feasibility study. That study costs $800,000 plus a variable cost proportional to the expected capital cost of the project. For example, for the $\ell_1$ in Table 2a and 2b, the expected feasibility cost is $27$ million.

A staged exploration program and management operating decisions

The company's geologists have developed a three-stage ($n \times 3$) sequential exploration program for the anomaly, each stage being one year in length. Exploration costs are incurred at the start of each stage. Exploration results upon completion of each stage are categorized as good, fair, or poor. This staging outcome is then combined with the prior information about the deposits to obtain an updated deposit description in a Bayesian framework described in detail below. The first stage of exploration costs $1$ million and consists of surface geological sampling, mapping, and reverse circulation drilling. The second stage of exploration costs $2$ million and involves detailed geochemical sampling and diamond drilling. The third stage of exploration costs $5$ million and includes further geological study, extensive diamond drilling, and specialized petrologic interpretation. Upon completion of exploration the firm must immediately either begin development or abandon the property. Managers will elect at the beginning of each stage to proceed with exploration, to defer the next stage of exploration for two years at a cost of $75$ thousand annually, or to allow the lease to lapse at no cost. If exploration is deferred, the exploration managers must decide at the end of a two-year deferral period to either abandon the lease at no cost or to start the next stage of exploration. Figure 5 delineates the exploration decision tree for both the gold and copper leases. The immediate decision is whether to proceed with exploration. Certainly, with cumulative exploration costs of $6$ million, an expected exploration cost of $2.5$ million, and expected gross exploration payoffs of roughly $1$ million for the gold property and $6$ million for the copper property, the

<table>
<thead>
<tr>
<th>Deposit type</th>
<th>Size (million tons)</th>
<th>Grade (g/t)</th>
<th>Gold (Moz)</th>
<th>Capital expenditure ($ million)</th>
<th>Development length (years)</th>
<th>Annual production (million t/year)</th>
<th>Life (years)</th>
<th>Unit operating cost ($/t)</th>
<th>DCF NPV ($ million)</th>
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<tr>
<td>World class</td>
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<table>
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<th>Deposit type</th>
<th>Size (million tons)</th>
<th>Grade (%)</th>
<th>Copper (billion lb)</th>
<th>Capital expenditure ($ million)</th>
<th>Development length (years)</th>
<th>Annual production (million lb/year)</th>
<th>Life (years)</th>
<th>Unit operating cost ($/lb)</th>
<th>DCF NPV ($ million)</th>
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| Table 2a. Characteristics of Possible Gold Projects |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Deposit type    | Size (million tons) | Grade (g/t) | Gold (Moz) | Capital expenditure ($ million) | Development length (years) | Annual production (million t/year) | Life (years) | Unit operating cost ($/t) | DCF NPV ($ million) |
| World class     | 720              | 1.00       | 23.20     | 720                           | 4                        | 40.0                              | 18         | 248                   | 8.0             |
| Large           | 100              | 1.20       | 6.20      | 300                           | 5                        | 16.0                              | 10         | 210                   | 8.1             |
| Midsize         | 36               | 1.30       | 1.50      | 120                           | 5                        | 6.0                               | 6          | 199                   | 8.5             |
| Small           | 6                | 1.40       | 0.27      | 28                            | 2                        | 1.5                               | 4          | 277                   | 10.7            |

| Table 2b. Characteristics of Possible Copper Projects |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Deposit type    | Size (million tons) | Grade (%) | Copper (billion lb) | Capital expenditure ($ million) | Development length (years) | Annual production (million lb/year) | Life (years) | Unit operating cost ($/lb) | DCF NPV ($ million) |
| World class     | 720              | 0.76      | 12.05             | 720                           | 4                        | 40.0                              | 18         | 0.48                  | 8.0             |
| Large           | 100              | 0.92      | 3.24              | 300                           | 3                        | 6.0                               | 10         | 0.40                  | 8.1             |
| Midsize         | 36               | 0.56      | 0.76              | 120                           | 3                        | 6.0                               | 6          | 0.39                  | 8.5             |
| Small           | 6                | 1.00      | 0.13              | 28                            | 2                        | 1.5                               | 4          | 0.48                  | 10.7            |
economics are not favorable at first appearance. However, as we will show, the geological and price uncertainty, combined with the ability to abandon exploration should either price or reserves drop, provides these properties with enough real option value to warrant at least the first stage of exploration in most price environments.

