Epidemiological studies of injuries at ski resorts have found that jumping generally poses a significantly greater risk of spine and head injuries to patrons. Jumping activities in resorts are now focused in terrain parks over man-made features which provides an opportunity to mitigate injury rates through better engineering design. However, the use of engineering design has been questioned by the NSAA due to the issue of rider variability [1], a view implicitly supported in a recent paper by Shealy, et al. [2] who studied jumper trajectories for two terrain park jumps and reported no correlation between the takeoff speeds and landing distances. While their theoretical analysis was flawed, the undeniable fact remains that rider actions can substantially influence the trajectory. In particular riders can “pop”, i.e. jump (or drop) before takeoff, thereby changing the initial velocity in both direction and magnitude. In this paper I expand on an earlier Newtonian analysis to include this effect and use the field data of Shealy et al. to constrain the range of realistic “pop” speeds. I find that “pop” speeds in the range of $-2.48 \rightarrow +1.12 \text{ m/s}$ account for the range of landing distances measured by Shealy, et al. Since the rider variability due to “pop” is bounded, similar Newtonian analyses can therefore provide bounds on the range of trajectories, landing positions, landing velocities, and equivalent fall heights which could be useful to winter terrain park designers.

I. INTRODUCTION

An epidemiological study of injuries at ski resorts by Tarazi, et. al. in 1999 found that jumping generally (whether in a terrain park or not) poses a significantly greater risk to patrons for certain classes of injuries involving the spine and head [3]. In 2011, Henrie, et al. reported that this class of injuries are twice as likely to occur inside terrain parks than outside [4]. These are consistent in that at most ski resorts today jumping features are generally located within terrain parks. In a 2011 thesis study focussed on terrain park injuries, Russell found that terrain park injuries were more severe when jumping features were involved [5]. Since terrain park jumps are man-made features, one may speculate whether better engineering design could mitigate injury rates associated with jumping in terrain parks. However, the practicality of using engineering design for terrain parks has been questioned. In particular, the 2008 NSAA Freestyle Terrain Notebook states that it is impossible to implement “standards”, i.e. quantifiable engineering practice, in the design of winter terrain park jumps due to variability in the snow conditions as well as the actions of the jumpers [1]. This position has been implicitly supported in two recent papers by Shealy and collaborators. First, in a two phase study in 2008 Shealy and Stone examined the trajectories of riders (snowboarders or skiers) over winter terrain park jumps and measured the accelerations riders experience upon landing [6]. In the first part of this study they found significant differences between their measurements and theoretical calculations based on a Newtonian model and concluded that “Elementary Newtonian physical laws for projectile motion cannot be used to predict the location that a skier/snowboarder will land based strictly on the speed of the skier/snowboarder at the point of take off and the geometry of the feature.” To make sense of this somewhat counter-intuitive remark, one must understand that by “takeoff speed” the authors do not mean the actual speed at takeoff, but rather, as they acknowledge at the end of their paper, the component of the takeoff velocity parallel to the ramp surface. They report that the vector nature of the takeoff velocity is a “source of error with the ballistic equations”; however, as pointed out previously, these are not errors with the ballistic equations, but rather incorrect values for the initial conditions used to find a particular solution of them. While SSSH made no further attempt to reconcile their results with theory, the authors still performed a useful service. By measuring the correlation between the component of the takeoff velocity parallel to the takeoff ramp and the total distance of the jump, the component of the takeoff velocity perpendicular to the takeoff surface, i.e. the “pop” speed, can be extracted. As shown below, this data has been used to constrain rider variability due to “pop” which will be helpful in the design of winter terrain park jumps. (However, caution is advised as some of their data could not be reconciled with theory regardless of the added “pop”.)
SSSH also criticized the theoretical models of Hubbard [8] and McNeil and McNeil [10] for assuming the jumper is an inert point mass. They further state that a “second source of error with the ballistic equations” is the possibility of aerial maneuvers which are neglected if an inert jumper is assumed. Neither of these criticisms has merit as both Hubbard and McNeil and McNeil calculate the trajectory of the center-of-mass (which is indeed a point), but this is not the same as assuming that the rider is a point particle because extended body effects such as drag and lift are included. Since these are the only factors that can influence the center-of-mass motion once the rider has left the surface, the model is generally valid. Furthermore, as shown below, since the lift/drag effects are small for jumps less than about 20 m; any changes in the lift/drag coefficients induced by aerial maneuvers will be perturbations on already small effects. Nevertheless, even for larger jumps or windy conditions where the drag/lift effect can be important, the Newtonian model is still valid and useful. As shown by McNeil, Hubbard, and Swedberg [11], one can calculate the range of possible trajectories by treating the full range of drag coefficients such that the trajectory resulting from any aerial maneuver will necessarily fall within this range.

