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Chapter 14

Using Real Options to Value and Manage Exploration

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Abstract

This paper considers the two main uncertainties facing an exploration manager, geological uncertainty and price uncertainty, within a real options framework. The paper presents two models of exploration. The first model considers only geological uncertainty and reveals how geological and economic factors influence exploration activity and the value of an exploration property. In particular, the model shows that exploration activity will logically respond to changes in the economic and technical environment. One outcome of this model is that higher geological uncertainty enhances exploration project value when initial resource estimates are small and harms projects that have quality resources. The second model considers both geological uncertainty and price uncertainty, and reveals important differences between exploration for commodities such as gold, which exhibit random walk prices and increasing future price uncertainty, and exploration for base metals, which exhibit reversion to a mean price level and saturating future price uncertainty. Under a value-maximizing policy gold exploration should, during periods of low and moderate prices, focus on promising green-fields deposits and deposits with previous positive exploration results while deferring geologically unpromising green-fields prospects until prices improve. This is because given gold's random walk price characteristic, there is always a chance that higher and higher gold prices could make up for poor geological potential. An expanded exploration policy that includes the deferred green-fields prospects is warranted when prices are higher. On the other hand, our model suggests that unpromising green-fields copper projects should not be kept in deferral mode during low prices, since future prices are capped by the trend of copper prices to a long-run mean. Green-fields copper projects should therefore either be explored immediately if promising or permanently abandoned if unpromising, regardless of current copper price. Since the geology and economics of the gold and copper examples have been designed to be as comparable as possible, these model differences in exploration behavior are driven entirely by the difference in price behavior of base metals, which exhibits reversion, and gold, which does not. That optimal exploration policy depends on the commodity brings to light the importance of modeling both geological and economic (price) uncertainty when valuing and managing exploration projects.

All of this is done within a real options analysis. Traditional discounted cash flow analysis ignores the nonlinearity of the payoffs to exploration and thereby undervalues exploration activity. The real options framework, which captures the nonlinearity of these payoffs, shows that in some cases where the traditional discounted cash flow value of an exploration project is negative, exploration activity can be value creating and should nevertheless proceed. Real option modeling thus provides economic validation of decisions that exploration managers often take in spite of traditional valuation criteria that recommend against taking such action.

Introduction

THE MAIN PURPOSE of exploration is to generate reserves. In this paper we stray from the Joint Ore Reserves Committee (JORC) very specific definitions of resource and reserve classifications and define reserves as the quantity of mineralization in a specific location that is of sufficient geological assurance and economic value to be profitably mined, either now or in the future, under currently forecast market and technical conditions. Thus, in our usage, the reserve quantity is not known with certainty, but is rather the mean of a probability density function representing possible quantities of mineral that might be extracted under the various economic and technical scenarios that could unfold given current information. A resource is the portion of that same mineralization that either has insufficient geological assurance or economic value to be considered a reserve at the time. Our generic definition allows us to discuss reserves and resources in a stochastic and inter-temporal sense, which is vital in a real options perspective.

Real options are options on real assets as opposed to financial assets. Howell et al. (2001) provide a readable introduction to real options. Early real options research incidentally focused on the option to develop a mine or explore for oil

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(Brennan and Schwartz, 1985; Paddock et al., 1988). The focus was on price and cost uncertainty, in contrast to the decision-analytic models of exploration dating back to the 1960s that considered only geological uncertainty and took the economic variables as known. Only recently have real options models of exploration quantified both price and geological uncertainty (e.g., Cortazar et al., 2003; Martzoukos, 2000, 2003), although one of the main technical artifacts of exploration that appears in the decision-analytic models, imperfect information from sampling, is still omitted. Our paper extends these recent models by introducing imperfect information into a real options model of exploration.

Given the increasingly sophisticated quantitative geological models of exploration there has been a call to add equally quantitative models of the economic parameters influencing exploration decisions (e.g., Lerche, 1997, p. 637). In this paper we seek to do just this. Our hope is that by quantitatively modeling exploration as an economic activity, exploration managers and geologists can make better decisions, decisions that add more value to the firm and the economy and that ultimately result in the more efficient generation of reserves. Our model also allows for the valuation of exploration projects, even those that are highly speculative.

Models are, by design, simplifications of reality. Our model is no exception; we have left out many of the complexities that geologists and exploration managers must wrestle with and that make the job of exploration management and valuation so difficult. That said, we include two of the largest drivers of exploration activity, mineral quantity uncertainty and mineral price uncertainty. The result is a tool for exploration planning and management that appears to explain certain real-world phenomena. Perhaps more importantly, the insights derived through quantification of the exploration process may lead to improvements in exploration decision-making.

Guiding Principles

In guiding our modeling efforts we look to five irrefutable facts that characterize exploration. First, exploration is a lumpy irreversible investment activity; it has an initial irreversible capital outlay over some finite period, followed by sequential options for further irreversible capital outlay over subsequent periods. Modeling exploration activity requires investment theory that characterizes investment as lumpy, irreversible, and sequential.

Second, dollars are spent on exploration only where there is some expectation of an economic return at the end of the exploration sequence. That expected return is principally the discovery of new reserves. This does not mean that every project needs to lead to the discovery of reserves. Indeed, economic failure, where the discovery is a resource too small or of too low a grade to be extracted at a profit, is the norm (Kesler, 1994, p. 56). Exploration has probabilities as low as 1 in 1,000 of moving beyond grassroots stages, and as low as 1 in 10 of finding an orebody in the few properties that warrant major exploration effort (Peters, 1978). This second irrefutable fact reveals that the appropriate economic model of exploration activity should be grounded in investment theory where expected (or mean) payoffs, rather than most likely payoffs, guide decision making.

Third, the payoffs to exploration are nonlinear—the payoff to success is proportional to the quality and quantity of reserves while the cost of failure is limited via abandonment options that can be exercised at any point in the exploration sequence. This means that our investment theory, in calculating the expected payoff, must adequately take into account the nonlinearity of the payoff function that is generated via the abandonment option.

Fourth, exploration investment is unlike most other investment activities due to the long time frame between the expenditure of capital and the realization of revenues. An analysis of 54 major base- and precious-metal deposits around the Pacific rim by Sillitoe (1995) reveals that the time from initial exploration spending to the discovery drill hole averaged 14 years for base metal deposits and 22 years for gold deposits. There is then an average of a further 13.5 years to first production for base metal deposits and 7 years to first production for gold deposits. That is, where exploration is successful there is an average of 27.5 years from initial spending to cash flow generation for base metal deposits. The average at gold deposits is 29 years. We do not know of any other private-sector economic activity with such a long lag between the start of investment spending and the beginning of income receipts. Under traditional discounting this distant income is worth almost nothing in present value terms at the start of an exploration program, causing Newendorp (1975) to recommend an undiscounted analysis of exploration cash flows. This is clearly unacceptable given that there is a cost of capital and investment risk. What we need is an investment model that can adequately discount far-off payments for cost of capital and risk without destroying their present value.

Finally, exploration investment is not investment in tangible capital such as buildings or machines. It is sequential investment in information, initially with the goal of locating a resource, and then with the goal of reducing the uncertainty over the quantity and quality of that resource to bring it into a reserve. Research and development (R&D) investment spending has this same characteristic. Models of R&D recognize that risk is not necessarily compounding over time, that multiple expansion and abandonment options exist during the development path, and that the main payoff to investment is information (Paxson, 2003). Our model should have these characteristics, too.

The first and second facts of exploration, that it is a lumpy and irreversible investment decision guided by expected payoffs, lend themselves to traditional discounted cash flow (DCF) analysis. However, facts three, four, and five—that exploration has nonlinear payoffs and abandonment options, has a long time frame between expenditure and cash flow generation, and has an outcome that is in intermediate stages updated information rather than cash flows—indicate that traditional DCF models are not likely to be suitable. An options framework, on the other hand, is a good candidate, as it deals explicitly with each of these

nuances of the economics of exploration. Indeed, exploration is an option on information, and it is difficult to value options with anything other than option pricing theory. Since at issue here are decisions over real assets, real option theory is ideal.

Exploration has long been guided by economics (e.g., Grayson, 1960; Kaufman, 1963; Davis et al., 1973; Peters, 1978; Megill, 1979). Expenditures in aggregate have been observed to vary with mineral prices, exploration costs, exploration success rates and mining firm profitability in a way that is consistent with exploration activity being undertaken in response to varying economic factors (Eggert, 1988; M.S. Enders and R.A. Leveille, unpub. data, 2004). Furthermore, considerable evidence exists that exploration is an economic activity that provides a positive return on investment (e.g., Mackenzie and Woodall, 1988; M.S. Enders and R.A. Leveille, unpub. data, 2004; Schodde, 2004). The application of real options represents a natural progression in the study of exploration and the enhancement of this economic return.

Some Preliminaries

Prior to developing our model we review some preliminaries for readers new to the concept of uncertainty and real options. Other readers can skip directly to the next section, where we present our analysis of the exploration process.

The nature of uncertainty in exploration

Real options (RO) models always specify quite clearly the nature of uncertainty to be considered. The two main sources of uncertainty affecting exploration decision-making are current geological uncertainty and future commodity price uncertainty. Geological uncertainty is, in the absence of exploration effort, constant through time. To a small extent one can learn more information about a mineral lease or property by learning of exploration results from surrounding leases. But for the most part, geological information must be purchased, thus making learning endogenous.

In the limit, it is technologically feasible to resolve most of the geological uncertainty in this regard; it is possible to drill the entire lease at 10-m spacing, generating, for all intents and purposes, an exact map of any mineralization within the lease boundary. A more reasonable sequence of exploration activities would be as follows: identify an area of likely mineralization, either through nature's signals such as outcrops, or through some preliminary information gathering such as soil sampling and trenching; drill in selected areas in the hope of further reducing rather than eliminating uncertainty; and then make decisions based on this new information. Broadly speaking, wherever there is motivation for planned spending to reduce uncertainty, agents are weighing up where to spend and when to stop spending. These decisions are based on the recognition that spending in some areas is likely to create more valuable information than spending in other areas, and that at some point an extra dollar spent drilling does not produce an incremental dollar's worth of information. Exploration is, baldly, an economic activity that produces information, motivated by cost-benefit analysis, and thus is amenable to analysis using fairly simple and well-established economic tools.

Economic uncertainty, on the other hand, is partially or fully resolvable only through waiting. This is called exogenous learning. For example, the London Bullion Market Association afternoon $US fixing price of gold on January 7, 2010, is uncertain as of writing this paper, but will have less uncertainty tomorrow, much less uncertainty on January 6, 2010, and will be known with certainty on January 8, 2010. One can spend money now to try and forecast the price on January 7, 2010, but no amount of money will resolve the uncertainty. Other uncertainties that affect the economics of a mine, such as mining cost inflation, environmental and social cost requirements, and taxation structures, also become evident over time. This difference between the two major areas of uncertainty (geological and economic) explains why previous models have either dealt with one or the other but not both.

The value/uncertainty relationship

Exploration creates information, and information has value whenever it enables better decision-making. For example, the more information about the reserve location, grade, and tonnage at mine development, the more likely that the mine and mill will be optimally sited, designed, and sized (Glanville, 1985; Gocht et al., 1988; Smith, 1997). The value penalty arising from uncertainty in the geology of a reserve is evident in the market. In the oil and gas industry, developed and producing proven reserves have three times the value of developed and producing probable reserves, and there is evidence that similar discounting occurs with mineral reserves (Davis, 2003). Traditional economic views of exploration, which see exploration as only increasing geological assurance as a technical requirement prior to extraction (e.g., McKelvey, 1974), fail to recognize this value/uncertainty relationship. In RO models, increasing geological assurance not only moves resources between the traditional inferred, indicated, and measured categories, but can turn a resource into what we call a reserve in this paper simply by reducing geological uncertainty and thereby assuring profitable extraction by increasing the likelihood that optimal mine and mill location and design decisions will be made.

Valuing future uncertain payoffs

The discounted cash flow (DCF) method is the most common method of calculating the net present value (NPV) of the payoffs from investment, be they the value of information gains within an exploration program or the payoffs created by any other investment. This technique aggregates time and risk adjustments into a single discount rate. That discount rate is used to bring the future payoffs to a present value so that their sum can be weighed against the present cost of the investment. Unfortunately, the use of DCF discount rates in the resource industries is often inherently flawed since it is common practice to use a single constant

discount rate across a wide range of projects, with perhaps an arbitrary boost for early stage exploration projects. Most projects have widely varying risk characteristics so that different discount rates are required. Yet, there is no easy way to determine the required project-specific discount rate (Fama, 1977; Myers and Turnbull, 1977). Even within a given project, the use of a constant discount rate over all cash flow periods presumes that cash flow risk is increasing in a specified manner that is unlikely to match the nature of the risk in the project (Robichek and Myers, 1966; Myers and Turnbull, 1977).

The RO method shares the same theoretical foundation as the DCF method but uses a risk adjustment that is unique to each project and the level of risk inherent in that project (Jacoby and Laughton 1992; Salahor, 1998; Samis et al., in press). In particular, each source of uncertainty affecting a project net cash flow is identified, the net cash flow risk that this generates is priced using market information or financial models, and then the risk-adjusted net cash flow is adjusted for the time value of money. For example, if a net cash flow is affected by price uncertainty, the RO technique replaces that uncertain price stream with its certainty equivalent, in this case a market-derived forward price. Geological uncertainty is treated in a similar manner. Because of their differing risk attributes, copper projects are discounted at a different rate than gold projects, and early stage projects are discounted at a different rate from late stage projects (Laughton and Jacoby, 1993). Discount rates tend to decline for the more distant cash flows, resulting in even the distant positive cash flows of an exploration project having significant positive present value.