**Geological uncertainty and information updating.**

Completing an exploration stage provides information that allows exploration managers to reassess whether it is beneficial to continue exploring. As in our earlier model, exploration provides imperfect information and does not resolve all uncertainty about the deposit. Figure 4 outlines this partial resolution of geological uncertainty over the life of the exploration program. Updating of deposit probabilities starts after the first stage of exploration, where stage 1 exploration moves the program to one of the outcomes in the second column in Figure 4. By the end of the program a final set of deposit-type probabilities exists, with resolution still imperfect (Δ > 0).

As in the first part of the paper, the updated probabilities at each stage are calculated with Bayes Theorem. Given the discrete nature of the deposit sizes in this example, the discrete version of Bayes Theorem gives the posterior probabilities:

\[
P(\text{Deposit} \mid \text{EA Outcome}) = \frac{P(\text{EA Outcome} \mid \text{Deposit}) \cdot P(\text{Deposit})}{\sum_j P(\text{EA Outcome} \mid \text{Deposit}_j) \cdot P(\text{Deposit}_j)}
\]

The index "\(j\)" indicates the type of deposit among the total number of deposit possibilities, \(n = 5\). The index "\(\bar{j}\)" indicates the exploration activity outcomes (EA Outcome) for a particular exploration stage. Each stage of exploration has three possible outcomes (\(j = \text{poor, fair, and good}\)), \(P(\text{Deposit}_j)\) is the probability that a particular deposit may occur given the current information available at the start of the exploration stage. When there is exploration measurement error, \(\sigma^2 > 0\) and \(P(\text{Deposit} \mid \text{EA Outcome})\) is a (0.3) due to error in the signals from exploration.

The numerator of the right-hand side of Bayes Theorem represents the joint probability that Deposit and Exploration Outcome both occur at an exploration stage, given past exploration outcomes. The first term of numerator, \(P(\text{EA Outcome} \mid \text{Deposit})\), is an indication of the quality of information provided by the current exploration stage and the exploration program as a whole. This is where
the geologist's confidence in the exploration measurements is injected into the model. This term stipulates the probability that the exploration stage would return a particular outcome given that a specific deposit is actually there.

Determining the prices $P(D|r)$ requires professional geological opinion from someone familiar with the geological setting and exploration program. As we noted before, the information is imperfect, but it is the best available information upon which to make a decision. The information is, of course, improved as exploration proceeds, which is the value of exploration activity.

Developing an exploration outcome probability tree for a multi-stage exploration program is not a simple task. Figure 4 illus trates this, despite extended effort working with geologists' expectations about this deposit, the tree still contains a few inconsistencies, even though by construction the expectations are unbiased. For example, after a fair-stage 1 result and a good stage 2 result the probability of a world-class deposit occurring is 1.2 percent, higher than the 1.1 percent probability of a world-class deposit occurring after a good stage 1 result and a good stage 3 result. While it would be reasonable for a fair-good sequence to yield a more optimistic resource view than a good-fair sequence, due to the superior information from stage 2 exploration, a fair-good sequence must lead to better prospects than a fair-good sequence. Geologists valuing an exploration program must imposing considerable effort to ensure that such inconsistencies are removed from the exploration tree.

**Mineral price uncertainty**

We model gold and copper prices over the life of the exploration program by one-factor geometric Brownian motion (so the case-gold), and mean reverting (in the case of copper) stochastic processes. Details of these processes can be found in Laughton and Jacoby (1995), Salber (1998), and Sams (2000). The parameters used to construct these price models are presented in Table 3.