Finally, SSSH confuse the general “Newtonian model” with a particular instance of the model. In common parlance, the “Newtonian model” consists of Newtonian dynamics under the governing forces (in this case gravity, friction, drag and lift), and as such, is unassailable. To present a particular solution of the model one must specify the parameters (e.g., mass, and friction, drag, and lift coefficients) and the initial conditions. Specifically, while in their example calculations McNeil and McNeil assumed the initial takeoff velocity was parallel to the takeoff surface, they ever assumed that this particular solution would be appropriate in general. Indeed, they explicitly stated that while the particular calculations presented are for an inert rider, rider actions such as “pop” could be included within the model by suitable adjustment of the initial conditions. In this paper I carry through on this suggestion by including the “pop” effect, present the general equations describing the trajectory of the center-of-mass of a jumper including the effects of drag and lift, and solve the equations numerically for three example jumps. I use the same jumps studied by SSSH as two of the example jumps and add a third (hypothetical) larger jump.

One additional systematic measurement error not treated by SSSH involves assuming that the distance measurements taken from a fixed location, e.g. from the center of the snowboard, do, in fact, measure the center-of-mass motion. Since the center of the board is not at the center-of-mass, these measurements need small corrections for changes in the orientation of the jumper from takeoff to landing. That is, the rider may start out with his snowboard parallel to the takeoff surface, but, if all goes well, ends up with his snowboard (approximately) parallel to the landing surface. This rotation leads to the fixed-location measurement (if taken on the board) systematically underestimating the center-of-mass distance. This effect is discussed and estimated below.

Of course, it would be convenient if one could ignore drag/lift and use the classic closed-form analytic solutions thereby avoiding cumbersome numerical analysis. Therefore, as a first application, I examine the degree to which drag/lift influence the trajectories for the three example jumps. Next, I include the “pop” effect and use the full theory (with drag/lift) to extract a range of realistic “pop” speeds from the data of SSSH. It was in the course of this analysis, that I discovered that several of the SSSH data had to be excluded because the speed-distance data could not be reconciled with theory for any value of the “pop” speed. Since, I show that at about the 10% level, the drag/lift effects can be safely neglected for jumps less than about 20 m, I then derive closed-form analytic expressions for the total distance traveled (as defined in SSSH) for tabletop jumps neglecting these effects. SSSH provided unit-dependent numerical equations for the same theoretical calculations of total jump distance but applied to their specific jumps. Here, I present the symbolic forms and in attempting to duplicate their numerical results find quantitative differences with their published equations that alter the results somewhat.

The paper is organized as follows. Section II gives the general Newtonian equations of motion for the center of mass of an extended object moving through the atmosphere. I include gravity, drag, and lift, but ignore other effects such as Coriolis and the Magnus force which are negligible for these applications. Three example jumps are treated. Numerical results illustrating the importance of drag and lift for each jump are then presented as well the influence of a stiff head or tail wind. From the landing velocity and landing slope angle we also calculate the equivalent fall height, $h$, [8, 10, 11], an important quantity that characterizes the injury risk due to impact (independent of the rider’s landing position). The “pop” effect is added to the model with the “pop” speeds constrained by the SSSH field data, and full numerical calculations illustrating this effect for the three example jumps are presented. (However, as previously noted, this analysis of their data revealed likely experimental errors in at least some of their measurements.) In Section III I treat the special case where drag and lift are neglected and analytic expressions are derived for the trajectory and landing conditions appropriate to tabletop jumps. I also derive expressions for the equivalent fall
height, present example calculations to illustrate fundamental weaknesses in the tabletop design (see also Swedberg and Hubbard [12]), and use the results to suggest an optimized design for the tabletop jump geometry that minimizes the fall height equivalent. I then revisit the work of SSSH and present new theoretical expressions for the total landing distance which differ quantitatively from theirs. The summary and conclusions follow. The two take-home messages are first, that while rider “pop” affects the total jump distance somewhat, it does so in an understandable and bounded fashion, and, second, that elementary Newtonian dynamics provides an excellent analysis tool that can be helpful to terrain park jump designers should they elect to use it.