Modeling Resource Uncertainty and the Value of Information

In modeling and valuing exploration activity, current geological uncertainty and future economic uncertainty are interwoven. The value of drilling another hole now is affected by the expected price of the mineral several years in the future, since this is the price that each unit of mineral found will ultimately fetch upon extraction. And the price of the mineral several years in the future depends on the supply forthcoming at that time, which depends on the amount of exploration activity now. Clearly this is a complex process, and one that will have to be simplified if we are to make any progress. As with earlier works (e.g., Kaufman, 1963; Newendorp, 1975; Howe, 1979, Ch. 10; Harris, 1990) we begin by taking prices as certain and focus only on initial geological uncertainty. We extend the earlier work by emphasizing the economics of exploration decisions. After developing this economic model of exploration decision-making, we add price uncertainty in the case study in the next section of the paper.

A model of resource uncertainty and the outcome of exploration

In this section of the paper we build a simple model of exploration through which particular management and valuation insights can be revealed. Our statistical model of the information gained by exploration is based largely on Campbell and Lindner (1983, 1985) and Kaufman (1963). To simplify matters we will consider exploration activity at the level of the project. We assume that exploration is guided by rational decision-making aimed at spending dollars only when there is an acceptable expected return to doing so.

At each point in time the company or government weighs up taking some discrete exploration activity at a given prospect, comparing the costs associated with this activity against the rewards that the activity is likely to produce. Rewards come in the form of new geological information. In all cases information will have value, in the sense that it will create better decisions on subsequent spending on exploration or development than in the absence of that information. In some cases the information will reveal that it is optimal to terminate an exploration program because the information likely to be gained from the next stage of exploration is not worth the cost; the project will either be deferred or abandoned, or it will move into development. In other cases the information will indicate that additional exploration spending is warranted. The value created by the improved information is discounted for risk and time such that it can be weighed against the exploration costs. Exploration creates wealth when the present value of expected benefits, appropriately discounted in a RO framework, exceeds the present value of expected costs.

To begin, consider a mineral lease that contains $R > 0$ units of economically recoverable mineral, though R may not be large enough to warrant development of a mine and therefore may not be a reserve. The level of R is unknown to the entity that owns the lease. Reconnaissance has revealed some information about the mineralization on the lease and is the basis for what we will call prior beliefs about the level of R. There is evidence that within a basin or geological region reserves have a log normal distribution (Kaufman, 1963; De Geoffrey and Wignall, 1985, p. 25–26; Harris, 1990, p. 300–301). A log normal distribution therefore seems to be a reasonable starting point when modeling geologists' inferences about R. Let these prior beliefs therefore be the log normal distribution $\tilde{R}_0 \sim f_L(\mu_0, \delta_0^2)$, where μ_0 is mean of the log of the beliefs and δ_0^2 is the variance of the log of beliefs. The tilde on $\tilde{R}_0$ suggests that it is a random variable, and the 0 subscript indicates time, with current time being zero. Given the properties of log normal distributions, the expected or mean size of the economic mineralization at time zero is

$$\bar{R}_0 = \exp[\mu_0 + 0.5\delta_0^2]. \tag{1}$$

The variance surrounding this estimate is

$$\mathrm{Var}(\tilde{R}_0) = \{\exp[2\mu_0 + \delta_0^2]\}\{\exp[\delta_0^2]-1\}, \tag{2}$$

which has the desirable property that more promising prospects have more variance surrounding the estimate of economic mineralization; the coefficient of variation is constant for all deposit sizes and is equal to $(\exp[\delta_0^2] - 1)^{0.5}$. Depending on the level of $\bar{R}_0$ and geological assurance sur-

rounding the estimate, $\bar{R}_0$ may define either a resource or reserve at this point. For concreteness, we assume that it is a resource due to a large $\mathrm{Var}(\tilde{R}_0)$. The initial beliefs about $\bar{R}_0$ and $\mathrm{Var}(\tilde{R}_0)$ contain composite information about the distribution of means of reported field sizes of reserves of this geological nature or area, the variance of possible field sizes associated with each mean, and information from early stage exploration (Kaufman, 1963). The initial resource estimate is almost certain to be wrong, with $\bar{R}_0 \neq R$. Nevertheless, it is the best information available and must be used when making decisions about whether or not to continue exploring the property.

A tractable case treats the next stages of exploration as a series of discrete sampling events, $i = 1, 2, \ldots, n$, ranging from geological mapping ($i = 1$) to pattern drilling ($i = n$). For now we assume that all n stages of exploration, if undertaken, must be completed sequentially with no option of going directly to development and no option for abandonment at any stage. If exploration is not undertaken the project must be abandoned. Only through exploration can enough information be gained to warrant proceeding with project development should the resource become a reserve. In essence, we assume that existing resource uncertainty is too high to warrant moving directly to development drilling, and that management must decide whether or not to commit now to an all-or-nothing exploration program to prove the resource.

In a departure from most decision-analytic exploration models, we treat exploration as Bayesian sampling from a continuous distribution rather than sampling from a simplified binomial or trinomial distribution.[1] In this sense our model will take on the flavor of a real options model, in which uncertain variables are modeled as continuous. Each stage of exploration generates an estimate X_i of the quantity of economic mineralization. These exploration outcomes randomly deviate from R because there is known error or noise σ_i^2 associated with sampling, akin to a detection probability of less than 1 (De Geoffrey and Wignall, 1985).[2] The level of σ_i^2 indicates the degree of sampling error in sampling stage i, typically modeled as being a decreasing function of exploration expenditure and exploration efficiency within the stage (De Geoffrey and Wignall, 1985, p. 75–76). Alternatively, stages with less sampling error could be treated as having a sample-equivalent size $n_i > 1$. Although σ_i^2 will probably decrease as each sampling stage becomes more targeted, to simplify matters we treat σ^2 as constant across each stage of exploration.

We noted above that δ_0^2 is the variance of the log of initial beliefs about the resource size. It is useful to think of δ_0^2 as hypothetically being generated by m previous stages of exploration given sampling error σ^2 in each stage, or $\delta_0^2 = \sigma^2/m$ (Kaufman, 1963, p. 162). That is, the initial resource uncertainty could equally be expressed as the outcome of m previous stages of exploration, each stage identical to the stages of exploration to follow. Then, define

$$\bar{X}_n = \frac{1}{n}\Sigma \log X_i, \tag{3}$$

as the log mean of the succeeding n-stage subsequent sampling exercise. This new information is combined with the previous information about $\tilde{R}_0$ to create an updated or posterior resource distribution $\tilde{R}_n$.

A priori, an unbiased sampling exercise is expected to return the prior resource estimate, with post-exploration uncertainty around this estimate going to zero as the number of stages (exploration effort) goes to infinity. These properties of the sampling program are brought about by the Bayesian assumption that sampling error introduces an independent log normal process with known precision. In other words, sampling will have an independent log normal distribution $f_L(\mu, \sigma^2)$ with process mean $\exp[\mu + 0.5\sigma^2] = R$ and process variance $\{\exp[2\mu + \sigma^2]\}\{\exp[\sigma^2]\text{-}1\}$. Kaufman (1963) shows that given this assumption the post-exploration resource under Bayesian updating is $\tilde{R}_n \sim f_L(\mu_n, \delta_n^2)$, where

$$\mu_n = \alpha\mu_0 + (1 - \alpha)\bar{X}_n, \tag{4}$$

$$\alpha = \frac{m}{m+n}, \; 1 - \alpha = \frac{n}{m+n}, \tag{5}$$

$$\delta_n^2 = \sigma^2/(m+n), \tag{6}$$

$$\bar{R}_n = \exp[\mu_n + 0.5\delta_n^2] = \exp[\mu_n + 0.5\sigma^2/(m+n)], \tag{7}$$

and

$$\mathrm{Var}(\tilde{R}_n) = \{\exp[2\mu_n + \delta_n^2]\}\{\exp[\delta_n^2]\text{-}1\}. \tag{8}$$

Equation (4) shows that the new information will be combined with the old via a linear weighting system. Equation (5) describes the weighting system, where more weight is placed on the sampling results when n is larger and m is smaller, perhaps due to a large sampling program or low sampling error.[3] Equation (6) reveals that the new resource variance parameter is less than the initial variance parameter since $\delta_n^2 = \sigma^2/(m+n) < \delta_0^2 = \sigma^2/m$ when $n > 0$. This is the essence of proving a resource. Equation (7) provides the new resource estimate, which will only change from the original estimate $\bar{R}_0$ if $\bar{X}_n \neq \mu_0 + 0.5\frac{\sigma^2}{m} = \mu_0 + 0.5\delta_0^2 = \ln \bar{R}_0$, that is, if the log sample mean is different from the log of the initial resource expectation. Equation (8) is the variance of the revised resource distribution. Some (e.g., Haskett, 2003) argue that exploration should always decrease variance, whereas Galli et al. (2001) suggest that a variance

[1]It is common practice to model exploration updating within a Bayesian framework (see Kaufman, 1963; Peters, 1978, p. 547–549; Goria et al., 2001, 2002; Armstrong et al., 2005). For an example of Bayesian updating given discrete sampling outcomes in mining exploration, see Howe (1979, p. 212–218).

[2]More sophisticated models treat the initial resource parameters *and* measurement error as unknown (Kaufman, 1972).

[3]Less sampling error will result in $m = \sigma^2/\delta_0^2$ being smaller.

increasing with mean resource size is a desirable property of any sampling program. Equation (8) shows that the variance of the resource estimate can increase if the revised mean of the log of resources, μ_n, increases, consistent with Galli et al.

Compare these outcomes to popular decision-analytic models of exploration activity that assume that exploration reveals a binomial positive or negative outcome R_{large} or R_{small} that completely resolves uncertainty. That is, sampling would produce X_1 (= X_2 = X_3 = … = X_n) = R_{large} or X_1 (= X_2 = X_3 = … = X_n) = R_{small}, with $\sigma^2 = 0$ (or $n = \infty$, $\alpha = 0$, and $\delta_n^2 = 0$ in this model). Of course, in this case there would be no need for sampling stages 2 through n, as all resource uncertainty would be resolved in stage 1. Another way of saying this is that sample outcomes are perfectly correlated or perfectly dependent. In our model, sampling instead updates the resource estimate $\bar{R}_0$ and the variance around that estimate without ever completely resolving uncertainty.

Exploration decisions must be made prior to having the information resulting from the exploration program, and so for our analysis we must treat the exploration outcome $\bar{X}_n$ as the random variable $\tilde{\bar{X}}_n$ with an anticipated mean $\bar{\tilde{X}}_n$ (a mean of possible means). Given the prior resource distribution $\tilde{R}_0$, sampling (with error) is a priori assumed to be from a log normal process $f_L(\tilde{R}_0, \sigma^2)$. Since expected exploration payoffs are based on the mean or expected resource, it is the preposterior distribution of the possible posterior resource means $\bar{R}_n$ (see eq. 7) that is of interest. Let the distribution of these possible updated resource means be $\tilde{\bar{R}}_n$. $\tilde{\bar{R}}_n$ will then incorporate prior information $\tilde{R}_0$ as well as the anticipated sampling outcome $\bar{\tilde{X}}_n$. Kaufman (1963) shows that given n stages of planned sampling the preposterior mean resource quantity has a log normal distribution

$$\tilde{\bar{R}}_n \sim f_L(\mu_0 + 0.5\frac{\sigma^2}{m+n}, \delta_{0n}^2), \qquad (9)$$

where

$$\delta_{0n}^2 = \delta_0^2 - \delta_n^2 = \frac{\sigma^2}{m} - \frac{\sigma^2}{m+n} = \delta_0^2 \frac{n}{(m+n)}. \qquad (10)$$

Equation (9) indicates that there are many possible resource quantities after exploration, only one of which will be realized. The distribution of these possible resource quantities is log normal. Equation (10) gives the variance parameter reflecting the range of possible resource quantities, with

$$\mathrm{Var}(\tilde{\bar{R}}_n) = \{\exp[2\mu_0 + \delta_0^2]\}\{\exp[\delta_{0n}^2]-1\}]. \qquad (11)$$

Comparing equation (11) to equation (2), we can see that exploration is expected to reduce the variance around the resource estimate since via equation (10) $\delta_{0n}^2 < \delta_0^2$.

Kaufman also shows that the anticipated mean of the updated resource estimate is

$$\bar{\tilde{\bar{R}}}_n = \exp[\mu_0 + 0.5\sigma^2/m] = \exp[\mu_0 + 0.5\delta_0^2]. \qquad (12)$$

Exploration should, if it is unbiased, not be expected to change the initial resource estimate. From equations (12) and (1),

$$\bar{\tilde{\bar{R}}}_n = \bar{R}_0, \qquad (13)$$

as desired. Endogenous or controlled learning that is not expected to change the prior belief about the mean of a random variable, as is the case here, is called pure learning (Martzoukos, 2003), and is often used to model the R&D process within firms. We mentioned the similarity between exploration and R&D above.

