The Inverting Effect of Curvature in Winter Terrain Park Jump Takeoffs

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Epidemiological studies of injuries at ski resorts have found that terrain parks, and jumps especially, pose a significantly greater head/neck injury risk to resort patrons than normal skiing activities [1–4]. One especially hazardous situation is when the jumper lands in an inverted position which can lead to catastrophic injury or death from spinal cord trauma. While jumpers can execute inverted maneuvers intentionally, curvature in the takeoff can lead to involuntary inversion. In this work we lay out the basic physics of this phenomenon assuming a rigid body model which simulates a stiff-legged jumper. I include an estimate of the partially compensating (forward) rotation due to the ground reaction force. I apply the results to an actual jump having a curved takeoff. For a jump trajectory with a landing just past the “knuckle”, the resulting net angle of inverting rotation is found to be about 60°, large enough to pose a potential injury risk. A mitigating takeoff design criterion adopted by the US Terrain Park Council based on human response times is also discussed and compared to the FIS standard for Nordic jumps.

I. INTRODUCTION

In an early 1999 study Tarazi, Dvorak, and Wing examined spinal cord injury data from two ski resorts over two seasons (1994-1996) and determined the rate of spinal cord injuries among snowboarders was about 40 per million snowboarder-days which is about four times the rate for skiers [1]. Furthermore, these researchers found that the majority (77%) of these injuries were sustained on jumps which they identified as “intrinsic” to snowboarding. The National Ski Areas Association reported approximately 16 million snowboarder-days in the U.S. for the 2000-2001 season [5]. Thus, one estimates that there were over 500 spinal cord injuries to snowboarders on terrain park jumps that year. Since this early study, there has been an increasing number of freestyle skiers migrating to terrain park jumps which are no longer the exclusive domain of snowboarders. In addition, the number of terrain parks available has increased dramatically in the last decade; so although there are no publicly published epidemiological data, if the Tarazi, et al. rates are still valid, there could be a higher number of spinal cord injuries today.

Safety concerns with terrain parks are further supported by more recent epidemiological studies. In a review of 24 articles between 1990-2004 from 10 countries Ackery, et al. [6] found evidence of an increasing incidence of traumatic brain injury and spinal cord injury in alpine skiing and snowboarding worldwide and noted that this increase coincided with “development and acceptance of acrobatic and high-speed activities on the mountain”. Henrie, et al. [3] have reported that one is twice as likely to suffer a head or spinal injury in a terrain park than outside the terrain park. Perhaps more disturbing however are the occasional catastrophic injuries in recreational terrain park jumps which since 2000 have included at least a dozen paralyzing or fatal events. In each of these catastrophic injury events it was reported that the rider landed on his or her head, shoulders, or back. Clearly, becoming inverted in an uncontrolled fashion during a jump carries significant risk for serious injury or death.

Several mechanisms exist by which one can become inverted. First, one may become inverted through rider actions such as intentionally inverting as, for example, in the execution of an inverted aerial maneuver. Inversion can also occur unintentionally as, for example, when the jumper unintentionally leans back prior to takeoff. This can occur when the friction coefficient drops suddenly prior to the takeoff which can happen when the terrain park staff salts the takeoff (lowering its friction coefficient) but not the transition and the nearby section of the approach. This effect can be qualitatively understood as follows. Assume the rider is facing toward the takeoff point (whether sliding downhill, horizontally or up the takeoff ramp). Because of the rearward friction force along the bottom of the board, to maintain quasi-equilibrium the rider must lean backwards from the apparent vertical so that the surface normal force creates a backward moment to counteract the forward one due to the friction force. Thus a sudden decrease in the friction coefficient will mean that the rider previously oriented to be in equilibrium on the higher friction surface will be out of equilibrium in the backward direction on the lower friction surface. Finally, a rider may become inverted unintentionally when there is (concave) curvature in the takeoff. Such curvature can impart an inverting angular velocity to the jumper, a phenomenon described as “back-seating” within the terrain park rider community (similar, but not to be confused with the term, “backseat” (posture), used by skiers). This last effect will be the primary focus of this work.