II. NEWTONIAN MODEL OF THE TRAJECTORY OF A TERRAIN PARK JUMPER

A. General Theory

Ballistics generally is the study of the motion of a projectile moving through the atmosphere and historically from the time of Galileo has been conducted primarily in the context of firearms and artillery [14]. In the skiing context the application of Newtonian physics to the motion of ski-jumpers and downhill racers has been extensively studied [15–19], but application specifically to winter terrain park jumps has only recently begun to attract attention [2, 6, 8–10, 20, 21]. Here the trajectories of jumpers from standard tabletop jumps will be treated. Fig. 1 defines the geometry and parameters for such a jump. Extension to alternative jump geometries, such as those proposed by Hubbard [8] and McNeil [10], is straightforward. One starts by considering the motion of an extended jumper’s body moving through the atmosphere under the influence of gravity, air drag, and lift only. Take the x-direction to be horizontal, the y-direction vertical, and the origin to be at the end of the takeoff ramp. The equations of motion describing the center of mass position vector are:

\[
\frac{d^2 \vec{r}(t)}{dt^2} = -g \hat{y} - \eta(t)|\vec{v} - \vec{w}| \left( \vec{v} - \vec{w} - \rho_{ld} \hat{s} \times (\vec{v} - \vec{w}) \right),
\]

where \(\hat{y}\) is the unit vector in the y-direction, \(\vec{v} = \frac{d\vec{r}}{dt}\) is the projectile velocity, \(\vec{w}\) is the wind velocity, \(g\) is the acceleration of gravity, \(\rho_{ld}\) is the lift to drag ratio, \(\hat{s}\) is the “sideways” direction unit vector of the jumper (defined below), and \(\eta(t)\) is the time-dependent drag parameter given by:

\[
\eta(t) = \frac{\rho C_d A(t)}{2m}
\]

FIG. 2: Jumper orientation unit vectors used to determine the direction of lift: \(\hat{n}\) is normal to snowboard surface, \(\hat{f}\) is the “forward” direction, and \(\hat{s} = \hat{f} \times \hat{n}\) is the “sideways” direction. Note that the velocity vector need not be in the “forward” (\(\hat{f}\)) direction. (color online)
the angle of the takeoff, this assumption is relaxed will require knowledge of the width profile of the terrain park landing surface. For inert jumpers the initial velocity is parallel to the end of the takeoff and is thereby determined by the initial speed, \( v_0 \), and lift coefficients. Lastly, once the parameters are given, a solution to the model requires specifying the initial conditions: \( \{ \vec{r}(0), \vec{v}(0) \} \). With the origin taken to be the end of the takeoff and taking \( t = 0 \) to be the moment of takeoff, one has \( \vec{r}(0) = \vec{0} \). In general, the initial velocity requires all three components, and indeed, many jumpers start a turn before the takeoff to provide angular momentum for a spin maneuver. Such spin maneuvers may give normal to the takeoff surface, \( \vec{v} \) varies with speed as well, but such effects are negligible for the range of speeds appropriate to terrain park jumping.