Calculating the value of exploration information

This statistical model of geological uncertainty resolution, which would seem to model real-world attributes of exploration and information updating quite well, is at the heart of the exploration program. Preposterior beliefs as described in equations (9) through (13) can now be used to generate expected benefits of the exploration sequence, which in turn are weighed against the costs of the sequence. If the anticipated benefits are greater than the anticipated costs, all in present value terms, the exploration program creates wealth and will be undertaken. If the anticipated costs outweigh the anticipated benefits, all in present value terms, the property will be abandoned. We will now show that although exploration is not expected to increase resources (eq. 13) it can nevertheless be valuable, even where the initially estimated resource $\bar{R}_0$ is too small to warrant mine development should more resource not be found.

Adding the economics to the model, let the present value of the expected cost of the n-stage exploration program be $C(n, \bar{R}_0; \sigma^2)$, where cost is increasing with the length of the exploration program n and the initial resource estimate $\bar{R}_0$, and decreasing in the noise associated with each sampling stage σ^2 (more detailed exploration is more expensive). Upon completion of the program the property may be developed if the resource becomes a reserve or abandoned if the resource is uneconomic. Given the prevailing mineral price, let the time $t(n)$ NPV of this option to develop the property after exploration be $\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2) \geq 0$, where the option value is increasing in posterior reserve size $\tilde{\bar{R}}_n$ and decreasing in posterior reserve uncertainty δ_n^2. The value discounting associated with δ_n^2 is not only a result of risk aversion as in Campbell and Lindner (1985), but also due to the fact that ex post facto suboptimally sized mines and mills are inefficient and can only be made more efficient through expensive expansion or contraction expenditures. Again, information has value.

Define resource quantity γ as the minimum that will lead to exercise of the mine development option upon completion of exploration. For a guideline on representative values of γ for various mineral types, see Gocht et al. (1988, p. 112), although γ will vary with resource quality and commodity price. The gross payoffs of the exploration program are now as shown in Table 1. If after exploration the updated resource size is $\tilde{\bar{R}}_n \leq \gamma$, the company or government abandons the property and $\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2) = 0$. If, on the other hand, after exploration the updated resource is $\tilde{\bar{R}}_n > \gamma$, $\tilde{\bar{R}}_n$ becomes a reserve and the firm or government will exercise the option to develop the property, gaining a

TABLE 1. Gross Payoffs from Exploration Given Posterior Resource Size $\tilde{\bar{R}}_n$

Posterior resource size	Gross payoff from exploration
$\tilde{\bar{R}}_n \le \gamma$	0
$\tilde{\bar{R}}_n > \gamma$	$\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2)$

project value of $\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2) > 0$. The gross project payoff is thus

$$\max(\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2), 0) \ge 0, \tag{14}$$

which is nonlinear (convex) in $\tilde{\bar{R}}_n$. The present value of the net exploration payoff can be represented as

$$\beta(n)\ \max(\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2), 0) - \mathrm{C}(n, \bar{R}_0; \sigma^2), \tag{15}$$

where $\beta(n)$ is a discount factor $e^{-rt(n)}$ that takes into account only the time value of money. We only discount for the time value of money because at the point of making the development decision resource risk has been reduced to δ_n^2, and the value effects of this remaining uncertainty have already been taken into account in the calculation of $\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2)$. The time discount factor $\beta(n)$ is increasing in the number of stages of exploration since more exploration takes more time: $\mathrm{d}t/\mathrm{d}n > 0$. However, even if the exploration program will take decades, the discount factor $\beta(n)$ does not reduce the present value of the ultimate payoff to almost zero, as it would if it included a risk premium, since it only adjusts for the cost of money.

An example will illustrate the model. Consider a small, late-stage gold project with an anticipated \$50 M development cost after n stages of exploration based on the initial resource estimate $\bar{R}_0$. Assume a six-stage exploration program ($n = 6$) that will take five years to complete and a continuously compounded time discount rate of $r = 5$ percent per year. With these parameters, $\beta(n) = 0.78$.[4] Assume that the present value of exploration costs is $\mathrm{C} = \$(^{10}/_6)\, n\, \bar{R}_0 = \$10\, \bar{R}_0$, probably reasonable for a property at this stage of development given that Schodde (2004) calculates a gold finding cost of \$32/oz through all stages of exploration. If $\delta_0^2 = 0.75$ and $\sigma^2 = 1.5$, then $m = 2$ and from equation (6), $\delta_n^2 = 0.1875$.[5] This reveals that there will be remaining resource uncertainty after exploration, although it is less than initial resource uncertainty. Assume that this remaining uncertainty is low enough that any economic mineralization can be mined, with a post-exploration resource value of \$50/oz given price and extraction cost expectations. The gross exploration payoff is then $\max(\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2), 0) = \max(50\, \tilde{\bar{R}}_n - \$50\ \mathrm{M}, 0)$, shown as line (1) of Figure 1 for $\tilde{\bar{R}}_n$ of between 50,000 oz and 2 Moz. The discounted (to stage 0) gross exploration payoff $\beta(n)\max(\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2), 0) = 0.78\ \max(\$50\ \tilde{\bar{R}}_n - \$50\ \mathrm{M}, 0)$ is shown as line (2) in Figure 1. These are the present value payoffs from exploration conditional on each updated resource estimate. For example, an updated reserve $\tilde{\bar{R}}_n = 1.25$ Moz will have a payoff with a present value of \$9.7 M, while an updated resource $\tilde{\bar{R}}_n = 950{,}000$ oz will have a zero present value payoff. In this example γ, the size at which the post-exploration resource is a reserve, is 1.0 Moz.

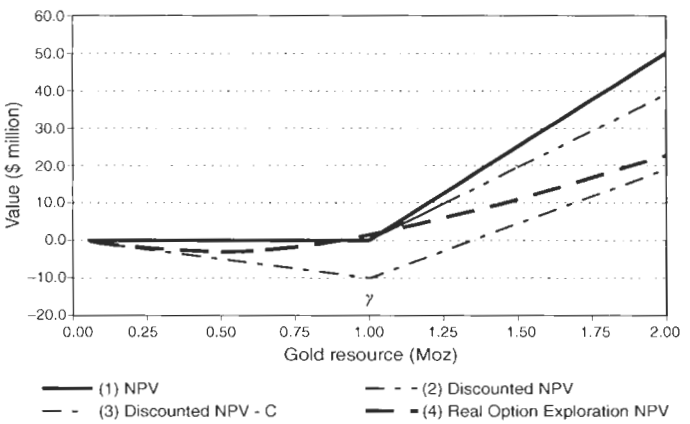

FIG. 1. Exploration project value as a function of the resource estimate. The vertical axis of the figure measures value, and the horizontal axis measures resource quantity in million of ounces of gold. Line (1) represents the NPV of the resource after exploration has been completed. For any resource level less than γ there is insufficient resource to warrant a mine. Resource levels greater than γ may be defined as reserves. Line (2) represents the present value of the post-exploration project at the start of exploration. Line (3) is the traditional discounted cash flow valuation of the exploration project, the discounted payoff from exploration less the exploration cost C. Line (4) is the real option calculation of the exploration project value.

The task is now to manage and value the option to invest in exploration, as of stage zero. Traditional DCF analysis would estimate the anticipated resource quantity after exploration $\tilde{\bar{R}}_n$, which, from equation (13), is the same as the prior resource estimate $\bar{R}_0$, calculate the anticipated gross exploration payoff after stage n,

$$\max(\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2), 0) = \max(\mathrm{NPV}_n(\bar{R}_0, \delta_n^2), 0),$$

discount that anticipated payoff for time and resource estimation uncertainty $\mathrm{Var}(\tilde{\bar{R}}_n)$, and deduct the present value of exploration costs. If we follow the standard finance assumption that resource uncertainty is unsystematic and can be diversified away and that the firm or government is risk neutral, the decision-maker can make a decision based on pure discounted expectations:

$$\mathrm{DCF}_0 = \beta(n)\ \max(\mathrm{NPV}_n(\bar{R}_0, \delta_n^2), 0) - \mathrm{C}(n, \bar{R}_0; \sigma^2). \tag{16}$$

[4]A traditional risk-adjusted discount rate of 15% would reduce $\beta(n)$ to 0.47, unfairly reducing project value by 40%. If exploration were to take 20 years, $\beta(n)$ would be 0.37 discounting only for time, compared with a $\beta(n)$ of 0.05 at a 15% discount rate.

[5]These parameters produce an initial resource coefficient of variation of 1.057, where a deposit with a prior expected resource of 1Moz has a 0.6% chance of R being greater than 5 Moz, a 36.2% chance of R being between 1 Moz, and a 63.2% chance of R being less than 1 Moz. The posterior resource estimate will have a coefficient of variation of 0.454 due to the reduced uncertainty from sampling.

Under the traditional DCF framework, exploration will proceed when the ordered pair $(\bar{R}_0, \delta_n^2)$ produces a positive DCF_0. Returning to the numerical example, $DCF_0 = 0.78 \max(\$50\bar{R}_0 - \$50\text{ M}, 0) - \$10\bar{R}_0$. Line (3) in Figure 1 shows DCF_0 values for prior resource estimates $\bar{R}_0$ of between 50,000 oz and 2 Moz. Exploration will proceed for any initial resource estimate of greater than 1.34 Moz since $DCF_0 > 0$ for these resource quantities. For initial resource estimates of less than 1.34 Moz, the DCF_0 of proceeding with exploration is negative, so the property will be abandoned.

In this DCF decision framework, projects where the probability of success is 1 in 10 or less, as is the case when initial resources are estimated at 500,000 oz or less, are far from satisfying the hurdle to go forward with exploration.[6] This proves problematic with intuition and evidence that exploration is often warranted when the chance of success is in that order. The essential problem with this DCF valuation approach is that the nonlinear (convex) payoff structure is ignored. This is remedied by the more comprehensive real options approach. Let $h(\tilde{\bar{R}}_n|\mu_0,\delta_0^2)$ represent the preposterior log normal distribution of expected resource quantities upon completion of exploration given the prior resource estimate parameters μ_0 and δ_0^2, where

$$P((\tilde{\bar{R}}_n|\mu_0,\delta_0^2) \le \gamma) = \int_0^{\gamma} h(\tilde{\bar{R}}_n|\mu_0,\delta_0^2)\,d\tilde{\bar{R}}_n$$

and

$$P((\tilde{\bar{R}}_n|\mu_0,\delta_0^2) > \gamma) = \int_{\gamma}^{\infty} h(\tilde{\bar{R}}_n|\mu_0,\delta_0^2)\,d\tilde{\bar{R}}_n = 1 - P((\tilde{\bar{R}}_n|\mu_0,\delta_0^2) > \gamma).$$

This last expression represents the probability that the resource will become a reserve after n stages of exploration conditional on μ_0 and δ_0^2. The preposterior real option value of expected benefits from exploration is[7]

$$\beta(n)\int_0^{\infty}\max(NPV_n(\tilde{\bar{R}}_n, \delta_n^2), 0)\,h(\tilde{\bar{R}}_n|\mu_0,\delta_0^2)\,d\tilde{\bar{R}}_n, \quad (17)$$

and the stage zero real options expected exploration property value is

$$RO_0 = \beta(n)\int_0^{\infty}\max(NPV_n(\tilde{\bar{R}}_n, \delta_n^2), 0)\,h(\tilde{\bar{R}}_n|\mu_0,\delta_0^2)\,d\tilde{\bar{R}}_n - C(n,\bar{R}_0;\sigma^2) \ge -C(n,\bar{R}_0;\sigma^2). \quad (18)$$

Exploration will go ahead whenever $RO_0 > 0$ conditional on the ordered pair (μ_0,δ_0^2). When $\max(NPV_n(\tilde{\bar{R}}_n, \delta_n^2), 0)$ is convex in $\tilde{\bar{R}}_n$, which is the case whenever $\gamma > 0$, the value in equation (18) will be greater than the value in equation (16) via Jensen's Inequality, and the resource size cutoff for continued exploration will be smaller than under the DCF approach, possibly even smaller than γ. This is a key insight from real options analysis.

For the numerical example above the real option exploration value, RO_0, is depicted in Figure 1 as line (4). While the log normal distribution of preposterior resource estimates lends itself to the potential for closed-form solutions for RO_0, we estimate RO_0 using a Monte Carlo simulation over 50,000 trials. All positive values on line (4) reveal the value generated by exploration, for without exploration all of this value would be lost (the project would be abandoned at stage 0). Now the exploration program will go ahead for any initial resource estimate greater than 900,000 oz, rather than the 1.34 Moz cutoff under traditional DCF analysis. If the current resource estimate is less than 900,000 oz the project will be abandoned, since at these levels going ahead with exploration costs more than the present value of any revenues that are likely from mining ultimate reserves. A 1.5 Moz resource has a current value of \$10.9 M, more than double the estimate using traditional DCF analysis. A 950,000-oz resource is now worth \$0.6 M, compared with being worthless via the traditional DCF approach. Since we have used the same discount factor $\beta(n)$ in both the DCF and real options calculations and valued stage n mineralization in both cases at \$50/oz, the difference between the line (4) and line (3) valuations in Figure 1 is solely due to the real options recognition that decisions about whether or not to develop the project are made conditional on the outcome of the exploration program, an option to develop, whereas the traditional DCF approach assumes that the decision is made at time 0, based on $\bar{R}_0$.[8] Of note, carrying out exploration at a $\bar{R}_0$ = 910,000-oz deposit has a 3 in 10 chance of exploration success, success defined as proving an expected 1 Moz or more. Exploration is nevertheless economic for properties with anything more than 900,000 oz of resources at stage 0 because of the high payoffs of an unexpected but large find, with the downside limited to exploration costs. The real options model is showing this and allowing for speculative exploration, whereas the traditional DCF model does not.

