1. Consider a hollow waveguide with a rectangular cross section of dimensions $a \times b$. It is oriented along the $z$-axis with the $x$-axis along the $a$ side and $y$-axis along the $b$ side. By $z$-translational symmetry, we may assume the electric and magnetic fields take the form:

$$\vec{E} = \vec{E}(x, y)e^{i(k_x x - \omega t)}$$

$$\vec{B} = \vec{B}(x, y)e^{i(k_x x - \omega t)}$$

(Note the difference between bold-faced and normal-faced vectors.)

a. Take the $\hat{x}$-component of Maxwell’s third equation and the $\hat{y}$-component of Maxwell’s fourth equation, and, using the forms above, solve for $E_y$ and $B_x$ in terms of $\{E_z, B_z\}$.

b. Consider the special case of transverse magnetic modes ($B_z = 0$). Assume the general solution of the $z$-component of the electric field is:

$$E_z = E_{z0} \left[ \alpha \sin(k_x x) + \beta \cos(k_x x) \right] \left[ \gamma \sin(k_y y) + \delta \cos(k_y y) \right],$$

where $E_{z0}$ is the amplitude of the wave. Apply the boundary condition that $E_{\|}$ is continuous across a boundary to find $\{\alpha, \beta, k_x\}$ in terms of $\{E_{z0}, B_{z0}\}$. 