Based on the author’s informal survey of current practice in the U.S., recreational terrain park jumps are created by resort staff or private contractors with little or no quantitative engineering considerations. Hard-earned experience has led to a steady improvement in the quality of terrain parks, but the quality of the products produced varies widely from resort to resort due to different hill topographical profiles, available snow resources, experience and skill.
level of the groomers, differing levels of maintenance, and capabilities of available equipment. Laudable fabrication training programs, such as “Cutter’s Camp” [7], increase the knowledge and skill level of the groomers and have helped considerably. Rider education programs such as Burton’s “Smart Style” have also helped [8], yet adoption of quantifiable engineering design of terrain parks has thus far been resisted by resorts citing the wide variability involved in snow conditions and rider actions.

While it is true that the users must bear the primary responsibility for and control over their safety while using terrain park jumps, ski resorts could employ engineering design principles to improve the quality and safety of their jumps as well as mitigate the negative consequences of poor rider decisions. Even though there is significant variability arising from changing snow conditions and rider decisions, the variations are bounded in understandable ways allowing one to engineer jumps that accommodate the variability. For example, snow/ski friction coefficients vary from about 0.04 to 0.12 [9, 10] and drag-area coefficients lie between 0.3 m$^2$ to 0.8 m$^2$ [11]. In addition, snow is a malleable surface causing jumps to evolve with the conditions and use, but such alterations are quantifiable and designs can accommodate the majority of these changes. Among rider actions, the most widely cited examples are riders checking their speed, spinning at takeoff, and adding “pop” (jumping up just before takeoff). Nevertheless, such effects have been shown to be similarly bounded [12–15]. There is a new consensus emerging that it will be practical to create safer designs within these bounds. Indeed, the Committee F-27 on Snow Skiing of the ASTM (formerly the American Society for Testing and Materials) has adopted recreational winter terrain park jumps as part of their purview. Since inversion is a primary contributing factor with catastrophic injuries, design aspects relating to inversion will very likely be considered in any terrain park standards. Curvature in the takeoff will be one such consideration.

This work uses dynamic modeling to study the role of curvature in the jump takeoff in inducing inverting rotations in jumpers. All such theoretical studies are approximate in nature. I have attempted to describe the approximations made and the range of practical applicability. The work uses a rigid-body model of the jumper to extend the earlier study by McNeil and McNeil [16]. Other dynamical modeling of trajectories includes that of Bohm and Senner [17] and Hubbard [18]; however these earlier studies do not treat the rotation of the jumper.

The paper is organized as follows. Section II defines the variables and terms needed to treat the motion of an extended object and applies fundamental Newtonian rigid-body dynamics to an inert rider jumping from a curved takeoff. The curvature of the end of the takeoff is characterized by its radius of curvature. The inverting rotational velocity imparted to the jumper is calculated along with the total inverting angle for the standard tabletop jump with concave curvature in the takeoff. The partially compensating (forward) rotation induced by the ground reaction force prior to takeoff is also calculated. In Section III the results are applied to a resort jump built several years ago. For a hypothetical takeoff speed that would carry the jumper two meters beyond the “knuckle” of a tabletop jump, the calculated inverting angle is substantial, $\sim 60^\circ$, and the total inverting angle of the jumper with respect to the normal of the landing surface is over $120^\circ$. There follows a discussion of suggested design criteria that may mitigate this effect. The last section contains a summary and conclusions.

## II. RIGID BODY MODEL OF ROTATION

### A. Theory

In this work I treat the jumper as a rigid body which approximates a stiff-legged jumper similar to what one might expect from a novice jumper. Of course, most experienced jumpers are not inert on the takeoff, but the rigid body model is still useful as it represents a “default” case that allows one to separate the effects of the design from the actions of the rider. Extensions to jumpers who “pop” from the takeoff or who alter their drag coefficient by altering their configuration in the air have been treated separately [14]. For the center of mass motion once airborne, I treat the body as freely falling with its angular momentum conserved. As shown in previous work [14], drag and lift alter their configuration in the air have been treated separately [14]. For the center of mass motion once airborne, I treat the model as still useful as it represents a “default” case that allows one to separate the effects of the design from the

\begin{equation}
 y_{J}(x) = x \tan \theta_T - \frac{g}{2v_0^2 \cos^2 \theta_T} x^2, \tag{1}
\end{equation}

where the subscript refers to the jumper’s $y$-coordinate, $\theta_T$ is the takeoff angle, $v_0$ is the takeoff speed, and $g = 9.81 \text{ m/s}^2$ is the gravity acceleration constant.