A Generalized Model of Exploration

The above analysis, while depicted in Figure 1 for the specific parameter values given in the example, is completely general. Given any gross payoff function, line (1), that is convex in current resource quantity, Jensen's Inequality will cause the real option resource value line (4) to be a smooth and strictly convex continuous function everywhere above line (3), the DCF resource valuation line. The real option value line is anchored at the origin and asymptotically approaches line (3) as initial resources become infinite. The exploration cutoff resource quantity will always be lower in the real options analysis than in the traditional DCF analysis, and the exploration project value will always be higher. Most significantly, exploration can be justified where initial resource estimates are less than required to support a mine, $\bar{R}_0 < \gamma$.

In addition, through some comparative static exercises the real options model provides rich insights into factors influencing exploration decisions:

[6]At an initial resource estimate of 500,000 oz, there is a 0.0972 chance that the posterior resource will be greater than 1 Moz given our model parameters.

[7]Compared to a DCF benefit estimate of $\beta(n)\max[NPV_n(\int_0^{\infty}\tilde{\bar{R}}_n h(\tilde{\bar{R}}_n|\mu_0, \delta_0^2)d\tilde{\bar{R}}_n, \delta_n^2), 0]$.

[8]In practice, the values calculated using the traditional DCF approach would probably be even lower than line (3) since this approach typically discounts remaining stage n resource risk more heavily than in our treatment.

1. Exploration project value is not monotone in initial resource quantity. Lines (3) and (4) are anchored at the origin and slope downwards and then upwards as initial resources increase (Fig. 1). Traditional project valuation shows the worst, or most uneconomic, project, i.e., the lowest point on line (3), to be that with an initial resource estimate of γ. Real options shows the most uneconomic project, the lowest point on line (4), to be that with an initial resource estimate of well less than γ. Real options are thus more forgiving in terms of small size resources. Line (4) shows real option exploration project value to be less sensitive to resource size than traditional DCF exploration project value, shown as line (3).

2. Additional stages of exploration always provide gross value through the value-of-information effect. Equation (6) shows that additional stages of exploration lower δ_n^2, which we have assumed will increase $\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2)$ for any $\tilde{\bar{R}}_n$. This causes γ to shift left and the slope of the upward-trending portion of line (1) to increase. In addition, via equation (10), more sampling stages increase the variance of the possible posterior resource quantities. This is a result that is not picked up in the traditional DCF analysis. Both effects will cause the real option value of the exploration property to increase: the first effect causes line (4) to rotate upwards, while the second effect causes the rotated line (4) to be less convex. Exploration creates value even though it is not expected to increase the quantity of the resource.

3. There is an optimal amount of exploration to commit to at stage zero. An increase in exploration effort increases $C(n, \bar{R}_0;\sigma^2)$, causing a downward rotation in lines (3) and (4) that offsets the gains outlined in the preceding paragraph. As a result, there will be a value-maximizing exploration effort n^* that will depend on the cost of exploration, the payoff value function $\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2)$, and initial resource beliefs.

4. Higher sampling precision, if it is costless, will increase exploration activity and the value of exploration. Higher sampling precision lowers σ^2, and, via the specification $\delta_0^2 = \sigma^2/m$, lowers m. From equations (10) and (11) this causes $\mathrm{Var}(\tilde{\bar{R}}_n)$ to increase, which will result in line (4) being less convex. The amount of and value of exploration projects will increase. The effects here are very similar to an increase in the number of sampling stages. Indeed, increased sampling precision and increased sampling size are imperfect substitutes in exploration value generation. Traditional DCF analysis would not pick up this effect.

5. Lower exploration costs increase exploration activity and the value of exploration. Lower exploration costs will rotate lines (3) and (4) upwards. The amount of and value of exploration projects will increase.

6. Higher margins upon extraction increase exploration activity and the value of exploration. Higher margins shift γ to the left and make the upward-trending portion of line (1) steeper, increasing the amount and value of exploration projects. The impact of higher margins on exploration is not identical to a decrease in exploration costs, but has the same broad effects. The impacts of mineral price on exploration activity have been noted (R.C. Schodde, unpub. data, 2003; Natural Resources Canada, 2004).

7. Higher interest rates will reduce exploration activity and lower the value of exploration. Higher interest rates will increase the $\beta(n)$ discount factor, rotating the upward sloping portion of line (3) clockwise and thereby rotating line (4) clockwise. Exploration activity will decrease, and the value of any given exploration project will also decrease. This assumes prices and costs are not affected by the change in interest rates. If they are, the impacts of interest rates will be combined with the previously noted impacts of prices (paragraph 6 above) and costs (paragraph 5 above) on exploration.

8. Higher initial resource uncertainty is not necessarily beneficial to exploration project value. Initial resource uncertainty is parameterized as δ_0^2. In examining the effects of an increase in δ_0^2 we wish to preserve the initial resource estimate $\bar{R}_0$, which means that via equation (1) an increase in δ_0^2 must be offset by a decrease in μ_0. That is, $\bar{R}_0 = \exp[\mu_0 + 0.5\delta_0^2]$ must be held constant. Equations (10) and (11) reveal that such a compensated increase in δ_0^2 results in an increase in $\mathrm{Var}(\tilde{\bar{R}}_n)$, with positive valuation impacts as in paragraph (4) above. But equation (6) reveals that an increase in δ_0^2 (a decrease in m) will increase δ_n^2, remaining resource uncertainty after exploration. This will lower $\mathrm{NPV}_n(\tilde{\bar{R}}_n, \delta_n^2)$. The ultimate impact on exploration property value is ambiguous.

To show this using the numerical example, assume that δ_0^2, the variance parameter on the initial resource estimate, rises from 0.75 to 1.50, with m consequently falling from 2 to 1. This increases the coefficient of variation on initial resources to 1.866 from 1.057. From equation (6) δ_n^2 rises from 0.1875 to 0.214. We assume that this increase in δ_n^2 lowers developed resource value from \$50/oz to \$40/oz due to the increased ex post facto suboptimality of the likely plant design associated with an increased δ_n^2. Exploration cost C remains unchanged. Figure 2 plots the original (lines 3 and 4) and new (lines 1 and 2) traditional DCF and real option valuations. Increasing initial resource uncertainty has increased exploration project real option value for projects of about 1.0 Moz or less (compare lines 2 and 4), while decreasing exploration real option project value for projects of greater than about 1.0 Moz. A 900,000-oz project has increased from a real options exploration value of –\$43,000 to a value of \$415,000, while a 1.1 Moz project has decreased in value from \$3 to \$2.4 M. The observation that increasing uncertainty does not always benefit option holders has been pointed out by Davis (2002). In that paper, as in this model, increasing uncertainty is most likely to be beneficial for out-of-the-money options, or, in Figure 2, initial resource estimates to the left of γ. Of course, in traditional DCF analysis, increasing uncertainty always reduces asset value (line 1 in Fig. 2 is everywhere on or below line 3). Yet an exploration property with 900,000 oz of certain resource is clearly less valuable in our example than a resource that has a mean of 900,000 oz and some variance around that mean that allows for reserves to be discovered. It is only through uncertainty, the possibility of ultimate resources being greater than 1 Moz in this case, that exploration becomes valuable at sites with small resource estimates.

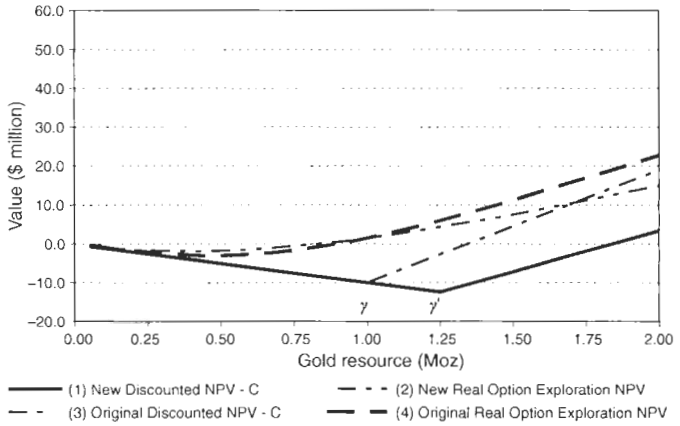

FIG. 2. Exploration project value when initial resource uncertainty is high. Higher resource uncertainty serves to make post-exploration resources less valuable, increasing the post-exploration resource size necessary for economic extraction from γ to γ'. Line (1) represents the new discounted payoff from exploration under higher initial resource uncertainty. Line (3) represents the original discounted payoff from exploration, which is the same as line (3) in Figure 1. Line (2) represents the new real option value of the exploration project, and line (4) represents the original real option value of the exploration project, which is the same as line (4) in Figure 1. Increased initial resource uncertainty has lowered the traditional exploration property value, since line (1) is everywhere below line (3). The real option value is higher under increased uncertainty when initial resources are below about 1 Moz, and lower when initial resources are above about 1 Moz.

While our model of exploration and learning is very simple, all of these insights are in accordance with what we would expect. Our main result is that exploration can have value even when the traditional DCF value of the investment is negative, but only for resource quantities above some lower bound; exploration of all projects regardless of initial resource expectations is not warranted. The framework also reveals that over time, as exploration costs rise, exploration will become less valuable and more likely to be curtailed on properties where resources are low. On the other hand, mineral price rises will tend to offset the effects of the higher exploration costs. We have modeled the real-world realization that increasing exploration precision and effort provides value that can offset its cost, and that exploration effort will respond to changes in macroeconomic variables such as exploration costs, mineral prices, and interest rates. Finally, those holding leases with low initial resource estimates would want higher initial resource uncertainty, whereas those holding high quality leases would not.

The model has an additional use. Having built a model of late-stage exploration project value, reconnaissance optimization now becomes possible given payoff line (4) (Fig. 1). To date such payoffs to reconnaissance have been presented qualitatively (J. Hronsky, unpub. data, 2004; R.C. Schodde, written commun., 2004) with a view to suggesting a minimum and maximum target size for reconnaissance. We now have a way to model the payoffs quantitatively.

Managing the Exploration Program under Price and Resource Uncertainty

While informative, the analysis to date has ignored economic (price) uncertainty and an important management alternative, the option to defer late-stage or even initial exploration in low-price environments. Project deferral can be a valuable alternative to project abandonment.

As noted earlier, price and reserve uncertainty are rarely modeled together, and exploration deferral is even more rarely modeled. To our knowledge the only other paper to model both reserve and price uncertainty and the option to defer exploration is Cortazar et al. (2003). In that model, there are 11 possible resource outcomes, R. Initially there is a prior expectation over these resource outcomes, $\tilde{R}_0$. Exploration over four stages lasting a total of eight years serves to resolve all uncertainty over R. A single poor exploration outcome results in project abandonment, whether this is optimal or not. Exploration can be costlessly and perpetually deferred at any stage pending improvement in the economic environment. Under these assumptions Cortazar et al. find considerable value to deferring exploration in low price environments. In their worked example a $1,000 M post-development prospect with a current NPV of –$6.29 M has a real option value of $3 M dollars, with the option to defer exploration contributing $1.6 M to the $3 M.

In this section we augment our model via a real-world case study that considers reserve uncertainty and price uncertainty. Our example and treatment is similar to that of Cortazar et al., only we have five possible discrete levels of R, assume that there is still uncertainty over R after exploration is completed, allow for repeated exploration upon a negative sampling outcome (since exploration information is imperfect), explicitly model resource updating in a Bayesian framework given exploration measurement error, impose costs while deferring exploration, and impose finite time periods over which exploration can be deferred. We use a discrete resource distribution because this is how the participants in the case study thought about possible exploration outcomes. While the original case study involved gold, we also fabricate a hypothetical copper exploration example because copper prices exhibit reversion to a long-term equilibrium price, whereas gold prices do not (Schwartz, 1997). This difference leads to markedly different exploration decisions in the various price environments.

Exploration lease background information

A company wishes to value and manage a geological anomaly that has been identified by an aerial geophysical survey during a green-fields exploration program. The geographical region in which the anomaly is located is large and has seen some previous exploration activity. A government geological database is publicly available with over 700 entries identifying various regional geological anomalies, the extent of exploration, and the results of exploration to date at each anomaly. Company geologists also have long exploration experience in the region. Based on this information, the senior geologists within the company grouped the possible deposit outcomes for the anomaly into five orebody size classifications: (1) a world-class (WC) deposit R_{WC} with a probability of 0.15 percent; (2) a large-scale (LS) deposit R_{LS} with a probability of 0.35 percent; (3) a mid-sized (MS) deposit R_{MS} with a probability of 2.0 percent;

(4) a small-sized (SS) deposit R_{SS} with a probability of 5.0 percent; and (5) a waste deposit (resources of less than γ, the minimum size needed to mine the resource) with a probability of 92.5 percent. Tables 2a and 2b list the geological and economic characteristics of each deposit size for each metal. The probabilities of each deposit size reflect the priors that will inform the analysis, a discrete version of the continuous deposit distribution $\tilde{R}_0 \sim f_L(\mu_0, \delta_0^2)$ presented above. Given the above probabilities and assuming no economic mineralization in the waste case, the gold lease has a time zero resource size $\bar{R}_0$ of 100,000 oz and the copper lease has a resource size $\bar{R}_0$ of 0.051 billion lb. While these estimates are by nature subjective, they are held to represent an accurate reflection of knowledge about the orebody given the information at hand. Exploration sampling will then serve to update these priors as in equations (4) through (8), with more weight being placed on the exploration outcome (i) the less the exploration measurement error, and (ii) the less the certainty over the original priors. At this stage, for instance, we would expect considerable weight to be given to the exploration outcomes.

