

**Homework #2 Solution**

The 1D Buckley-Leverett problem was solved using the Explicit Upstream Weighted finite difference formulation. Considering the simplest form for waterflood of a linear core initially saturated with oil and at residual water saturation, the material balance equation (Equation 1) presented by Buckley and Leverett (1941) is,

$$\frac{\partial S_w}{\partial t} = -u_t \frac{\partial f_w}{\partial x} \tag{1}$$

where,

$S_w$ : water saturation

$t$ : time

$x$ : distance along path of flow

$u_t$ : total interstitial velocity,  $u_t = \frac{q_w}{A\phi}$

$f_w$ : water fractional flow,  $f_w = \frac{\lambda_w(S_w)}{\lambda_w(S_w) + \lambda_o(S_w)}$

$q_w$ : water injection rate

$A$ : cross-sectional area of the core

$\phi$ : porosity

mobility of water is,  $\lambda_w(S_w) = \frac{k_{rw}(S_w)}{\mu_w}$

mobility of oil is,  $\lambda_o(S_w) = \frac{k_{ro}(S_w)}{\mu_w}$

and the relative permeability is calculated as a function of saturation using,

$$k_{rw} = k_{rw} * \left( \frac{S_w - S_{wr}}{1 - S_{orw} - S_{wr}} \right)^{n_w}$$

$$k_{ro} = k_{ro} * \left( \frac{S_o - S_{orw}}{1 - S_{orw} - S_{wr}} \right)^{n_o}$$

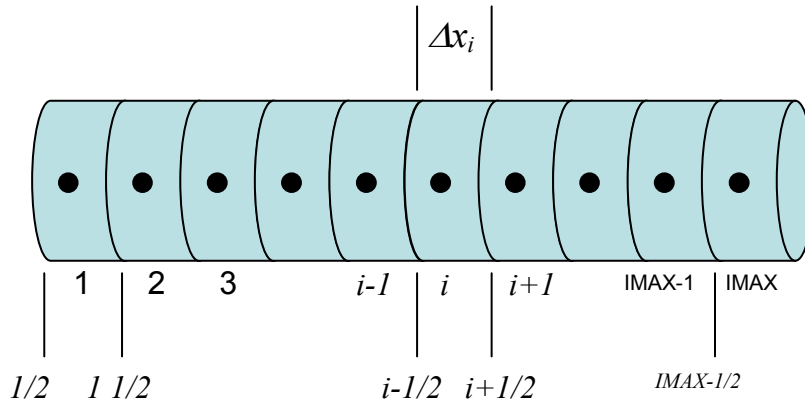
Considering the space discretization shown in Figure 1, Equation 1 can be expressed in its discretized form (that applies for each space partition) as,

$$\frac{S_{w,i}^{n+1} - S_{w,i}^n}{\Delta t} = -u_t \frac{f_{w,i}^n - f_{w,i-1}^n}{\Delta x} \tag{2}$$

To improve the computation process and avoid using an imaginary grid block at the left of the first node, the following equation is used for the first node,

$$\frac{S_{w,1}^{n+1} - S_{w,1}^n}{\Delta t} = -u_t \frac{f_{w,1}^n - f_{w,1/2}^n}{\Delta x/2} \tag{3}$$

These equations (Eqs. 2 and 3) can be solved explicitly for water saturation at the new time step (n+1).



**Figure 1. 1-dimensional core space discretization**

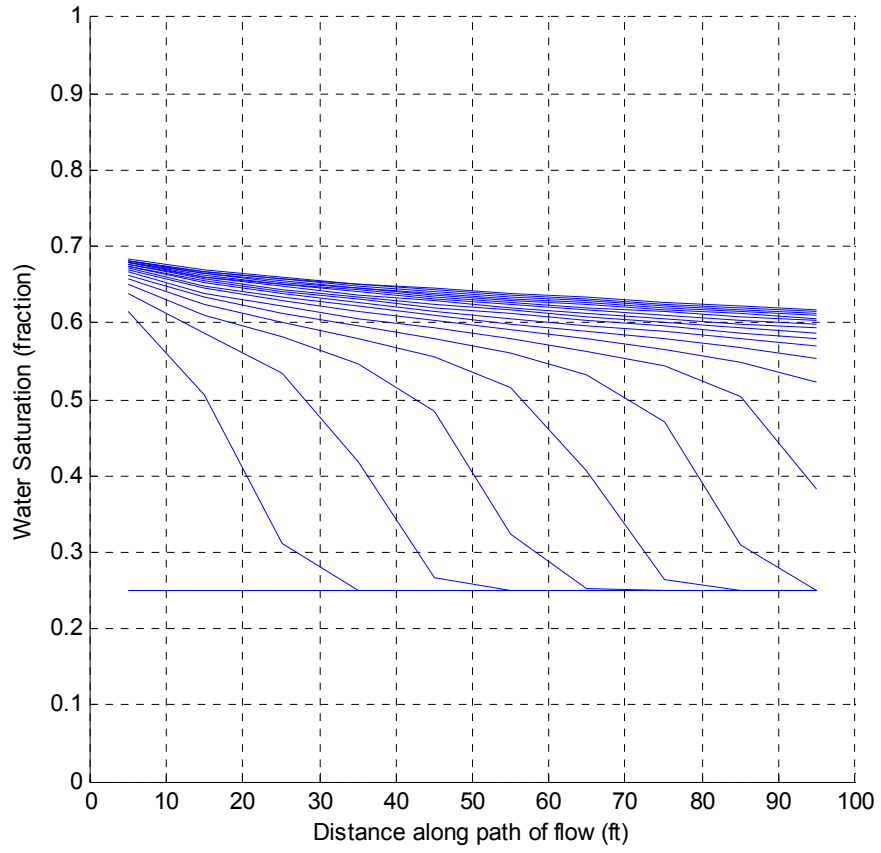
The Explicit Upstream Weighted Formulation described to solve the 1-dimensional Buckley-Leverett problem was coded in a MATLAB script, shown at the end of this document.

Table 1 shows the values of water saturation for the 10 grid blocks at 10 days.

**Table 1. Water saturation values for the 10 grid blocks at 10 days**

<b>Grid block index</b>	1	2	3	4	5	6	7	8	9	10
<b>Sw (fraction)</b>	0.68313	0.66975	0.66058	0.65302	0.64641	0.64045	0.63497	0.62986	0.62503	0.62045

A plot of water saturation profile as a function of distance along path of flow at every five time steps is shown in Figure 2. The initial condition (at t = 0) corresponds to the horizontal line of constant water saturation equal 0.25 (i.e. Swr). The figure shows an increase of water saturation with time and how the water injection front propagates along the 1-dimensional core.



**Figure 2. Water saturation profile along the path of flow at every five time steps**