I will consider inverting rotation for trajectories calculated for standard tabletop jumps which have become the most widely used design. This jump, sketched in Fig. 1, consists of a start, approach, transition, takeoff, deck, landing,
and run-out. The lip of the takeoff is a distance, $H$, above the deck which has length, $D$, before transitioning to a straight landing. In this work the takeoff is assumed to have a radius of curvature, $R$. Some experienced park riders speculate that such curvature in the takeoff commonly shows up in small jumps where there is limited snow from which to construct the transition and landing which leads some builders to combine the two. Curvature can also develop in the takeoff due to heavy use if not regularly maintained. The deck is assumed to be horizontal, but generalization to sloped decks is straightforward. The intersection of the deck and the landing is called the “knuckle” and the intersection region between the landing and the run-out is called the “bucket”. The use of the tabletop form in the examples presented here should not be considered an endorsement of this form. Researchers have shown that the tabletop design is not optimal from the point of view of its potential impact hazard as characterized by an equivalent fall height [16, 18–21]. Nevertheless, the focus of this work will be the effect of the curved takeoff on the jumper with the other components of the jump only affecting the total inversion angle through the time of flight.

The equation of the deck and landing surface for the tabletop jump is given by:

$$y_L(x) = \begin{cases} -H, & x < D \\ -\tan \theta_L(x - D) - H, & x \geq D. \end{cases}$$

(2)

where the subscript refers to the deck/landing surface’s $y$-coordinate, $\theta_L$ is the landing angle as shown in Fig. 1. For a jump that covers a horizontal distance of $x_L$ the time of flight is given by:

$$t_f = \frac{x_L}{v_0 \cos \theta_T}.$$

(3)

The curvature of the takeoff is characterized by the radius of curvature, $R$. The radius of curvature is defined as the radius of the osculating circle that best encompasses the last few meters of the takeoff. It can be estimated by measuring the length of a chord, $L_c$, drawn near the end of the takeoff and the perpendicular distance, $d_c$, between the midpoint of the chord and the takeoff surface as shown in Fig. 1. A discussion of selecting a practical length for the chord is given later. From these two measurements the radius of curvature is given by:

$$R = \frac{4d_c^2 + L_c^2}{8d_c}.$$

(4)

Assuming the rider is a rigid body during the takeoff implies that the takeoff velocity is parallel to the takeoff surface at the lip. Here I consider rotations about the $z$-axis (i.e. perpendicular to the xy-plane); therefore a positive rotation will tend to orient the rider toward the rear (backward or opposite the direction of travel) and a negative rotation will tend to orient the rider toward the front (forward or in the direction of travel). The instantaneous backward angular speed while on the curved portion near the end of the takeoff is given by the takeoff speed, $v_0$, divided by the radius of curvature:

$$\omega = \frac{v_0}{R}.$$

(5)

One can intuitively understand this relation by recognizing that a rider moving at constant speed, $v_0$, on a vertical circular track of radius, $R$, will rotate by $2\pi$ for each revolution. Since the time to execute one revolution is $T = \frac{2\pi R}{v_0}$, the angular speed of the rider is $\omega = \frac{2\pi}{T} = \frac{v_0}{R}$ [22]. Once the jumper leaves the surface, no further torques can be exerted so the jumper’s angular momentum is conserved. Again, assuming a rigid body jumper, the moment of inertia is constant so the angular velocity is constant. Thus, the total inverting rotation is given by the angular velocity times the time of flight,

$$\phi = \omega t_f = \frac{x_L}{R \cos \theta_T},$$

(6)

where a positive sign means an inverting rotation. The inverting rotation is proportional to the horizontal jump distance and inversely proportional to the radius of curvature. For a straight takeoff the radius of curvature is infinite giving $\phi = 0$. As can be seen from Eq. 6, so long as the center of mass covers a distance, $x_L$, the inverting rotation angle does not depend upon the landing orientation of the jumper.