The NPV values presented in the last column of Tables 2a and 2b are calculated using the traditional DCF cash flow approach that is common in the mining industry. The gold project NPV values are calculated with a forecast gold price of \$400/oz and a risk-adjusted discount rate of 8 percent/yr continuously compounded. The copper project NPV values are calculated with a forecast price of \$1.00/lb and a discount rate of 12 percent/yr continuously compounded. These NPVs assume that the n stages of exploration are complete and that a particular deposit type with reserve R denoted in column 4 of Table 2a or 2b has been discovered. The expected gross exploration payoff, $\text{NPV}_n(\bar{R}_0, \delta_n^2)$, is roughly \$1 M for the gold case and \$6 M for the copper case. Prior to development the project is also subject to a feasibility study. That study costs \$300,000 plus a variable cost proportional to the expected capital cost of the project. For example, for the $\bar{R}_0$ in Tables 2a and 2b, the expected feasibility cost is \$2.7 M.

A staged exploration program and management operating decisions

The company's geologists have developed a three-stage ($n = 3$) sequential exploration program for the anomaly, each stage being one year in length. Exploration costs are incurred at the start of each stage. Exploration results upon completion of each stage are categorized as good, fair, or poor. This sampling outcome is then combined with the prior information about the deposit to obtain an updated deposit description in a Bayesian framework described in detail below. The first stage of exploration costs \$1 M and consists of surface geochemical sampling, mapping, and reverse circulation drilling. The second stage of exploration costs \$2 M and involves detailed geochemical sampling and diamond drilling. The third stage of exploration costs \$3 M and involves a further geochemical study, extensive diamond drilling, and specialized geologic interpretation. Upon completion of exploration the firm must immediately either begin development or abandon the property.

Managers can elect at the beginning of each stage to proceed with exploration, to defer the next stage of exploration for two years at a cost of \$75,000 annually, or to allow the lease to lapse at no cost. If exploration is deferred, the exploration managers must decide at the end of a two-year deferral period to either abandon the lease at no cost or to start the next stage of exploration. Figure 3 delineates the exploration decision tree for both the gold and copper leases. The immediate decision is whether or not to proceed with exploration. Certainly, with cumulative exploration costs of \$6 M, an expected feasibility cost of \$2.7 M, and expected gross exploration payoffs of roughly \$1 M for the gold property and \$6 M for the copper property, the

TABLE 2a. Characteristics of Possible Gold Projects

Deposit type	Size (million t ore)	Grade (g/t)	Gold (Moz)	Capital expenditure (\$ million)	Development horizon (years)	Annual production (million t ore)	Production life (years)	Unit operating cost (\$/oz)	Unit operating cost (\$/t)	DCF NPV (\$ million)
World class	720	1.00	23.20	720	4	40.0	18	248	8.0	765.5
Large	160	1.20	6.20	300	3	16.0	10	210	8.1	339.2
Mid-size	36	1.30	1.50	120	3	6.0	6	199	8.3	68.5
Small	6	1.40	0.27	28	2	1.5	4	237	10.7	2.6

TABLE 2b. Characteristics of Possible Copper Projects

Deposit type	Size (million t ore)	Grade (%)	Copper (billion lb)	Capital expenditure (\$ million)	Development horizon (years)	Annual production (million t ore)	Production life (years)	Unit operating cost (\$/lb)	Unit operating cost (\$/t)	DCF NPV (\$ million)
World class	720	0.76	12.05	720	4	40.0	18	0.48	8.0	859.0
Large	160	0.92	3.24	300	3	16.0	10	0.40	8.1	462.6
Mid-size	36	0.96	0.76	120	3	6.0	6	0.39	8.3	105.8
Small	6	1.00	0.13	28	2	1.5	4	0.48	10.7	13.2

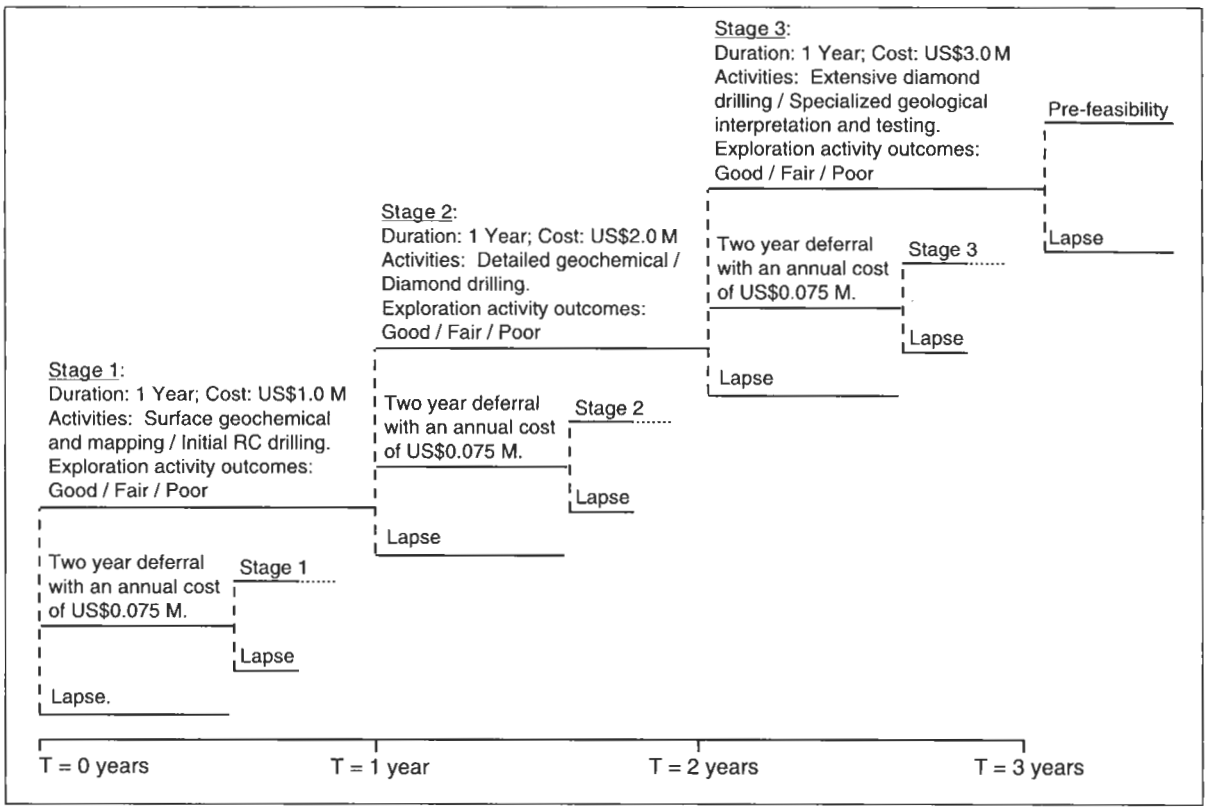

FIG. 3. Exploration decision tree for both the gold and copper leases. At each of three stages of exploration the decision is whether to proceed with exploration, to defer exploration for two years, or to let the lease lapse (abandon the lease). If deferral is chosen the decision at the end of the deferral period becomes whether to proceed with that stage of exploration or let the lease lapse. At the end of the third stage of exploration the lease moves immediately into prefeasibility/feasibility and development. Deferral costs $0.075 M per year. Stage 1 exploration costs $1 M, stage 2 costs $2 M, and stage 3 costs $3 M.

economics are not favorable at first appearance. However, as we will show, the geological and price uncertainty, combined with the ability to abandon exploration should either price or reserves drop, provides these properties with enough real option value to warrant at least the first stage of exploration in most price environments.

Geological uncertainty and information updating

Completing an exploration stage provides information that allows exploration managers to reassess whether it is beneficial to continue exploring. As in our earlier model, exploration provides imperfect information and does not resolve all uncertainty about the deposit. Figure 4 outlines this partial resolution of geological uncertainty over the life of the exploration program. Updating of deposit probabilities starts after the first stage of exploration, where stage 1 exploration moves the program to one of the outcomes in the second column in Figure 4. By the end of the program a final set of deposit-type probabilities exists, with resolution still imperfect ($\delta_3^2 > 0$).

As in the first part of the paper, the updated probabilities at each stage are calculated with Bayes Theorem. Given the discrete nature of the deposit sizes in this example, the discrete version of Bayes Theorem gives the posterior probabilities:

$$P(\text{Deposit}_I|\text{EA Outcome}_j) = \frac{P(\text{EAOutcome}_j \mid \text{Deposit}_I)\cdot P(\text{Deposit}_I)}{\sum_{I=1}^{N} P(\text{EAOutcome}_j \mid \text{Deposit}_I)\cdot P(\text{Deposit}_I)}$$

The index "I" indicates the type of deposit among the total number of deposit possibilities, N = 5. The index "j" indicates the exploration activity outcomes (EAOutcome) for a particular exploration stage. Each stage of exploration has three possible outcomes (j = poor, fair, and good). $P(\text{Deposit}_I)$ is the probability that a particular deposit may occur given the current information available at the start of an exploration stage. When there is exploration measurement error, $\sigma^2 > 0$ and $P(\text{Deposit}_I \mid \text{EA Outcome}_j) \in (0,1)$ due to error in the signals from exploration.

The numerator of the right-hand side of Bayes Theorem represents the joint probability that Deposit_I and Exploration Outcome_j both occur at an exploration stage, given past exploration outcomes. The first term of numerator, $P(\text{Exploration Outcome}_j \mid \text{Deposit}_I)$, is an indication of the quality of information provided by the current exploration stage and the exploration program as a whole. This is where

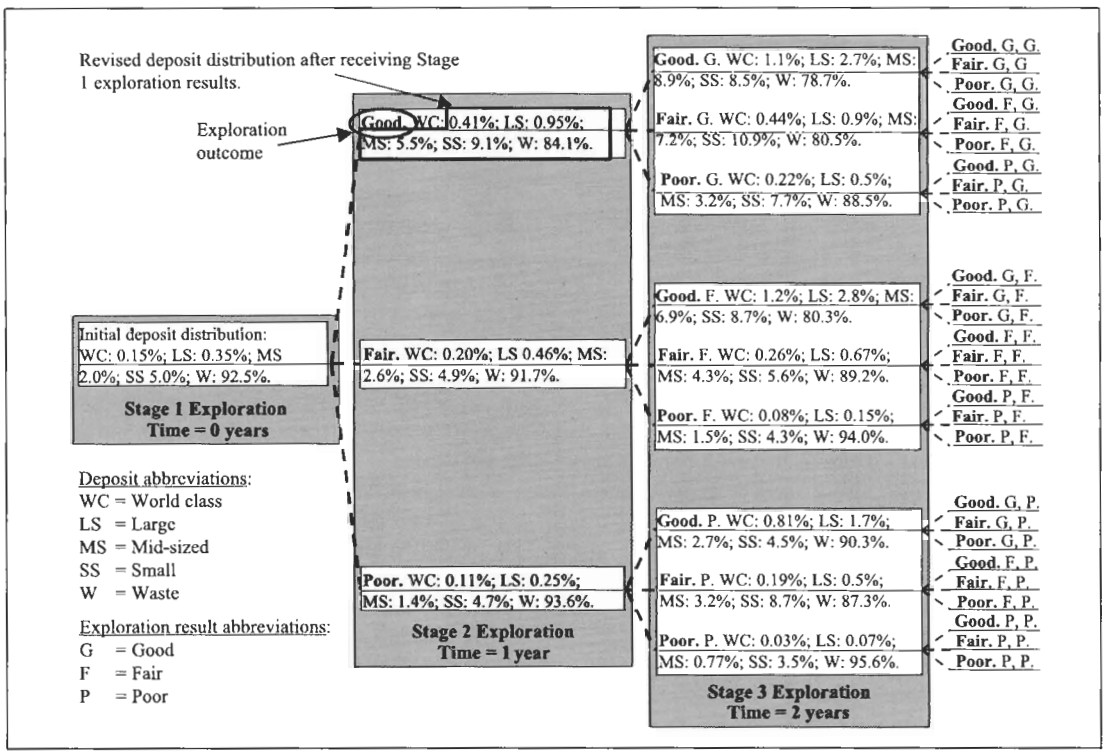

FIG. 4. Resolution of exploration uncertainty. Undertaking a stage of exploration takes one year and results in either a good, fair, or poor exploration outcome. The boxes hold the deposit probabilities at the start of each exploration stage $i = 1, 2, 3$ conditional on previous exploration results. Each exploration outcome serves to update the assessed probabilities of the resource being a certain deposit type. In a break with most models of exploration, uncertainty over the deposit type remains even after the 3rd stage of exploration. The final probabilities over the five deposit types for the 27 possible stage-three exploration outcomes are not shown.

the geologist's confidence in the exploration measurements is injected into the model. This term stipulates the probability that the exploration stage would return a particular outcome given that a specific deposit is actually there.

Determining the priors $P(\text{Deposit}_I)$ requires professional geological opinion from someone familiar with the geological setting and exploration program. As we noted before, the information is imperfect, but it is the best available information upon which to make a decision. The information is, of course, improved as exploration proceeds, which is the value of exploration activity.