Figure 5 depicts the time zero gold price forecast and forecast confidence boundaries for the next 10 years when the spot price of gold is $400/oz. The gold price model incorporates a flat expected nominal gold price, reflecting a lease rate equal to the bond rate of 5 percent. The gold price model has two important characteristics. First, gold price uncertainty increases with the forecast horizon. This is highlighted by confidence boundaries that continue to move apart. Second, a price shock results in a shift of price.

$^*$Gold lease rates are usually less than the bond rate. Our parameter values are chosen to be consistent with the price forecast that we observe practicing using in DCF calculations.

<table>
<thead>
<tr>
<th>Stage 1 Exploration</th>
<th>Time = 8 years</th>
<th>Stage 2 Exploration</th>
<th>Time = 5 years</th>
<th>Stage 3 Exploration</th>
<th>Time = 3 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>WC</td>
<td>Medium</td>
<td>WC</td>
<td>Medium</td>
<td>WC</td>
<td>Medium</td>
</tr>
<tr>
<td>MS</td>
<td>Small</td>
<td>SS</td>
<td>Small</td>
<td>SS</td>
<td>Small</td>
</tr>
<tr>
<td>W</td>
<td>White</td>
<td>W</td>
<td>White</td>
<td>W</td>
<td>White</td>
</tr>
<tr>
<td>Exploration result</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>Great</td>
<td>F</td>
<td>Fair</td>
<td>C</td>
<td>Poor</td>
</tr>
<tr>
<td>P</td>
<td>Poor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4. Resolution of exploration uncertainty.** Undergoing a stage of exploration takes one year and results in either a good, poor, or no exploration outcome. The boxes hold the deposit probabilities at the start of each exploration stage.

$,$1, 2, 3 conditional on previous exploration results. Each exploration outcome serves to update the assessed probabilities of the resource being a certain deposit type. In a break with most models of exploration, uncertainty over the deposit type remains even after the 5th stage of exploration. The final probabilities over the five deposit types for the 27 possible-stage exploration outcomes are not shown.
Table 5. Parameters for the Gold and Copper Price Processes

<table>
<thead>
<tr>
<th>Price model parameter</th>
<th>Gold</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current market price ($)</td>
<td>400</td>
<td>1.00</td>
</tr>
<tr>
<td>Long-run price median ($)</td>
<td>N/A</td>
<td>0.05</td>
</tr>
<tr>
<td>Current of the price median</td>
<td>0.72</td>
<td>0.00</td>
</tr>
<tr>
<td>Price volatility (%)</td>
<td>12</td>
<td>23.3</td>
</tr>
<tr>
<td>Correlation between metal and market uncertainties:</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>Price of market risk, P(Risk)</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Ratio of mean reversion (half-life, years)</td>
<td>N/A</td>
<td>1.875</td>
</tr>
<tr>
<td>Nominal risk-free rate (%)</td>
<td>5.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

expectations. This is illustrated (Fig. 5) in a scenario where the gold price decreases from $400/oz to $320/oz over the first two years of exploration. The revised price forecast and confidence boundaries due to the 20 percent price shock are outlined starting in year two. The real options model takes all such price scenarios and revised expectations into account on a probability-weighted basis when calculating the time zero value of an asset.

Gold price uncertainty is assumed to have a low correlation with general financial market uncertainty (i.e., being a low Capital Asset Pricing Model beta). This is consistent with the gold mining industry practice of using low risk-adjusted discount rates. The long-dashed lines in Figure 5 delineate the risk-adjusted (RA) gold price expectations used in the RO computations. When the gold price is $400/oz at time zero, the risk adjustment trims a few dollars off the gold price forecast as compensation for risk. Similarly, if gold price in year two turns out to be $320/oz, a revised risk-adjusted gold price expectation is used thereafter, which again trims a few dollars off the gold price forecast at time two for risk. These long dashed lines are also known as the gold forward price, as quoted on NYMEX or COMEX. In each price forecast the compensation for risk increases the further into the future the price is forecast.