As can be seen in Fig.1(a), even without any backward rotation, a jumper whose orientation starts normal to the takeoff angle, $\theta_T$, upon reaching the landing surface at angle, $\theta_L$, will find himself at an angle of $\theta_T + \theta_L$ with respect to the normal of the landing surface. Therefore, the magnitude of the total inversion angle, $\Phi$, with respect to the normal to the landing surface is given by

$$\Phi = \phi + \theta_T + \theta_L.$$

(7)
Having $\Phi$ greater than $90^\circ$ means the jumper will land on his back or head. Typically $\theta_T$ and $\theta_L$ are between $15^\circ$ and $30^\circ$ which means a relatively small additional inverting rotation (between $30^\circ$ and $60^\circ$) will result in a $\Phi$ greater than $90^\circ$. A jumper executing an air maneuver will alter his moment of inertia and the calculation of the inverting rotation angle is more complicated. As a qualitative example, suppose a jumper starts the jump in an approximately upright position at takeoff and subsequently bends down to grab his board as is commonly done. In this case the jumper’s moment of inertia about the inverting axis of rotation will decrease with a corresponding increase in the rotation speed and total angle of rotation.

### B. Extended body effects

There are two refinements to the previous theory arising from the finite extent of the jumper and snowboard/skis. First, since the rotations are about the center of mass; therefore the radius of curvature in Eq. 6 should be taken from the center of the osculating circle to the center of mass, not the takeoff surface. For a given takeoff posture of the jumper with height, $h$, the center of mass is located at approximately $h/2$; so to approximate this effect we replace $R$ by $(R - h/2)$ in Eq. 6:

$$\phi \simeq \frac{x_L}{(R - h/2) \cos \theta_T}.$$  

(8)

Second, assuming the rider has not ‘popped’ off the takeoff surface, there will be a compensating forward rotation induced by the normal ground reaction force acting on the rear section of the snowboard/skis once the center of mass has passed the end of the takeoff and the front of the snowboard/skis starts to fall while the back is still on the takeoff surface. To get an estimate of this effect, I treat the snowboard/skis as rigid. Of course, real snowboards/skis are flexible and have camber, but the effect of the flex will always reduce the magnitude of this effect compared to the rigid case. A highly flexible snowboard/ski will not deliver as much torque on the jumper as a stiff board as some of the energy must necessarily go into deforming the snowboard/skis. In the case of a stiff board with high camber, one can obtain an upper bound on the effect by making the extreme assumption that the camber is sufficiently strong that only the rear of the snowboard/skis is in contact with the takeoff surface once the center of mass has crossed the end of the takeoff thereby delivering a maximum torque to the jumper. However, as shown below for the example evaluated here, this effect increases the compensating (forward) rotation speed by less than 2%

As before, let $(x(t), y(t))$ be the time-dependent horizontal and vertical coordinates, respectively, of the center of mass with the origin at the end of the takeoff. Let $\varphi(t)$ be the time-dependent azimuthal angle of the bottom of the long axis of the snowboard/skis about the z-axis, that is, the angle of the snowboard/skis with respect to the horizontal (x-) axis (with positive sense being above the horizontal axis). For example, as can be seen in Fig. 1(a), upon takeoff we have the initial condition, $\varphi(0) = \theta_T$. Ignoring air drag, the equations of motion governing the rigid body motion from the moment the center of mass passes the end to the takeoff while the snowboard/skis are in contact with the takeoff are:

$$x''(t) = -\frac{N}{m} \left( \frac{y(t) + \mu x(t)}{\sqrt{x(t)^2 + y(t)^2}} \right),$$

(9)

$$y''(t) = -g + \frac{N}{m} \left( \frac{(x(t) - \mu y(t))}{\sqrt{x(t)^2 + y(t)^2}} \right),$$

(10)

$$\varphi''(t) = -\frac{12}{m h^2} \left( \frac{\sqrt{x(t)^2 + y(t)^2} + \mu h}{2} \right),$$

(11)

where $g$ is the gravitational acceleration constant, $N$ is the normal ground reaction force on the snowboard/skis, $m$ is the mass of the jumper, and $\mu$ is the friction coefficient. In Eq. 11, the moment of inertia for rotations about the center of mass has been approximated by the cylindrical expression, $\frac{1}{12} m h^2$. Since these equations apply while the snowboard/skis are in contact with the takeoff, we have the additional constraint,

$$\varphi(t) = \arctan \frac{y(t)}{x(t)}.$$  

(12)