Devising an exploration outcome probability tree for a multi-stage exploration program is not a simple task. Figure 4 illustrates this; despite extended effort working with geologists' expectations about this deposit, the tree still contains a few inconsistencies, even though by construction the expectations are unbiased. For example, after a fair stage 1 result and a good stage 2 result the probability of a world-class deposit occurring is 1.2 percent, higher than the 1.1 percent probability of a world-class deposit occurring after a good stage 1 result and a good stage 2 result. While it would be reasonable for a fair-good sequence to yield a more optimistic resource view than a good-fair sequence due to the superior information from stage 2 exploration, a good-good sequence must lead to better prospects than a fair-good sequence. Geologists valuing an exploration program must expend considerable effort to ensure that such inconsistencies are removed from the exploration tree.

Mineral price uncertainty

We model gold and copper prices over the life of the exploration program by one-factor geometric Brownian motion (in the case of gold) and mean reverting (in the case of copper) stochastic processes. Details of these processes can be found in Laughton and Jacoby (1993), Salahor (1998), and Samis (2000). The parameters used to construct these price models are presented in Table 3.

Figure 5 depicts the time zero gold price forecast and forecast confidence boundaries for the next 10 years when the spot price of gold is \$400/oz. The gold price model incorporates a flat expected nominal gold price, reflecting a lease rate equal to the bond rate of 5 percent.[9] The gold price model has two important characteristics. First, gold price uncertainty increases with the forecast horizon. This is highlighted by confidence boundaries that continue to move apart. Second, a price shock results in a shift of price

[9]Gold lease rates are usually less than the bond rate. Our parameter values are chosen to be consistent with the price forecasts that what we observe practitioners using in DCF valuations.

TABLE 3. Parameters for the Gold and Copper Price Processes

Price model parameter		Gold	Copper
Current spot price	($/oz or $/lb)	400	1.00
Long-term price median	($/oz or $/lb)	N/a	0.95
Growth rate of the price median	(%/yr)	-0.72	0.00
Price volatility	(%/yr)	12	23.3
Correlation between metal and market uncertainty, ρ		0.1	0.8
Price of market risk, PRiskMkt		0.4	0.4
Rate of mean reversion (half life, years)		N/a	1.878
Nominal risk-free rate (%/yr, continuously discounted)		5.0	5.0

expectations. This is illustrated (Fig. 5) in a scenario where the gold price decreases from $400/oz to $320/oz over the first two years of exploration. The revised price forecast and confidence boundaries due to the 20 percent price shock are outlined starting in year two. The real options model takes all such price scenarios and revised expectations into account on a probability-weighted basis when calculating the time zero value of an asset.

Gold price uncertainty is assumed to have a low correlation with general financial market uncertainty (i.e., has a low Capital Asset Pricing Model beta). This is consistent with the gold mining industry practice of using low risk-adjusted discount rates. The long dashed lines in Figure 5 delineate the risk-adjusted (RA) gold price expectations used in the RO computations. When the gold price is $400/oz at time zero, the risk adjustment trims a few dollars off the gold price forecast as compensation for risk. Similarly, if gold price in year two turns out to be $320/oz, a revised risk-adjusted gold price expectation is used thereafter, which again trims a few dollars off the gold price forecast at time two for risk. These long dashed lines are also known as the gold forward price, as quoted on NYMEX or COMEX. In each price forecast the compensation for risk increases the further into the future the price is forecast.

Figure 6 outlines the expected prices and confidence boundaries for the copper price model. A key characteristic of the copper price model is price reversion, the result of supply/demand influencing price behavior. The copper price model parameters in Table 3 assume price reversion to a long-term equilibrium of $0.98/lb. Mean reversion causes the long-term forecast of copper price to be largely unaffected by the level of copper price in year zero or year two; after a hypothetical price shock of –55 percent between year zero and year two the year two forecast of copper price in year 10 is almost the same as the year zero forecast of price in year 10.

Price reversion also affects risk discounting. In the presence of reversion, price uncertainty almost saturates (stops growing) in the long-term. The confidence boundaries in Figure 6 exhibit this in that the 80 percent confidence range for long-term copper prices eventually stabilizes with an upper boundary of $1.37/lb and a lower boundary of $0.68/lb. Reflecting this stabilization of price risk, the difference between the expected copper price and the risk-adjusted expected copper price (or the forward price, again delineated by the long-dashed lines) increases initially and then stabilizes with the saturation of copper price uncertainty. In a DCF framework, this risk adjustment effect could only be achieved by using a declining discount rate as the cash flows become more distant.

Valuation results

The real option method folds all of this geological and

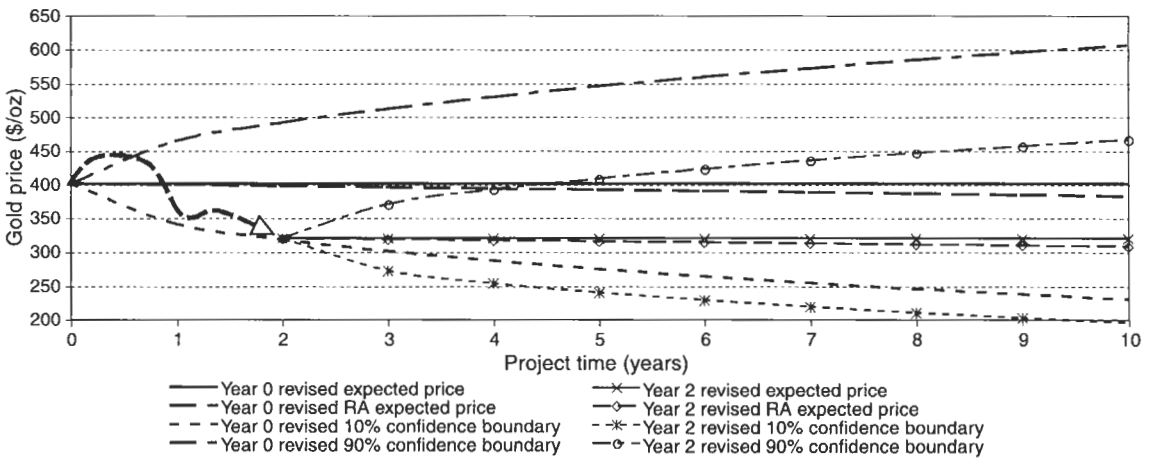

FIG. 5. Gold price model. The vertical axis shows the price of gold, and the horizontal axis shows years from the start of the exploration project. The solid line delineates gold price expectations, or the price mean, given a current (stage zero) spot price of $400/oz. The dashed lines outline the 90 and 10 percent confidence boundaries around this forecast, based on gold prices following a random walk stochastic process. The figure also shows the risk-adjusted (RA) expected price, which is also the forward gold price curve. If the path of gold follows a random walk between year 0 and year 2 that results in a year 2 price of $320/oz, the updated expectations and confidence boundaries around that expectation, as well as the forward curve, will be as depicted. Because of the random walk nature of gold prices the confidence boundaries on any price forecast are always increasing as the forecast horizon increases.

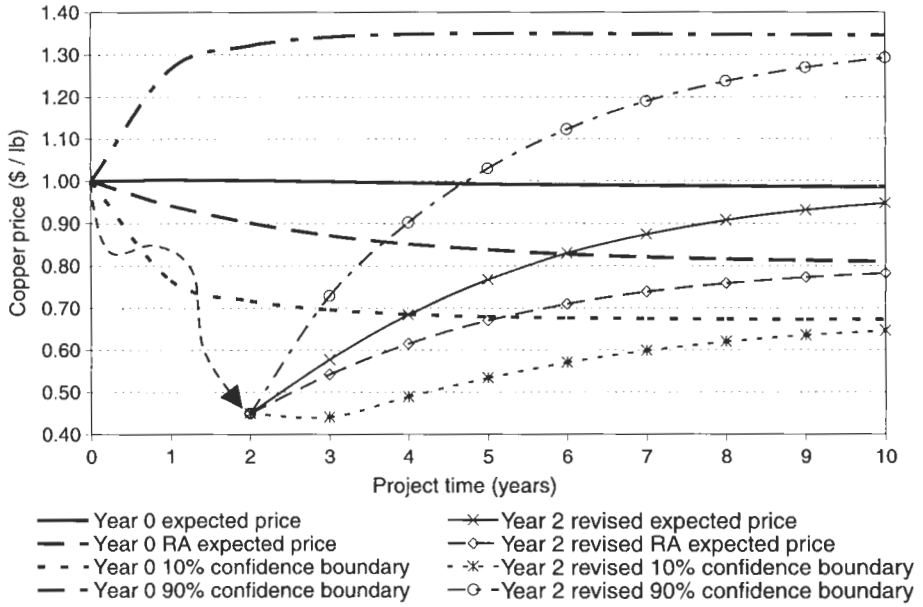

FIG. 6. Copper price model. The vertical axis shows the price of copper, and the horizontal axis shows years from the start of the exploration project. The solid line delineates copper price expectations, or the price mean, given a current (stage zero) spot price of $1.00/lb. The dashed lines outline the 90 and 10 percent confidence boundaries around this forecast, based on copper prices following a mean reverting stochastic process. The figure also shows the risk-adjusted (RA) expected price, which is also the forward copper price curve. If the path of copper follows a downward path between year 0 and year 2 that results in a year 2 price of $0.45/lb, the updated expectations and confidence boundaries around that expectation, as well as the forward curve, will be as depicted. Note that the short-term downward price drift does not change greatly the long-term expected copper price. This is because of the mean reversion of copper price. Price reversion also causes the uncertainty surrounding future price expectations to saturate, as shown by the parallel nature of the long-term price forecast confidence boundaries.

price uncertainty into the lease value calculation, where optimal decision-making is assumed across the equivalent of thousands of price and reserve scenarios. The calculations were completed with a non-commercial Visual C++ program. The program uses a directed graph algorithm to construct the underlying exploration decision tree. Given an initial exploration state, the algorithm considers the decisions possibilities at each state and then generates a set of future exploration states based on these decisions. This process continues at each new exploration state generated until either a lapse exploration state (abandonment) or a lease development state is reached. A description of the directed graphing algorithm can be found in Cormen et al. (1990).

Once the decision tree is built, exploration outcomes and their occurrence probabilities and associated posterior deposit probabilities are attached to each decision node. Exploration outcome paths are then generated to each of the conditional exploration outcomes at future exploration decision nodes.

Given the stochastic nature of prices, the valuation algorithm requires dynamic programming and the Projected Successive-Over-Relaxation (PSOR) finite difference method to approximate the underlying DCF or RO valuation equation. A general discussion of the underlying RO mine valuation equation can be found in Samis (2000) and the PSOR finite difference approximation can be found in Wilmott et al. (1993). The value calculation starts by calculating the NPV for each deposit outcome R_{WC}, R_{LS}, R_{MS}, and R_{SS} using real option discounting of expected cash flows. These values use the economic parameters specified in Tables 2a, 2b, and 3 and incorporate an option to abandon the operation once it is developed and in production. Since reserve uncertainty remains at the end of each of the 27 exploration outcome paths, the terminal value of each exploration outcome path weights these NPVs by the probabilities associated with the particular exploration outcome path to come up with 27 $\mathrm{NPV}_3(\bar{\bar{R}}_3, \delta_3^2)$ values. From this the expected feasibility cost is deducted to obtain the gross expected exploration payoffs.

Dynamic programming is then used to work recursively back through the exploration tree. For each exploration outcome decision node, the present value of continuing exploration is calculated using the PSOR approximation of the underlying valuation equation over a range of mineral prices. The approximation starts with the expected NPV of the lease one period into the future (i.e., at the end of the current exploration phase) and incorporates the costs associated with the current exploration phase. A comparison is then made between the present value of continued exploration and the present values of deferring exploration for two years or letting the lease lapse at each mineral price within the price range. The lease NPV is the maximum value generated by continuing exploration, deferring exploration, or abandoning the lease. The valuation calculation moves to the decision nodes at the next earlier time period once all decision nodes have been valued at a particular time.

This numerical approach takes into account all possible price and deposit outcomes, and, working recursively from the end of the exploration program, calculates a probability-weighted initial value given optimal decision-making as new price and reserve information unfolds over time.

Time 0 DCF and Real Option lease valuations

A time zero DCF NPV can be calculated for each lease using the economic data in Tables 2a, 2b, and 3. Assuming a flat price forecast of $400/oz for gold and $1.00/lb for copper, we noted previously that the time zero expected gross payoff at the end of exploration was $1 M for the gold property and $6 M for the copper property. Bringing these payoffs back to time zero, deducting the present value of the $2.7 M feasibility study, and deducting the present value of the sequential exploration expenditures at time $t = 0$, 1 and 2, gives a gold lease value of –$6.7 M and a copper lease value of –$2.8 M. Neither project is economically viable under this valuation technique.

When mineral price and reserve uncertainty, appropriately discounted within a real option framework, is combined with the ability to manage the exploration process both leases have a positive RO value. Given a time zero gold price of $400/oz and copper price of $1.00/lb (the same assumptions as in the DCF calculation), the gold lease has a time zero RO value of $0.575 M, and the copper lease has a time zero RO value of $0.191 M. In essence, the copper and gold deposits in this case study are equivalent to deposits of less than 1.35 Moz and greater than 900,000 oz in size in the hypothetical example in Figure 1. Static DCF valuations calculate negative resource values, while RO valuations calculate positive values. This is the standard problem with DCF analysis – valuable exploration that can be conducted in stages is quantified as a negative NPV endeavor when the assumption is made that all exploration must be conducted sequentially. Even if we allow for a complete dynamic analysis using decision trees that take into account geological and price uncertainty, the copper lease has no value at $1.00/lb copper when discounted using a DCF framework. The gold lease has a value of $0.197 M, one-third of the value calculated in real options model. The differences here are due to the fact that real options analysis takes into account the risk-reducing impacts of flexibility and price mean reversion, while a decision tree approach that uses an exogenously specified discount rate is no better than the standard DCF technique in accurately reflecting the risk attributes of the projects.