Figure 9 outlines the expected prices and confidence boundaries for the copper price model. A key characteristic of the copper price model is price reversions, the result of supply/demand influencing price behavior. The copper price model parameters in Table 5 assume price reversions to a long-term equilibrium of $0.98/lb. Mean reversion causes the long-term forecast of copper price to be largely unaffected by the level of copper price in year zero or year two; after a hypothetical price shock of -50 percent between year zero and year two the two year forecast of copper price in year 10 is almost the same as the year zero forecast of price in year 10.

Price reversion also affects risk discounting. In the presence of reversion, price uncertainty almost saturates (stops growing) in the long-term. The confidence boundaries in Figure 6 exhibit this in that the 90 percent confidence range for long-term copper prices eventually stabilizes with an upper boundary of $1.57/lb and a lower boundary of $0.68/lb. Reflecting this stabilization of price risk, the difference between the expected copper price and the risk-adjusted expected copper price (or the forward price, again delineated by the long-dashed lines) increases initially and then stabilizes with the saturation of copper price uncertainty. In a DCF framework, this risk adjustment effect could only be achieved by using a declining discount rate as the cash flow becomes more distant.

**Validation results**

The real option method tracks all of this geological and

![Fig. 5. Gold price model. The vertical axis shows the price of gold, and the horizontal axis shows years from the start of the exploration project. The solid line delineates gold price expectations, on the price mean, given a current (large zero) spot price of $400/oz. The dashed lines outline the 80 and 10 percent confidence boundaries around this forecast, based on gold prices following a random walk dynamic process. The figure also shows the risk-adjusted (RA) expected price, which is also the forward gold price curve. If the path of gold follows a random walk between year 0 and year 2 that result in a price of $320/oz, the updated expectations and confidence boundaries will reflect that expectations, as well as the forward curve, will be as depicted. Because of the random walk nature of gold prices the confidence boundaries on any price forecast are always increasing as the forecast horizon increases.](image-url)
price uncertainty into the lease value calculation, where optimal decision-making is assured in the equivalent of thousands of price and reserve scenarios. The calculations were completed with a non-commercial Visual C++ program. The program uses a directed graph algorithm to construct the underlying exploration decision tree. Given an initial exploration state, the algorithm considers the decisions possibilities at each state and then generates a set of future exploration states based on these decisions. This process continues at each new exploration state generated until either a lapse exploration state (abandonment) or a lease development state is reached. A description of the directed graphing algorithm can be found in Cormen et al. (1996).

Once the decision tree is built, exploration outcomes and their occurrence probabilities and associated price or deposit probabilities are attached to each decision node. Exploration outcome paths are then generated to each of the conditional exploration outcomes at future exploration decision nodes.

Given the stochastic nature of prices, the valuation algorithm requires dynamic programming and the Projected Successive Over-Relaxation (PSOR) finite difference method to approximate the underlying OCP or RO valuation equation. A general discussion of the underlying RO finite valuation equation can be found in Samis (2008) and the PSOR finite difference approximation can be found in Wilmott et al. (1995). The value calculation starts by calculating the NPV for each deposit outcome \( R_{PK}, R_{RP}, R_{AD}, \) and \( R_{BG} \) using real option discounting of expected cash flows. These values use the economic parameters specified in Tables 2a, 2b, and 3 and incorporate an option to abandon the operation once it is developed and in production. Since reserve uncertainty remains at the end of each of the 27 exploration outcome paths, the terminal value of each exploration outcome path weights these NPVs by the probabilities associated with the particular exploration outcome path to come up with 27 NPVs, \( \sum_j NPV_j \) values. From this the expected feasibility cost is deducted to obtain the gross expected exploration payoffs.