Taking two derivatives of the constraint equation and using Eq. 11 gives:

$$\frac{N}{m} = -\frac{h^2 (x(t)^2 + y(t)^2)(x(t) y''(t) - y(t) x''(t)) - 2 (y(t) x'(t) - x(t) y'(t))(x(t) x'(t) + y(t) y'(t))}{12 \left( \sqrt{x(t)^2 + y(t)^2} + \mu h \right)(x(t)^2 + y(t)^2)^2}$$  

(13)
Substituting this expression in the remaining equations yields two coupled second order differential equations for \( x(t) \) and \( y(t) \) subject to the initial conditions,

\[
\begin{align*}
  x(0) &= 0, \\
  y(0) &= 0, \\
  x'(0) &= v_0 \cos \theta_T, \\
  y'(0) &= v_0 \sin \theta_T,
\end{align*}
\]

where \( v_0 \) is the takeoff speed (at the point where the center of mass crosses the end of the takeoff). These two equations can be solved numerically over the time period that the board remains in contact with the end of the takeoff and the final (forward) rotational speed due to the ground reaction force determined. The rotational speed induced by the ground reaction force partially compensates for the inverting rotation induced by the curved takeoff treated in the previous section. An example calculation of both effects is provided in Section D.

C. Discussion

Since a realistic jump takeoff does not have constant curvature, a determination of the radius of curvature requires choosing a chord length. Since the inverting effect is induced through the snowboard/ski, it is reasonable to take the length of a snowboard/ski, say about 2 m, for a practical length of the chord used to determine the radius of curvature.

If curvature in the takeoff should be avoided, the question naturally arises as to how long should the straight portion of the takeoff prior to the lip be. There must be some curvature between the approach and the takeoff since the rider must go from a basically downhill direction to an upward direction at takeoff. According to Schmidt and Lee [24] balance is a learned automatic response; so one can estimate that the time needed to recover normal balance after traversing a curved section should be of the order of this automatic human response time (\( \tau \sim 0.2 \text{s} \) [23, 24]). The US Terrain Park Council [25] has adopted the value of 1.57 \( \sim 0.3 \text{s} \) as a jump design criterion which states that the final section of the takeoff should be straight for 0.3 s times the takeoff speed. This is slightly more conservative than the standard of 0.25 s times the takeoff speed adopted by the International Ski Federation (FIS) for Nordic jumps [26].

For large Nordic jumps the takeoff speed is about 26 m/s which implies a straight section of 6.5 m. One can quantify the USTPC criterion as follows. Let \( (x_L, y_L) \) be the coordinates of the designed or expected landing point (taking the origin to be the lip of the takeoff). Using Eq. 1 for the trajectory, one finds that the minimum straight section of the takeoff under the USTPC criterion, \( S_{USTPC} \), is given by,

\[
S_{USTPC} = 1.5\tau \frac{v_0}{\cos \theta_T} = 1.5\tau \frac{x_L}{\cos \theta_T} \sqrt{\frac{g}{2(tan \theta_T x_L - y_L)}},
\]

where \( \tau \) is the balance response time (\( \sim 0.2 \text{s} \)) and \( \theta_T \) is the takeoff angle.

D. Example calculation

To illustrate the effect of curvature on the takeoff for an inert jumper, consider the larger of the example jumps shown in Fig. 2(a). The takeoff angle is \( \theta_T \approx 35^\circ \) and the landing angle is \( \theta_L \approx 28^\circ \). The curvature in the takeoff in the last couple of meters is evident. As discussed above, such a takeoff will tend to put the jumper in the “backseat” if no compensating maneuver is used. Experienced jumpers (not executing an inverted aerial) are able to accommodate this effect by giving themselves a compensating forward rotation upon takeoff. To get a length scale, I estimate that the person standing in the photo has a height of approximately 1.8 m. From this scale, I estimate the deck length to be \( D \approx 5.36 \text{m} \) and the vertical height of the takeoff above the deck to be \( H \approx 0.44 \text{m} \). Similarly, from the chord length \( (L_c \approx 1.7 \text{m}) \) and the perpendicular distance from the midpoint of the chord to the takeoff surface \( (d_c \approx 0.045 \text{m}) \), and using Eq. 4, I estimate the radius of curvature to be approximately \( R \approx 8.1 \text{m} \).