Aerial maneuvers. Given some specific maneuver, the frontal area, \( A(t) \), for that maneuver could be measured or estimated and incorporated in the numerical solutions; however, since we are interested only in the bounds of the effect of drag/lift, I treat it as constant over the course of one jump and examine the bounds by taking limiting cases. The effect of drag/lift due to any maneuver will then lie within these bounds. Similarly, the wind velocity can vary with time (i.e. gusts), but will be treated as constant for the duration of each jump. The range of values for the physical parameters used in this study are given in Table I.

\[
\rho(Y) = \rho_0 \frac{T_0}{T} e^{-\frac{T}{T_0} \beta Y}
\]

(3)

where \( Y \) is the altitude above sea level in meters, \( T_0 \) is the reference temperature (298.15 K), \( T \) is the absolute temperature (\( T = T_C + 273.15 \) K, where \( T_C \) is the temperature in celsius), \( \rho_0 = 1.1839 \text{ kg/m}^3 \), and \( \beta = 1.151 \times 10^{-4} \text{ m}^{-1} \). For example, this relation gives the air density at Mammoth Mountain Ski resort (where SSSH carried out part of their study) as \( \rho(3370 \text{ m}) = 0.885 \text{ kg/m}^3 \) at \( T_C = -10 \) C (SSSH did not report a temperature.). In principle, \( C_d \) varies with speed as well, but such effects are negligible for the range of speeds appropriate to terrain park jumping.

The direction of the lift force is determined by the shape and orientation of the jumper with respect to the velocity through the air. For specificity, we take a jumper (snowboarder or skier) such as shown in Fig. 2 which also shows the unit vectors, \( \{ \hat{n}, \hat{f}, \hat{s} \} \) that define the orientation of the jumper, i.e. \( \hat{n} \) is normal to snowboard surface, \( \hat{f} \) is the “forward” direction, and \( \hat{s} = \hat{f} \times \hat{n} \) is the “sideways” direction. Assuming the lift is dominated by the flat snowboard surface, the direction of the lift will be \( \hat{s} \times (\vec{v} - \vec{w}) \). In the applications considered here, to keep the examples simple no maneuvers will be treated; thus, the “sideways” direction is fixed, and the lift vector will always lie in the xy-plane perpendicular to the jumper’s velocity vector. In general the drag factor, \( \eta(t) \), is a function of time since the jumper can change his frontal area by executing aerial maneuvers. Given some specific maneuver, the frontal area, \( A(t) \), for that maneuver could be measured or estimated and incorporated in the numerical solutions; however, since we are interested only in the bounds of the effect of drag/lift, I treat it as constant over the course of one jump and examine the bounds by taking limiting cases.

The above model, i.e. Newtonian dynamics under the governing forces of gravity, friction, drag, and lift, is quite general. In order to apply the model one must provide values for the parameters, i.e. the mass and friction, drag and lift coefficients. Lastly, once the parameters are given, a particular solution to the model requires specifying the initial conditions: \( \{ \vec{r}(0), \vec{v}(0) \} \). The direction of the lift force is determined by the shape and orientation of the jumper with respect to the velocity through the air. For specificity, we take a jumper (snowboarder or skier) such as shown in Fig. 2 which also shows the unit vectors, \( \{ \hat{n}, \hat{f}, \hat{s} \} \) that define the orientation of the jumper, i.e. \( \hat{n} \) is normal to snowboard surface, \( \hat{f} \) is the “forward” direction, and \( \hat{s} = \hat{f} \times \hat{n} \) is the “sideways” direction. Assuming the lift is dominated by the flat snowboard surface, the direction of the lift will be \( \hat{s} \times (\vec{v} - \vec{w}) \). In the applications considered here, to keep the examples simple no maneuvers will be treated; thus, the “sideways” direction is fixed, and the lift vector will always lie in the xy-plane perpendicular to the jumper’s velocity vector.

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The effect of drag/lift due to any maneuver will then lie within these bounds. Similarly, the wind velocity can vary with time (i.e. gusts), but will be treated as constant for the duration of each jump. The range of values for the physical parameters used in this study are given in Table I.