Dynamic programming is then used to work recursively back through the exploration tree. For each exploration outcome decision node, the present value of continuing exploration is calculated using the PSOR approximation of the underlying valuation equation over a range of mineral prices. The approximation starts with the expected NPV of the lease one period into the future (i.e., at the end of the current exploration phase). A comparison is then made between the present value of continued exploration and the present values of deferring exploration for two years or leasing the lease lapse at each mineral price within the price range. The lease NPV is the maximum value generated by continuing exploration, deferring exploration, or abandoning the lease. The valuation calculation moves to the decision nodes at the next earlier time period once all decision nodes have been evaluated at a particular time.
This numerical approach takes into account all possible price and deposit outcomes, and, working recursively from the end of the exploration program, calculates a probabil- ity-weighted initial value given optimal decision-making as new price and reserve information unfolds over time.

Time 9 DCF and Real Option three valuations.

A time zero DCF NPV can be calculated for each lease using the economic data in Tables 2a, 2b, and 3. Assuming a flat price forecast of $400/oz for gold and $1,00/ib for copper, we noted previously that the time zero expected gross payoffs at the end of exploration was $1,00,000 for the gold property and $60,000 for the copper property. Bringing these payoffs back to time zero, deducting the present value of the $2,74 million feasibility study, and deducting the present value of the sequential exploration expenditures at time t = 0, 1 and 2, gives a gold lease value of $67,74 million and a copper lease value of $2,8 million. Neither project is economically viable under this valuation technique.

When mineral price and reserve uncertainty, appropriately discounted within a real option framework, is combined with the ability to manage the exploration process both leases have a positive RO value. Given a time zero gold price of $400/oz and copper price of $1,00/ib (the same assumptions as in the DCF calculation), the gold lease has a time zero RO value of $9,575,000, and the copper lease has a time zero RO value of $61,191,000. In essence, the copper and gold deposits at this case study are equivalent to deposits of less than 1.35 Moz and greater than 900,000 oz in size (in the hypothetical example in Figure 1). Static DCF valuations calculate negative reserve values, while RO valuations calculate positive values. This is the standard problem with DCF analysis — valuable exploration that can be conducted in stages is quantified as a negative NPV endeavor when calculated using dynamic analysis with decision trees that take into account geological and price uncertainty, the copper lease has no value at time 1/ib copper when discounted using a DCF framework. The gold lease has a value of $9,157,000, one-third of the value calculated in real options model. The differences here are due to the fact that real options analysis takes into account the risk-reducing impacts of flexibility and price mean reversion, while a decision tree approach that uses an exogenously specified discount rate is no better than the standard DCF technique in accurately reflecting the risk attributes of the projects.

Figure 7 shows the time zero real option exploration projects NPVs and optimal exploration decision for the gold lease over a range of spot prices. The RO method indicates that when price is below $314/oz, lease has no value and should be abandoned since holding costs are likely to outweigh the present value of any eventual mineral revenues. Exploration should be deferred when the gold price is between $314/oz and $456/oz, paying holding costs in the mean time. Exploration should start immediately when the gold price is above $456/oz. At $400/oz, the gold lease value of $9,575,000 is due to the ability to defer stage 1 exploration for two years, waiting to see if gold price rises before committing to the first stage. If it were not for the flexibility inherent in the sequential exploration process the lease would have no value and would be abandoned. The time zero real option exploration project NPVs and stage 1 exploration decisions for the copper lease over a range of spot prices are delineated in Figure 8. This lease is abandoned if the copper price is below $0.32/ib, whereas stage 1 exploration is started if the price is above this level. Stage 1 exploration is not deferred at any price. The intuition here is that, with saturating price uncertainty, the value of waiting for more price information is small— the operator knows where the price level will go with reasonable certainty. Given a $75,000 cost per period for waiting, the wait for the small amount of price information that will be revealed over time is not optimal in our example. The time zero RO NPV s for the gold and copper leases vary differently with spot price because of the different pricing models. The gold lease valuation has a convex shape similar to a financial call option because gold is assumed to follow a random walk price process, which is also a standard price model assumption for financial stocks. The copper lease valuation has a concave shape at high prices because of price reversion. The copper price is assumed to revert to $0.98/ib, so when the copper price is above this level there is the expectation that price will fall in the future. This results in a diminishing marginal value of higher spot copper prices. Schwartz (1997) finds a similar effect in a development copper property valuation. In practice, one would therefore expect to see gold lease values being less sensitive than copper leases to price movements at low prices, but having more sensitivity than copper to price movements at high prices.