Figure 7 shows the time zero real option exploration project NPVs and optimal exploration decisions for the gold lease over a range of spot prices. The RO method indicates that when price is below $314/oz, this lease has no value and should be abandoned since holding costs are likely to outweigh the present value of any eventual mineral revenues. Exploration should be deferred when the gold price is between $314/oz and $456/oz, paying holding costs in the mean time. Exploration should start immediately when the gold price is above $456/oz. At $400/oz, the gold lease value of $0.575 M is due to the ability to defer stage 1 exploration for two years, waiting to see if gold price rises before committing to the first stage. If it were not for the flexibility inherent in the sequential exploration process the lease would have no value and would be abandoned.

The time zero real option exploration project NPVs and stage 1 exploration decisions for the copper lease over a range of spot prices are delineated in Figure 8. This lease is abandoned if the copper price is below $0.52/lb, whereas stage 1 exploration is started if the price is above this level. Stage 1 exploration is not deferred at any price. The intuition here is that, with saturating price uncertainty, the value of waiting for more price information is small—the operator knows where the price level is going with reasonable certainty. Given a $75,000 cost per period for waiting, waiting for the small amount of price information that will be revealed over time is not optimal in our example.

The time zero RO NPVs for the gold and copper leases vary differently with spot price because of the different pricing models. The gold lease valuation has a convex shape similar to a financial call option because gold is assumed to follow a random walk price process, which is also a standard price model assumption for financial stocks. The copper lease valuation has a concave shape at high prices because of price reversion. The copper price is assumed to revert to $0.98/lb, so when the copper price is above this level there is the expectation that price will fall in the future. This results in a diminishing marginal value of higher spot copper prices. Schwartz (1997) finds a similar effect in a development copper property valuation. In practice, one would therefore expect to see gold lease values being less sensitive than copper leases to price movements at low prices, but having more sensitivity than copper to price movements at high prices.

Time 1 Real Option NPV payoff diagrams (prior to the start of stage 2 exploration)

Once exploration begins, the RO value of both leases changes in response to the assessment of geological information gained from completing an exploration stage. Figures 9 and 10 outline the RO lease NPVs over a range of mineral prices and results (poor, fair, or good) of stage 1 exploration. In our example, good or fair results increase the lease NPV while poor results decrease the lease NPV.

The figures also show decisions about stage 2 exploration once the stage 1 outcome is revealed. The option to explore again upon poor stage 1 exploration results adds value to the lease in all but low price environments.[10]

Of interest in Figure 10 is the virtual insensitivity of copper lease value to copper price. This again is due to price reversion. It is price reversion that also prevents the project from being abandoned at low copper prices when the stage 1 outcome is good or fair—low prices will revert to the mean, and it is therefore better to pay maintenance costs

[10]One of the best examples of this is the 75 drill holes at the Hemlo gold deposit in Canada that came up showing no commercial mineralization. The 76th hole found the deposit (Mining Journal, 2005). See also Smith (2005) for the value of the option to drill again.

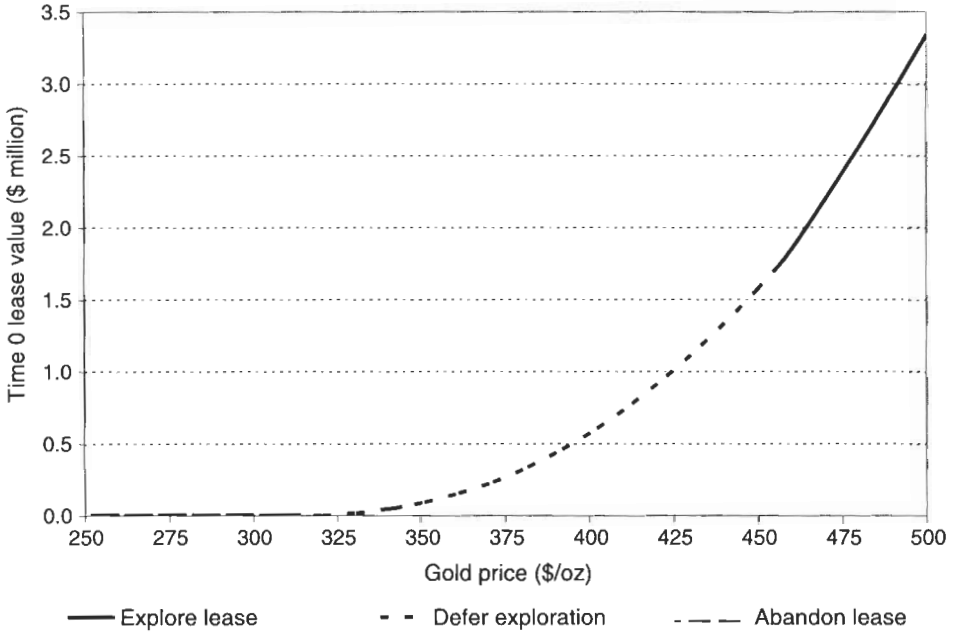

FIG. 7. Time 0 gold lease real option NPV (prior to the start of stage 1 exploration). The vertical axis measures lease value, and the horizontal axis measures current gold price. Lease value is increasing in current gold price. The figure also shows the value-maximizing exploration decisions as a function of current gold price and the implicit gold price forecast (see Fig. 5). At all gold prices below $314/oz this gold lease should be abandoned. At prices between $314/oz and $456/oz stage 1 exploration should be deferred two years, at a cost of $75,000 per year, and then reevaluated. At prices above $456/oz stage 1 exploration should proceed immediately. At a gold price of $400/oz the exploration lease has a real option value of $0.575 M, though stage 1 exploration should be deferred for two years and then reassessed.

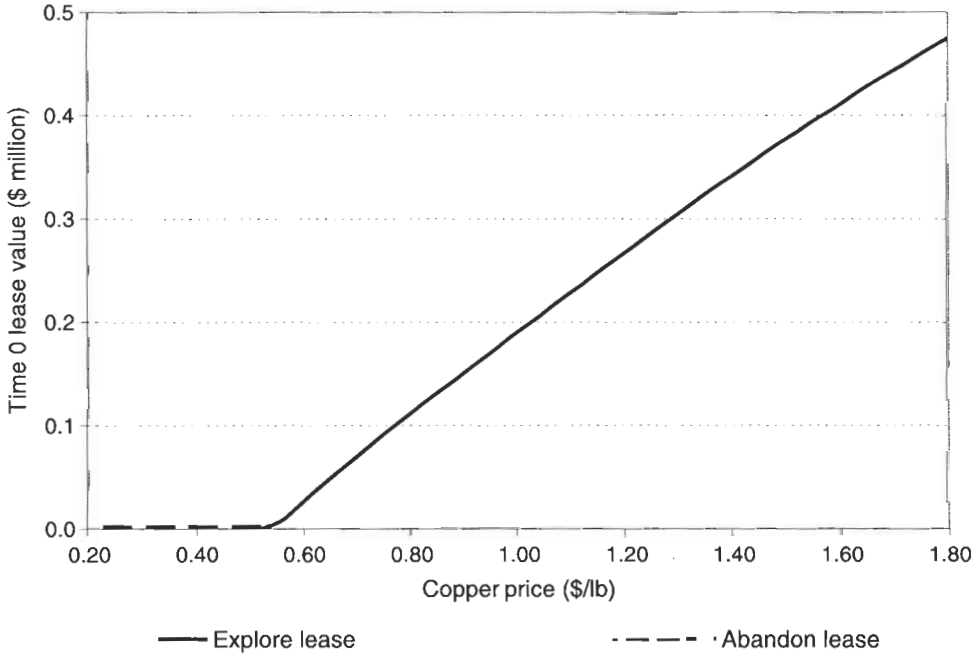

FIG. 8. Time 0 copper lease real option NPV (prior to the start of stage 1 exploration). The vertical axis measures lease value, and the horizontal axis measures current copper price. Lease value is increasing in current copper price. The figure also shows the value-maximizing exploration decisions as a function of current copper price and the implicit copper price forecast (see Fig. 6). At all copper prices below $0.52/lb this copper lease should be abandoned. At prices above $0.52/lb stage 1 exploration should proceed immediately. At a copper price of $1.00/lb the exploration lease has a real option value of $0.191 M, and stage 1 exploration should be started immediately. In contrast to gold leases, copper leases should either be explored immediately or abandoned since there is no long-term price information to be gained by deferring the exploration decision.

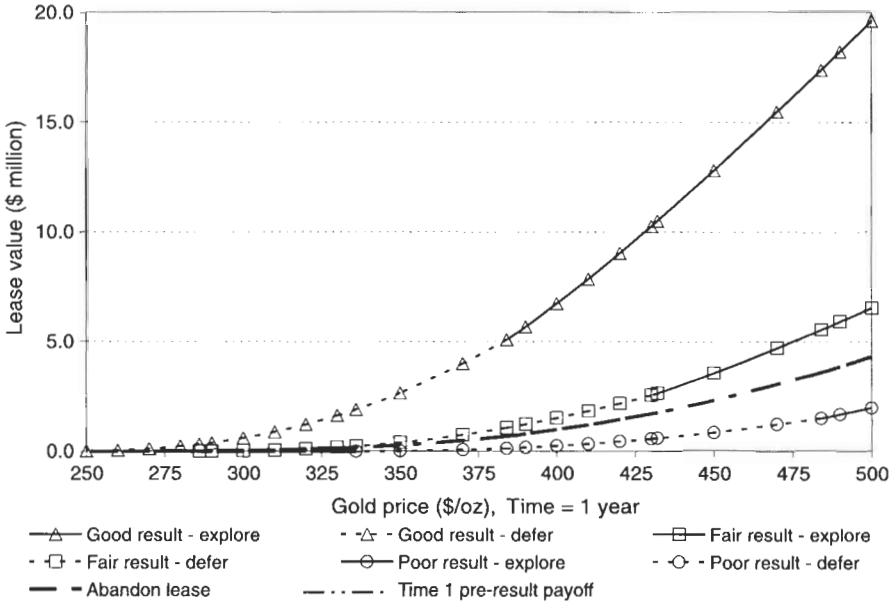

FIG. 9. Gold lease real option NPV at the end of stage 1 exploration (time 1). As a result of the outcome of stage 1 exploration the value of the lease is updated. The top curve shows the time 1 value of the lease after a good stage 1 exploration outcome. The next lower curve shows the value of the lease after a fair stage 1 exploration outcome. The bottom curve shows the value of the lease after a poor stage 1 exploration outcome. The dashed line with no markers shows the time 1 expected value of the lease prior to the information gained from stage 1 exploration. In this case, good and fair stage 1 exploration outcomes serve to increase the value of the lease, while a poor outcome decreases the value of the lease. The figure also shows the optimal stage 2 exploration decision given current (time 1) gold price and stage 1 exploration outcome. Gold prices at which stage 2 exploration is deferred decrease from $484/oz to $434/oz to $384/oz as stage 1 exploration outcomes improve from poor to fair to good. Similarly, the abandonment trigger price decreases from $336/oz to $286/oz to $240/oz (not shown) as stage 1 exploration results improve from poor to fair to good.

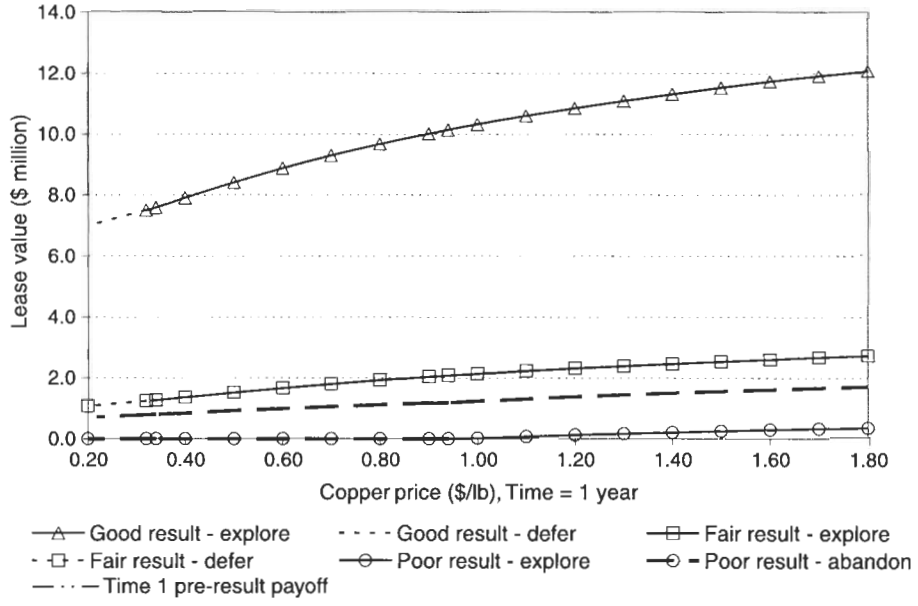

FIG. 10. Copper lease real option NPV at the end of stage 1 exploration (time 1). As a result of the outcome of stage 1 exploration the value of the lease is updated. The top curve shows the time 1 value of the lease after a good stage 1 exploration outcome. The next lower curve shows the value of the lease after a fair stage 1 exploration outcome. The bottom curve shows the value of the lease after a poor stage 1 exploration outcome. The dashed line with no markers shows the time 1 expected value of the lease prior to the information gained from stage 1 exploration. In this case good and fair stage 1 exploration outcomes serve to increase the value of the lease, while a poor outcome decreases the value of the lease. The Figure also shows the optimal stage 2 exploration decision given current (time 1) copper price and stage 1 exploration outcome. If the outcome of stage 1 exploration is poor, stage 2 exploration is undertaken if the copper price is greater than $0.94/lb. Otherwise, the lease is abandoned. If stage 1 exploration reveals good or fair results, stage 2 exploration is deferred if copper price is less than $0.34/lb., and undertaken immediately if copper price is higher than this. A property with fair or good results in stage 1 exploration is never abandoned.

and wait for prices to rise rather than to abandon the property if there is adequate resource potential.