B. Modelling “Pop”

The above model, i.e. Newtonian dynamics under the governing forces of gravity, friction, drag, and lift, is quite general. In order to apply the model one must provide values for the parameters, i.e. the mass and friction, drag and lift coefficients. Lastly, once the parameters are given, a particular solution to the model requires specifying the initial conditions: \( \{ \vec{r}(0), \vec{v}(0) \} \). With the origin taken to be the end of the takeoff and taking \( t = 0 \) to be the moment of takeoff, one has \( \vec{r}(0) = \vec{0} \). In general, the initial velocity requires all three components, and indeed, many jumpers start a turn before the takeoff to provide angular momentum for a spin maneuver. Such spin maneuvers may give the jumper an initial \( z \)-component of velocity (transverse to the downhill direction); however, to keep things simple, the \( z \)-component of the initial velocity is taken to be zero in the examples presented here. Analyzing jumps where this assumption is relaxed will require knowledge of the width profile of the terrain park landing surface. For inert jumpers the initial velocity is parallel to the end of the takeoff and is thereby determined by the initial speed, \( v_0 \), and the angle of the takeoff, \( \theta_T \); \( \vec{v}(0) = \{ v_0 \cos \theta_T, v_0 \sin \theta_T, 0 \} \). This was the particular case treated by McNeil and McNeil [10].

We now treat the jumper’s “pop” by adding to the “no-pop” initial velocity, \( \vec{v}_0 \), an additional velocity component normal to the takeoff surface, \( \vec{v}_p \), that is \( \{ -v_p \sin \theta_T, v_p \cos \theta_T, 0 \} \). Since the ski/snow interface has a relatively low friction

\[
\begin{array}{|c|c|c|}
\hline
\text{Parameter} & \text{Symbol} & \text{Units} & \text{Value Range} \\
\hline
\text{Acceleration of gravity} & g & \text{m/s}^2 & 9.81 \\
\text{Mass of jumper} & m & \text{kg} & 75 \\
\text{Height of jumper} & H_J & \text{m} & 1.7 \\
\text{Drag coefficient times frontal area of jumper} & C_dA & \text{m}^2 & 0.279 - 0.836 \\
\text{Density of air} & \rho & \text{kg/m}^3 & 0.90 - 1.18 \\
\text{Coefficient of kinetic friction} & \mu & \text{dimensionless} & 0.04 - 0.12 \\
\text{Lift to drag ratio} & \rho_{ld} & \text{dimensionless} & 0.0 - 0.1 \\
\hline
\end{array}
\]
### Table II: Tabletop Jump Parameters.

<table>
<thead>
<tr>
<th>Jump</th>
<th>Deck Length (D) (m)</th>
<th>Height of Takeoff (h) (m)</th>
<th>Takeoff Angle (\theta_T) (°)</th>
<th>Landing Angle (\theta_L) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.1</td>
<td>0.6</td>
<td>26</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>12.3</td>
<td>2.0</td>
<td>21</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>20.0</td>
<td>3.0</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

coefficient, the jumper cannot add a substantial component of velocity parallel to takeoff surface. The “pop” will alter the initial velocity vector (speed and direction) accordingly:

\[
v_0 \rightarrow v_{0+p} = \sqrt{v_0^2 + v_p^2},
\]

\[
\theta_T \rightarrow \theta_{T+p} = \theta_T + \delta \theta_p
\]

(4)

where \(\delta \theta_p = \arctan(v_p/v_0)\) and where the \((\ldots +p)\)-subscript denotes the initial conditions appropriate to the “pop” case. Equivalently, the initial velocity vector with “pop” can be written in component form,

\[
v_{0+p} = \{v_0 \cos \theta_T - v_p \sin \theta_T, v_0 \sin \theta_T + v_p \cos \theta_T, 0\}.
\]

(5)

Starting from the initial conditions, the equations of motion can be integrated numerically to find the jumper’s trajectory as a function of time. Fig. 1 shows the standard tabletop jump consisting of a takeoff inclined at angle, \(\theta_T\), at a height, \(H\), above a flat deck of length, \(D\), transitioning to a straight landing surface inclined at angle, \(\theta_L\). Of course, any shape jump can be accommodated, but the simple straight line geometry used in the tabletop form allows one to find analytic solutions for landing parameters in the absence of drag/lift as shown by Swedberg [20]. I perform illustrative calculations for three jumps. The two jumps studied by SSSH [2], are identified here as example jumps A and B with a hypothetical larger jump, C, added. (The A and B jumps here are referred to as the “smaller” and “larger” jumps, respectively, in SSSH.) The parameters used for the three jumps are given in Table II. The equations were solved numerically using the NDSolve function in Mathematica [24] and checked with a fourth-order Runge-Kutta code.