Time 1 Real Option NPV payoff diagrams (prior to the start of stage 2 exploration)

Once exploration begins, the RO value of both leases changes in response to the assessment of geological information gained from completing an exploration stage. Figures 9 and 10 outline the RO lease NPVs over a range of mineral prices and results (poor, fair, or good) of stage 1 exploration. In our example, good or fair results increase the lease NPV while poor results decrease the lease NPV. The figures also show decisions about stage 2 exploration once the stage 1 outcome is revealed. The option to explore again upon poor stage 1 exploration results adds value to the lease in all but low price environments. Of interest in Figure 10 is the virtual insensitivity of copper lease value to copper price. This again is due to price reversion. It is price reversion that also prevents the project from being abandoned at low copper prices when the stage 1 outcome is good or fair—low prices will revert to the mean, and it is therefore better to pay maintenance costs.

18One of the best examples of this is the 75 drill holes at the Hondo gold deposit in Canada that came up showing no commercial mineralization. The 75th hole formed the deposit Mining Journal (1995). See also Smits (2005) for the rules of the option to drill again.
Fig. 7. Time 0 gold lease real option NPV (prior to the start of stage 1 exploration). The vertical axis measures lease value, and the horizontal axis measures current gold price. Lease value is shown in current gold price. The figure also shows the value-maximizing exploration decision as a function of current gold price and the implied gold price forecast (see Fig. 5). At all gold prices below $318/oz, this gold lease should be abandoned. At prices between $341/oz and $416/oz, stage 1 exploration should be deferred two years, at a cost of $75,000 per year, and then reevaluated. At prices above $416/oz, stage 1 exploration should proceed immediately. At a gold price of $400/oz, the exploration lease has a real option value of $0.375 M, though stage 1 exploration should be deferred for two years and then reevaluated.

Fig. 8. Time 0 copper lease real option NPV (prior to the start of stage 1 exploration). The vertical axis measures lease value, and the horizontal axis measures current copper price. Lease value is increasing in current copper price. The figure also shows the value-maximizing exploration decision as a function of current copper price and the implied copper price forecast (see Fig. 6). At all copper prices below $0.52/lb, this copper lease should be abandoned. At prices above $0.53/lb, stage 1 exploration should proceed immediately. At a copper price of $1.00/lb, the exploration lease has a real option value of $0.191 M, and stage 1 exploration should be started immediately, in contrast to gold leases, copper leases should only be explored immediately or abandoned since there is no long-term price information to be gained by deferring the exploration decision.
Fig. 9. Gold lease real option NPV at the end of stage 1 exploration (time 1). As a result of the outcome of stage 1 exploration the value of the lease is updated. The top curve shows the time 1 value of the lease after a good stage 1 exploration outcome. The next lower curve shows the value of the lease after a fair-stage 1 exploration outcome. The bottom curve shows the value of the lease after a poor stage 1 exploration outcome. The dashed line with no markers shows the time 1 expected value of the lease prior to the information gained from stage 1 exploration. In this case, good and fair stage 1 exploration outcomes serve to increase the value of the lease, while a poor outcome decreases the value of the lease. The figure also shows the optimal stage 2 exploration decision given current (time 1) gold price and stage 1 exploration outcome. Gold prices at which stage 2 exploration is deferred decrease from $884/oz to $286/oz to $240/oz (not shown) as stage 1 exploration outcomes improve from poor to fair to good. Similarly, the abandonment trigger price decreases from $536/oz to $286/oz to $240/oz (not shown) as stage 1 exploration results improve from poor to fair to good.