The evolution of lease values and the price triggers for exploring, waiting, or abandoning are presented in decision tree form in Figures 11 and 12. Gold lease values at all points are based on a then current price of \$400/oz and copper lease values assume a current price of \$1.00/lb. Note that it is optimal to explore again in certain price environments given one or even two stages of poor exploration outcomes. Poor exploration results are much more damaging on copper leases than on gold leases, with copper leases being abandoned in these cases because there is little chance that a low reserve base will be offset by skyrocketing prices. If there is some probability of resources being high, however, as with a fair exploration outcome, low copper prices do not deter immediate exploration due to the expected mean reversion of prices by the time the production of the majority of the resource is underway. Gold, on the other hand, must have high prices to go ahead with exploration, since that price process does not have any mean reversion and low prices will likely remain low (Fig. 5).

The final columns in Figures 11 and 12 show the prices at which the feasibility (S, for scoping) study should be undertaken given that the previous three stages of exploration have been undertaken. If price is below this level the project should be abandoned because the quantity of the resource has remained below the resource/reserve cutoff γ. As in the model in the previous section, metal price and ore quality together determine γ; poor exploration results can nevertheless produce reserves when price is high enough such that γ is a low value. For example, a good/poor/poor gold exploration outcome produces a sufficiently high estimate of $\bar{R}_n$ to define this a reserve, as used in this paper, as long as the gold price is above \$390/oz upon completion of exploration.

Model differences between gold and base metal exploration

This real options model of exploration reveals important differences between exploration for gold, which exhibits random walk prices and increasing future price uncertainty, and for base metals, which exhibit reversion to a mean price level after a price shock and saturating price

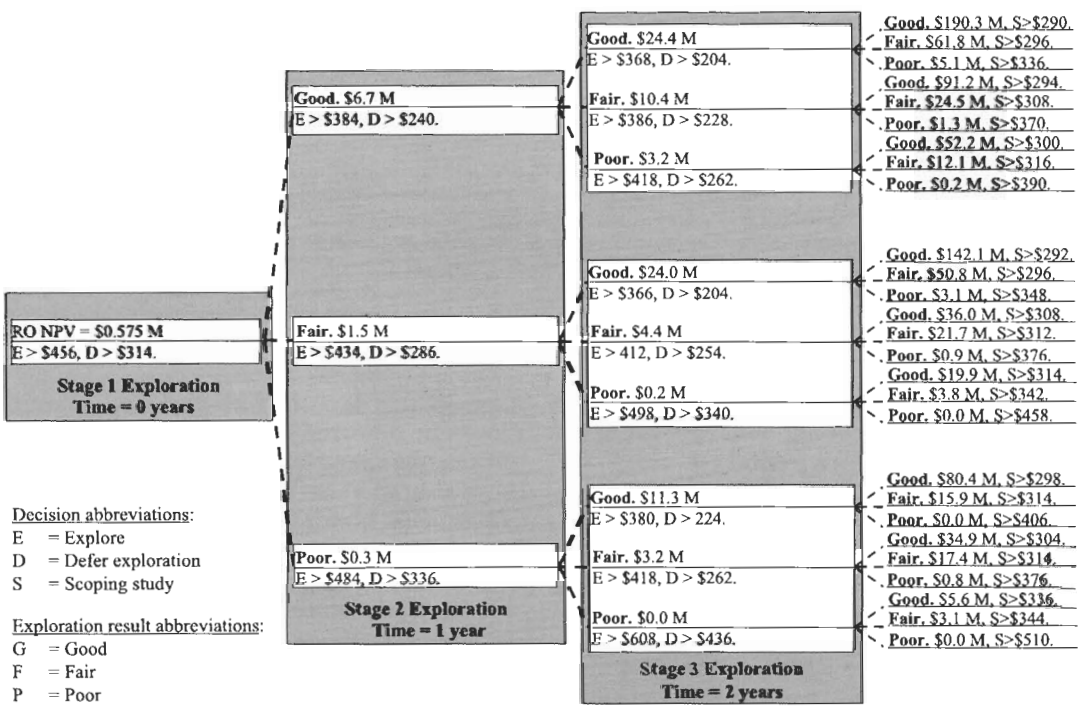

FIG. 11. Evolution of real option NPV and decision rules for the gold lease when the gold price is \$400/oz. This figure shows in decision tree format the information contained within Figure 9 for all stages of exploration. The first column shows that at \$400/oz gold, the exploration property is worth \$0.575 M at time 0. Exploration should proceed for spot gold prices above \$456/oz, and should be deferred for spot prices of between \$456/oz and \$314/oz. At prices of less than \$314/oz the lease should be abandoned. The second column shows the same information contingent on a \$400/oz gold price at time 1 and the outcome of stage 1 exploration. Column 3 shows the information contingent on a \$400/oz gold price at time 2 and the outcome of stage 1 and 2 exploration. Column 4 shows the information contingent on a \$400/oz gold price at time 3 and the outcome of stage 1, 2, and 3 exploration. In column 4 a \$400/oz gold price and 3 good exploration outcomes yield a time 3 project value of \$190.3 M, and scoping (prefeasibility/feasibility/development) should proceed for any gold price greater than \$290/oz. On the other hand, a \$400/oz gold price and three fair exploration outcomes yield a time 3 project value of \$21.7 M, and scoping (prefeasibility/feasibility/development) should proceed for any time 3 gold price greater than \$312/oz.

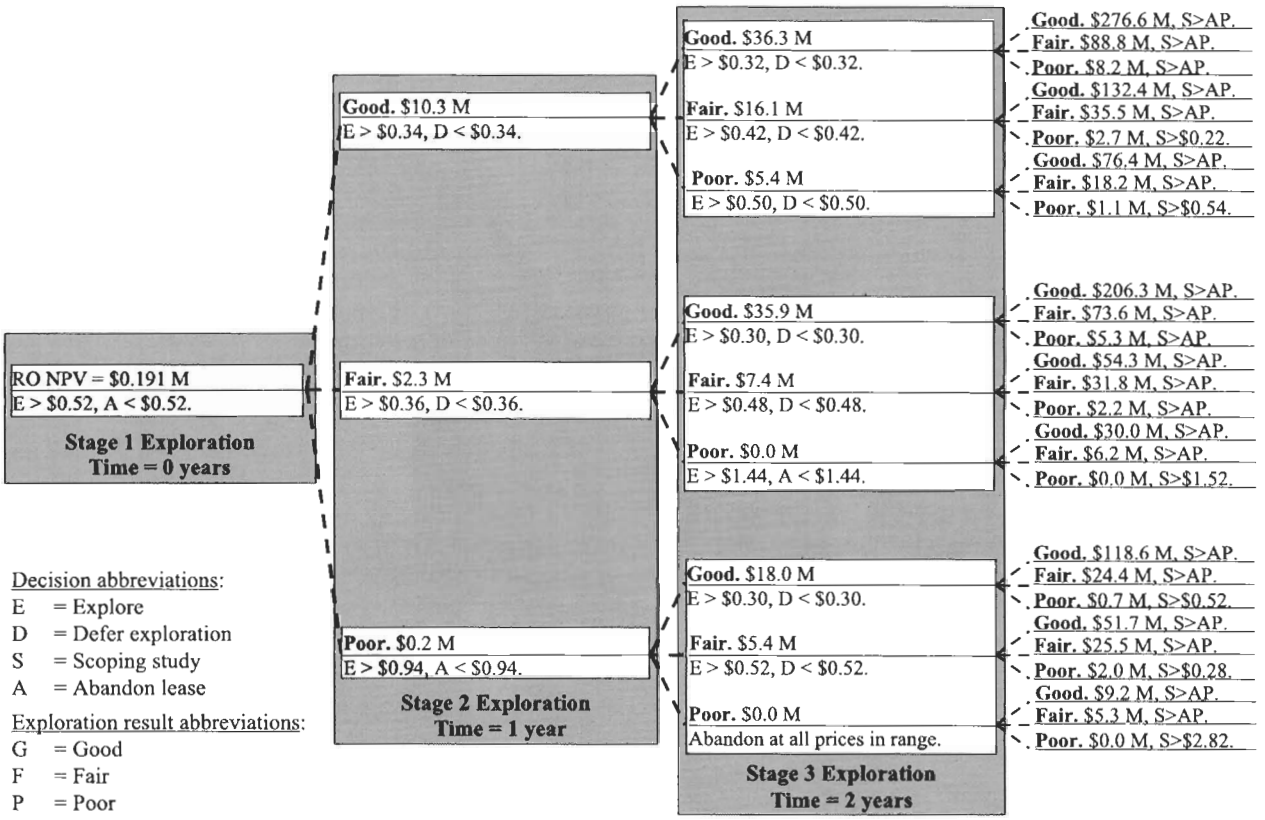

FIG. 12. Evolution of real option NPV and decision rules for the copper lease when the copper price is $1.00/lb. This figure shows in decision tree format the information contained within Figure 10 for all stages of exploration. The first column shows that at $1.00/lb copper, the exploration property is worth $0.191 M at time 0. Exploration should proceed for spot copper prices above $0.52/lb, and should be abandoned for spot prices of $0.52/lb or less. The second column shows the same information contingent on a $1.00/lb copper price at time 1 and the outcome of stage 1 exploration. Column 3 shows the information contingent on a $1.00/lb copper price at time 2 and the outcome of stage 1 and 2 exploration. Column 4 shows the information contingent on a $1.00/lb copper price at time 3 and the outcome of stage 1, 2 and 3 exploration. In column 4 a $1.00/lb copper price and three good exploration outcomes yield a time 3 project value of $276.6 M, and scoping should proceed regardless of the copper price. Two poor exploration outcomes lead to project abandonment regardless of the copper price.

uncertainty. For example, our case study supports a gold exploration policy that focuses on deposits with previous positive exploration results during periods of moderate prices, while deferring geologically unpromising deposits, and an expanded exploration policy that includes the deferred green-fields deposits when prices are higher. The intuition here is that unpromising deposits may ultimately become valuable due to increasing gold prices, and that they should be kept in reserve until price improves. Real-world evidence of gold exploration activity in the 1990s supports this trend. On the other hand, our model suggests that unpromising green-fields copper projects not be kept in deferral mode waiting for higher prices, since prices are capped by the trend to a long-run mean; green-fields copper projects should either be explored if promising or abandoned if unpromising. Given this we would not expect green-fields copper exploration expenditures to be sensitive to the price cycle, but rather an ongoing activity as long as price exceeds some minimum level (col. 1 of Fig. 12). M. S. Enders and R. A. Leveille (unpub. data, 2004) report exactly this policy at Phelps Dodge. Our model also shows that later stage copper deposits with positive exploration outcomes should be deferred in low price environments, since they will become valuable once price reverts to the mean, and so these should show some exploration response to price swings. This, combined with constant spending on green-fields projects across the price cycle, would then be revealed as green-fields exploration constituting a higher proportion of total expenditure during the bottom of the price cycle and a lower proportion at the peak of the price cycle. There is ample evidence that this is broadly the case for base metals. In addition, we would expect to see stage 2 gold exploration spending respond to movements in gold prices, while we would expect stage 2 copper exploration spending for fair and good outcome properties to be ongoing for all but very low prices (Figs. 11 and 12). Empirically, gold exploration expenditure does seem to be more responsive to price than base metal exploration expenditure (R. C. Schodde, unpub. data, 2003).

Since the geology and economics of our gold and copper examples have been designed to be as comparable as possible, these model differences are driven entirely by the dif-

ference in price behavior of base metals, which exhibit price reversion, and gold, which does not. If one wants to model and value exploration, and to thereby explain the differences that are found between base metal and gold exploration, such pricing details must be included in the model of exploration. Modeling only the uncertainty in a resource is unlikely to capture these effects.

Conclusions

This paper models the two main uncertainties facing an exploration manager, geological uncertainty and price uncertainty, in a real options framework aimed at directing exploration decisions such that project value is maximized. Both models show that in some cases where the traditional discounted cash flow value of the project is negative, exploration activity can be value creating and should proceed. The first model considers only geological uncertainty and reveals the many geological factors influencing exploration activity and the value of an exploration property. The second model adds price uncertainty. The second model reveals important differences between exploration for gold, which exhibits random walk prices and increasing future uncertainty, and the exploration for base metals, which exhibit reversion to a mean price level and saturating price uncertainty.

Our models justify the aggressive decisions that exploration managers often make, even in the face of contrary advice from traditional discounted cash flow models. That the valuation models are now giving the right signal is a significant step forward in managing and valuing exploration projects.

Acknowledgments

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