To obtain a typical trajectory, assume the initial takeoff speed is sufficient to carry the jumper to a point 2 m beyond the “knuckle” which is often described as the “sweet spot”, considered the most desirable landing location. From the ballistic equations of motion one determines the takeoff speed to be about 7.1 m/s with a time of flight of \( t_f \approx 1.27 \text{s} \). To treat the finite size refinements we must specify the size of the jumper and snowboard/ski. As a concrete example, take the jumper to be 1.65 m high riding on a 1.5 long snowboard. From Eq. 5 the above values give an inverting angular speed due to the curvature of about 55.8\(^\circ\) per second. Numerically solving Eqs. 9 and 10 using Eq. 13 for \( N/m \), we calculate the compensating (forward) rotational speed induced by the ground reaction force
for our example snowboarder to be about 10.4° per second (independent of the mass). Combining these gives a net inverting rotational speed of 45.4° per second. Using Eq. 8 gives a net takeoff-induced inverting angle of \( \phi \simeq 57.4° \) upon landing. When the takeoff and landing angles are included, the total inverting angle of the jumper with respect to the normal of the landing surface is about 120.4° which is substantial enough to present an injury hazard. Under the USTPC criterion, Eq. 15, the straight section of the takeoff before the lip in this example should be at least 2.1 m to mitigate a takeoff-induced inversion risk for this jump.

III. SUMMARY AND CONCLUSIONS

Epidemiological studies of ski and snowboard injuries have determined that terrain park jumps pose a significant risk for head and spinal injuries to resort patrons. The most devastating jump-related injuries nearly always involve the jumper landing in an inverted position on his/her head, neck, or back. There are several ways for a jumper to become inverted, including his/her own actions at the takeoff that lead to inversion either voluntarily or involuntarily. As discussed previously, parsimonious salting practices may also contribute to the jumper leaving the takeoff in an unbalanced “backseat” posture. In addition, and the focus of this work, concave curvature in the last section of the takeoff can induce an inverting rotation if no compensating actions are taken by the rider. Using a rigid-body model of the jumper, approximating a stiff-legged novice, the total angle of inverting rotation induced by a curved takeoff can be derived. Example rigid-body model calculations of realistic jumps with curved takeoffs (including the partially compensating forward rotation caused by the ground reaction force) show that the resulting inverting angle can be substantial, \( \sim 60° \), giving a total inverting angle of the jumper with respect to the normal of the landing surface over 120°. These results suggest that takeoffs should be designed with a minimum straight section before the lip in order to reduce the risk of inverted landings. Accordingly, the USTPC has adopted a criterion specifying that the end of the takeoff be straight for at least 0.3 s times the nominal takeoff speed.

Safety in terrain park jumps is a partnership between the jumper and the resort. The jumper should follow the well-publicized “Smart Style” advice to “ride within your ability” and “look before you leap; land on your feet”. Resorts cannot control the rider’s actions, and thus the rider must bear the primary responsibility for the consequences of his/her own actions. However, certain factors such as salting practices and the design and maintenance of the takeoff can be controlled by resorts so as to reduce the risk of potentially hazardous inversions.

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FIG. 1: (a) Schematic of the standard table top jump with (potentially dangerous) curvature of radius, $R$, in the takeoff. Also shown is a schematic jumper who takes off oriented normal to the end of the takeoff. If he does not change orientation during the jump, he will land in a backward orientation with respect to the normal of the landing surface by the sum of the takeoff and landing angles, $\theta_T + \theta_L$, as shown. (b) Close-up of takeoff showing supplementary constructs of a chord of length, $L$, and a distance, $d_c$, from center of chord to takeoff. These parameters are used to calculate the radius of curvature as described in the text.

FIG. 2: (a) Example of a jump with a curved takeoff along with (b) a schematic trace describing the jump parameters from which the radius of curvature can be estimated.