Fig. 10. Copper lease real option NPV at the end of stage 1 exploration (time 1). As a result of the outcome of stage 1 exploration the value of the lease is updated. The top curve shows the time 1 value of the lease after a good stage 1 exploration outcome. The next lower curve shows the value of the lease after a fair-stage 1 exploration outcome. The bottom curve shows the value of the lease after a poor stage 1 exploration outcome. The dashed line with no markers shows the time 1 expected value of the lease prior to the information gained from stage 1 exploration. In this case good and fair stage 1 exploration outcomes serve to increase the value of the lease, while a poor outcome decreases the value of the lease. The figure also shows the optimal stage 2 exploration decision given current (time 1) copper price and stage 1 exploration outcome. If the outcome of stage 1 exploration is poor, stage 2 exploration is undertaken if the copper price is greater than $0.94/lb. Otherwise, the lease is abandoned. If stage 1 exploration reveals good or fair results, stage 2 exploration is deferred if copper price is less than $0.84/lb, and undertaken immediately if copper price is higher than this. A property with fair or good results in stage 1 exploration is never abandoned.
and wait for prices to rise rather than to abandon the property if there is adequate resource potential.

The evolution of lease values and the price triggers for exploring, waiting, or abandoning is presented in decision tree form in Figures 11 and 12. Gold lease values at all points are based on a then current price of $400/oz and copper lease values assume a current price of $5.00/lb. Note that it is optimal to explore again in certain price environments given one or even two stages of poor exploration outcomes. Poor exploration results are much more damaging on copper leases than on gold leases, with copper leases being abandoned in these cases because there is little chance that a low reserve base will be offset by skyrocketing prices. If there is some probability of resources being high, however, as with a fair exploration outcome, low copper prices do not deter immediate exploration due to the expected mean reversion of prices by the time the production of the majority of the resource is underway. Gold, on the other hand, must have high prices to go ahead with exploration, since that price process does not have any mean reversion and low prices will likely remain low (Fig. 5).

The final columns in Figures 11 and 12 show the prices at which the feasibility (S, for scouting) study should be undertaken given that the previous three stages of exploration have been undertaken. If price is below this level the project should be abandoned because the quantity of the resource has remained below the reserve/reserve cut-off P_. As in the model in the previous section, metal price and quality together determine P_; poor exploration results can nevertheless produce reserves when price is high enough such that γ is a low value. For example, a good/poor/poor gold exploration outcome produces a sufficiently high estimate of R2 to define this a reserve, as used in this paper; as long as the gold price is above $390/oz upon completion of exploration.

Model differences between gold and base metal exploitation

This real options model of exploration reveals important differences between exploration for gold, which exhibits random walk prices and increasing future price uncertainty, and for base metals, which exhibit reversion to a mean price level after a price shock and saturating price

