General instructions: Answer all of the "Quickies" (question 1) and TWO of the remaining problems. Read all the questions and answer the easier questions first. Clearly indicate which problem you do NOT want graded. If none is indicated, only the first two problems will be graded. Show all work.

1. (30) “Quickies”
   a. (7) Sketch a reference frame with a point source located at $\vec{r}'$ and field point at $\vec{r}$ and, referring to your sketch, define “retarded time”.

   b. (7) Write down the general expressions for the retarded potentials, $V$ and $\vec{A}$.

   c. (7) Explain why the sky is blue.

   d. (9) Consider a proton with rest mass, $m_p \simeq 1$ Gev/c$^2$, and lab kinetic energy, $T_{Lab} = 2$ GeV, colliding with another proton at rest. Use the invariance of the total 4-momentum squared to find the center-of-mass momentum, $p_{CM}$. Give your answer in GeV/c units.
2. (35) The vector potential of an oscillating magnetic quadrupole in the radiation zone is found to be \( \vec{A}(r, t) = A_0 \sin \theta \cos \frac{\theta \sin \omega t}{r} \hat{\phi} \) where \( t_r = t - r/c \).

a. Find the magnetic field, \( \vec{B} \), in the radiation zone (to first order in \( 1/r \)). Show all work.

b. Use \( \vec{E} = c\vec{B} \times \hat{r} \) to find the Poynting vector, \( \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \).

c. Integrate the time-averaged Poynting vector through the surface of a large sphere centered on the radiating quadrupole to find the total power radiated.
3. (35) Consider a semi-circular segment of wire or radius, $R$, in the upper half x-y plane centered on the origin as shown above. At $t = 0$ a counter-clockwise current is turned on which varies as: $I(t) = I_0 t/\tau$ where $\tau$ is a constant.

a. Find the vector potential, $\vec{A}(t)$, at the origin. Show all work.

b. Consider a field point near the origin at $\vec{r} = x\hat{x}$ for $x \ll R$. From the arbitrary current element shown on the figure above draw the relative position vector $(\vec{r} - \vec{r}')$ and find the retarded time for that element’s contribution to the vector potential to first order in $x/R$. 
4. (35) A charged loop of radius, $R$, lies in the xy-plane and is centered at the origin and carries a charge density, $\lambda(\phi, t) = \lambda_0 \sin(\phi - \beta(t))$, where $\beta$ measures the orientation of the loop as shown in the figure. The loop rotates with constant angular velocity, $\beta = \omega t$; so the charge density at time, $t$, is given by $\lambda(\phi, t) = \lambda_0 \sin(\phi - \omega t)$, where $\phi$ is measured with respect to the x-axis and identifies an arbitrary charge element of the loop.

a. Sketch a charge element at some arbitrary angle, $\phi$, on the figure above and find the retarded time for the contribution that the charge element makes to the retarded potential at a point on the x-axis, $\vec{r} = x\hat{x}$, to first order in $R/x \ll 1$.

b. Find $\sin(\phi - \omega t_R)$ to lowest order in $R/\lambda$ where $\lambda = c/\omega$ is the reduced wavelength.

c. Write down an integral expression for the vector potential, $\vec{A}(x, t)$, to lowest order in $1/x$. (For this situation $d^3r' \vec{J}(r', t_R) \rightarrow \omega R \lambda(\phi, t_R) d\phi \hat{\phi}.$)