**FIG. 3:** Example calculations of the drag and lift effects for the three sample jumps (parameters given in Table II). A drag-area coefficient of 0.557 m\(^2\) was used, and the initial speeds for each jump respectively are 35.8 km/h (9.94 m/s), 43.5 km/h (12.08 m/s), 54.0 km/h (15.0 m/s). (color online)

### III. RESULTS

#### A. The Effect of Drag and Lift

First I examine the effect of air drag/lift for the three example jumps. Hoerner [25] provides product drag coefficient times frontal area factors for adult humans for the subsonic speeds appropriate to terrain park jumps. Hoerner lists drag-area coefficients in the range of 0.279 m\(^2\) for a tucked position, 0.557 m\(^2\) for a sideways standing position, and 0.836 m\(^2\) for a face-on standing position. Here the effect for a snowboarder riding sideways to the direction of travel is considered; so the standing-sideways drag-area coefficient (0.557 m\(^2\)) is used. Maneuvers in the air can change the
drag coefficient, but as shall be shown the effects are small. There is no published data on the lift to drag ratio for snowboarders; so it was estimated based on free-fall aerodynamic trajectories of sky-divers wearing snowboards which gave a crude estimated value of \( \rho_{\text{air}} \approx 0.10 \). This should be adequate as the final results are not very sensitive to this quantity. The density of air is taken to be 0.868 kg/m\(^3\) which is appropriate to the Mammoth Mountain terrain park at an altitude of 3370 m at a temperature of \( T_0 = -10 \) C. Lower altitudes will have larger air densities according to Eq. 3. The mass of the jumper was fixed in all cases to 75 kg. The tangential component of the takeoff speeds for the A and B jumps were taken from the illustrative examples taken from Fig. 5 of SSSH, namely, 35.8 km/h (9.94 m/s) for the A jump and 43.5 km/h (12.08 m/s) for the B jump. The tangential component of the takeoff speed for the large jump was arbitrarily fixed at 54 km/h (15 m/s) to result in a jump distance beyond the knuckle. For these examples where only the effect of drag/lift is studied, the jumpers are treated as inert (i.e. no “pop”) so the takeoff velocity vector is directed parallel to the takeoff ramp.

Fig. 3 shows the resulting calculated trajectories with and without drag/lift. In each figure the solid line is the trajectory without ‘pop’, the thin dashed line is the jumper trajectory with positive ‘pop’, and the fat dashed line is the jumper trajectory with negative ‘pop’. The ‘pop’ speed values which were extracted from the data [2] for each case are given in the respective caption.

SSSH noted the problem with associating center-of-mass coordinates with the data they collected based on the speed of the feet of the jumpers. The relationship between these two quantities depends on the posture and orientation of the rider. To estinate this effect I assume the jumper takes off and lands in the same posture, i.e. riding sideways, but undergoes a slight forward rotation that keeps the jumper’s snowboard parallel to the landing surface. For cases where the jumper lands on the sloped landing surface, this results in the center-of-mass x-coordinate traveling an extra distance of approximately,

\[
\Delta D_t \approx \frac{H_f (\sin \theta_T + \sin \theta_L)}{2 \cos \theta_L},
\]

where \( H_f \) is the height of the jumper. We use this shift in comparing with the SSSH data using a nominal rider height of 1.7 m. Since this data was not reported, this estimate will introduce a systematic correction to the total distance measurements of approximately +0.5 m.