**Fig. 11.** Evolution of real option NPV and decision rules for the gold lease when the gold price is $400/oz. This figure shows the decision tree format the information contained within Figure 9 for all stages of exploration. The first column shows that at $400/oz gold, the exploration property is worth $5.057 M at time 0. Exploration should proceed for spot gold prices above $500/oz, and should be deferred for spot prices below $350/oz and $500/oz. At prices of less than $350/oz the lease should be abandoned. The second column shows the same information contingent on a $400/oz gold price at time 3 and the outcome of stage 3 exploration. Column 3 shows the information contingent on a $400/oz gold price at time 3 and the outcome of stage 1, 2, or 3 exploration. Column 4 shows the information contingent on a $400/oz gold price at time 3 and the outcome of stage 1 and 2 exploration. Column 5 shows the information contingent on a $400/oz gold price at time 3 and the outcome of stage 1, 2, and 3 exploration. In column 6 a $400/oz gold price and 3 good exploration outcomes yield a time 3 project value of $390.3 M, and scouting (feasibility/feasibility/development) should proceed for any gold price greater than $790/oz. On the other hand, a $400/oz gold price and 3 good exploration outcomes yield a time 3 project value of $317.3 M, and scouting (feasibility/feasibility/development) should proceed for any time 3 gold price greater than $312/oz.
uncertainty. For example, our case study supports a gold exploration policy that focuses on deposits with previous positive exploration results during periods of moderate prices, while deferring geologically unpromising deposits, and an expanded exploration policy that includes the deferred greenfields deposits when prices are higher. The intuition here is that unpromising deposits may ultimately become valuable due to increasing gold prices, and that they should be kept in reserve until price improves. Real-world evidence of gold exploration activity in the 1990s supports this trend. On the other hand, our model suggests that unpromising greenfields copper projects not be kept in deferral mode waiting for higher prices, since prices are capped by the trend for a long run mean, greenfields copper projects should either be explored if promising or abandoned if unpromising. Given this we would not expect greenfields copper exploration expenditures to be sensitive to the price cycle, but rather an ongoing activity as long as price exceeds some minimum level (col. 1 of Fig. 12). M. S. Enders and R. A. Leveille (unpub. data, 2004) report exactly this policy at Phelps Dodge. Our model also shows that later stage copper deposits with positive exploration outcomes should be deferred in low price environments, since they will become valuable once price reverts to the mean, and so these should show some exploration response to price swings. This, combined with constant spending on greenfields projects across the price cycle, would then be revealed as greenfields exploration constituting a higher proportion of total expenditure during the bottom of the price cycle and a lower proportion at the peak of the price cycle. There is some evidence that this is broadly the case for base metals. In addition, we would expect to see stage 2 gold exploration spending respond to movements in gold prices, while we would expect stage 2 copper exploration spending for fair and good outcomes to be ongoing for all but very low prices (Figs. 11 and 12). Empirically, gold exploration expenditure does seem to be more responsive to price than base metal exploration expenditure (R. C. Schodle, unpub. data, 2003).

Since the geology and economics of our gold and copper examples have been designed to be as comparable as possible, these model differences are driven entirely by the dif-
ference in price behavior of base metals, which exhibit price reversals, and gold, which does not. If one wants to model price and value, and to thereby explain the differences that are found between base metal and gold exploration, such pricing details must be included in the model of exploration. Modeling only the uncertainty in a resource is unlikely to capture these effects.

Conclusions
This paper models the two main uncertainties facing an exploration manager, geological uncertainty and price uncertainty, in a real options framework aimed at directing exploration decisions such that project value is maximized. Both models show that in some cases where the traditional discounted cash flows of the project is negative, exploration activity can be value creating and should proceed. The first model considers only geological uncertainty and reveals the many geological factors influencing exploration activity and the value of an exploration property. The second model adds price uncertainty. The second model reveals important differences between exploration for gold, which exhibits random walk prices and increasing future uncertainty, and the exploration for base metals, which exhibit reversion to a mean price level and saturating future uncertainty.

Our models justify the aggressive decisions that exploration managers often make, even in the face of contrary advice from traditional discounted cash flow models. That the valuation models are now giving the right signal is a significant step forward in managing and evaluating exploration projects.

Acknowledgments
We owe immense gratitude to Richard Schodde of WMC for thought-provoking discussions of exploration econometrics, and to Gordon Kaufman, William Navidi, and Luiz Tenorio for statistical advice. Michael Sumit would also like to thank Julian Verbeek of RSG Consulting for helping develop an earlier version of this model. Two anonymous referees, editors Jeff Hedlundquist, Michael Doggett, and Jack Parry, and Margaret Armstrong, Beverley Harris, and Irina Khindanova offered useful suggestions that have shaped the paper’s exposition. We also thank Spiriton Mashakaudzirira for useful comments on an earlier draft.

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