A loop of radius, $R$, is centered at the origin and lies in the xy-plane. The loop consists of two oppositely-charged half loops of uniform linear charge density, $\pm \lambda_0$. Let $\beta(t)$ be the angle with respect to the x-axis to one of the joints where the half-loops are joined (as shown). Initially, $\beta(t = 0) = 0$ and $\frac{d\beta}{dt}(t = 0) = 0$; so at this time the loop is oriented with the positive side centered on the positive y-axis, that is,

$$\lambda(\theta, t = 0) = \begin{cases} +\lambda_0, & 0 \leq \theta < \pi \\ -\lambda_0, & \pi \leq \theta < 2\pi \end{cases}.$$ 

where $\theta$ identifies an element of the loop. The loop rotates in the $+\hat{z}$ direction with an angular velocity which increases linearly in time according to, $\omega(t) = \frac{d\beta}{dt} = \alpha t$. The general formula for the retarded vector potential, $\vec{A}$, is:

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}', t_R) d^3r'}{|\vec{\rho}_R|},$$

where $\vec{\rho}_R = \vec{r} - \vec{r}'(t_R)$ with the retarded time given by $t_R = t - |\vec{\rho}_R|/c$.

a. Through what angle will the loop have rotated after time, $t$?

b. What is the retarded time for a point on the z-axis?

c. For this case, we can write $d^3r'\vec{J}(\vec{r}', t_R) \rightarrow R\omega(t_R)\lambda(\theta, t_R)d\theta$, where $\lambda(\theta, t_R)$ is the linear charge density at time $t_R$, similar to the $t = 0$ case shown above. Find $\lambda(\theta, t_R)$.

d. Find the vector potential, $\vec{A}(z, t)$, for a point on the z-axis.