The change in vertical orientation is accomplished by having a slight forward rotation at the point of takeoff which means the feet will be moving slower than the head. This will introduce another error in the use of speed measurements of the feet to represent center-of-mass speeds. For the larger of the jumps studied in SSSH (jump B) the needed rotation is the sum of the takeoff and landing angles, i.e. 51\(^\circ\). The time of flight for the mean jump distance for jump B is 1.53 s; so the angular rotation speed, \( \omega_f \), is about \( \omega_f = 33^\circ/s = 0.58\text{rad/s} \). Although many jumpers do not fully rotate so as to land flat on the landing surface, to get an upper bound on this effect assume the jumper leaves the takeoff with this (forward) rotational speed which means that his center-of-mass is moving faster than his feet by approximately \( \omega_f H_f/2 \). Using the nominal values from the tables, the speed correction to the SSSH measured value is about \( +0.49\text{m/s} \) compared to the SSSH average takeoff speed of 12.1 m/s. Thus the upper bound for the increase in the jump distance due to this effect is about 2.3 m (14%). However, nearly all jumpers perform some maneuver that can reduce their moment of inertia thereby increasing their angular speed. A careful treatment of this effect will require film analysis which is not available.

The role of wind is examined next. Consider a 20 mph wind blowing in the x-direction. The change in the jumper’s trajectory for both the head wind and tail wind cases are shown in Fig. 4 for both the three example jumps. For these cases the effect of drag including the 20 mph wind on the total horizontal distance travelled varies from -8.8% \( \rightarrow +3.5% \) for the A jump, -7.1% \( \rightarrow +4.6% \) for B jump, to -14.4% \( \rightarrow +8.5% \) for the C jump. For the case of a 30 mph wind (not shown) the change in the total horizontal distance travelled varies from -14.9% \( \rightarrow +4.5% \) for the A jump, -20.6% \( \rightarrow +7.0% \) for B jump, to -23.2% \( \rightarrow +10.2% \) for the C jump. The effect of a head wind is fractionally much more dramatic than that of a tail wind which suggests that terrain park staff should be aware of the additional hazard to landing short (e.g. on the deck) when a significant head wind is present.
B. The Effect of “Pop”

Next consider the effect of “pop”. Fig. 5 of SSSH plots the distance traveled versus the component of the takeoff velocity parallel to the takeoff ramp (i.e., the quantity they call the “takeoff speed” which is referred to here as tangential takeoff speed, $v_0$) for the A and B jumps. One should be able to use this data to estimate a range of “pop” speeds since for each $v_0$ there is a range of measured total distances. They illustrate the maximum range of variation due to “pop” by selecting data at $v_0 = 35.8$ km/h for the A jump and $v_0 = 43.5$ km/h for the B jump. From Fig. 5 of SSSH I estimate that for the A jump at $v_0 = 35.8$ km/h the (corrected) maximum and minimum landing distances (beyond the knuckle) are about 6.4 m and about 0.5 m, respectively. Similarly, for the B jump at $v_0 = 43.5$ km/h SSSH report about 6.4 m and about -1.0 m (i.e., on the deck) for the (corrected) maximum and minimum landing distances, respectively. Using these parameters and applying Eq. 4 to determine the initial conditions with “pop”, I then adjust the “pop” speed to reproduce the measured total ground distance traveled. In this way I determine the range of “pop” speeds for the A jump to be -2.48 m/s to +0.83 m/s, and -0.65 m/s to +1.12 m/s for the B jump. For the hypothetical larger C jump the “pop” speed was arbitrarily varied from -1.0 m/s to +1.0 m/s. The trajectories from these calculations are shown in Fig. 5 and the results for relevant kinematic quantities given in Table III.

In the next section where drag and lift effects are neglected, an analytic closed-form expression for the total distance traveled is derived, Eq. 12. Using the initial conditions appropriate to the “pop” case, Eq. 4, one can invert this analytic expression to find the “pop” speed versus total distance traveled which can be used to convert Fig. 5 of SSSH into a plot of “pop” speed versus tangential takeoff speed. The data from SSSH’s Fig. 5 was not made available by the authors; so I extracted as many of the data points as could be individually identified from the plot directly. Consider the case of the B jump (SSSH, Fig. 5b). Fig. 6 shows the “pop” speeds extracted from the B jump (SSSH, Fig. 5b) as a function of the tangential takeoff speed. It shows a clear anti-correlation between the rider’s tangential speed at takeoff and the “pop” speed the rider elects to add suggesting that the riders may have some sense of the speed needed to land just beyond the knuckle and use “pop” to make last second adjustments.

However, some of the measurements extracted from Fig. 5 of SSSH could not be reconciled with theory. As one example, consider the left-most point in SSSH, Fig. 5b which has a landing distance (beyond the knuckle) of approximately +0.9 m (or 13.3 m for the total distance) associated with a “takeoff speed” (i.e., tangential component of takeoff velocity), $v_0 \approx 30$ km/h. Barring a wind gust in excess of 100 mph, there is no value of “pop” that a rider add in order to reach that distance with a tangential component of takeoff velocity equal to 30 km/h. In fact, even ignoring drag, the maximum landing distance results from the (humanly impossible) “pop” speed of 8.7 m/s, and even that will only carry the rider a total distance of 11.1 m. Given the length of the deck, there is no physical way for a rider with a tangential takeoff speed of only 30 km/h to clear the knuckle. Several other data suffer the same problem.

Assuming there were no equipment or transcription errors, one is left to speculate on the origins of possible systematic errors in their data collection method. One such possibility is a change in the orientation of the rider’s snowboard (or skis) over the last 25 cm gap between the timing gates placed on the takeoff as the rider initiates a spin move. If the gates are triggered by, say the front boot, then such a shift in the snowboard (or skis) orientation will increase the measured time interval even though the tangential component of the takeoff velocity is not significantly changed, thereby inducing a systematic underestimation of that component of the takeoff velocity.

IV. ANALYTIC EXPRESSIONS

A. Total Jump Distance

As shown in Fig. 3, the effect of drag/lift scales with the size of the jump. In terms of the total distance, including drag/lift alters the drag free results by about 3% to 9%. Aerial maneuvers generally reduce the frontal area relative to the position at takeoff and would therefore result in even small drag effects. Therefore, for jumps less than 20 m in size it appears to be a reasonable approximation to ignore drag/lift which allows one to derive reliable analytic
TABLE III: Results of jump distance variation due to “pop”. All results include the effects of drag and lift. $H_{pop}$ is the equivalent jumping height needed to give the “pop” speed, $v_p$. $\delta \theta_T$ is the change in the takeoff angle due to the ‘pop’, $x_L$ is the horizontal jump distance, $\Delta x_L$ is the fractional change in the horizontal jump distance due to the “pop”, and $h$ is the equivalent fall height upon landing. The various jump parameters are given in Table II.

<table>
<thead>
<tr>
<th>Jump</th>
<th>$v_p$</th>
<th>$H_{pop}$</th>
<th>$\delta \theta_T$</th>
<th>$x_L$</th>
<th>$\Delta x_L$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-2.48</td>
<td>-0.313</td>
<td>-14.2</td>
<td>7.00</td>
<td>-40.7</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.00</td>
<td>0.0</td>
<td>11.81</td>
<td>0.0</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>+0.83</td>
<td>+0.035</td>
<td>+4.84</td>
<td>12.54</td>
<td>+6.12</td>
<td>1.46</td>
</tr>
<tr>
<td></td>
<td>-0.65</td>
<td>-0.021</td>
<td>-3.08</td>
<td>12.70</td>
<td>-22.0</td>
<td>0.026</td>
</tr>
<tr>
<td>B</td>
<td>0.0</td>
<td>0.00</td>
<td>0.0</td>
<td>16.28</td>
<td>0.0</td>
<td>0.556</td>
</tr>
<tr>
<td></td>
<td>+1.12</td>
<td>+0.064</td>
<td>+5.30</td>
<td>18.48</td>
<td>+13.5</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>-1.0</td>
<td>-0.051</td>
<td>-3.81</td>
<td>22.33</td>
<td>-11.5</td>
<td>0.336</td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>0.00</td>
<td>0.0</td>
<td>25.24</td>
<td>0.0</td>
<td>0.979</td>
</tr>
<tr>
<td></td>
<td>+1.0</td>
<td>+0.051</td>
<td>+3.81</td>
<td>27.16</td>
<td>+7.62</td>
<td>1.64</td>
</tr>
</tbody>
</table>

results. For an inert rider (no “pop”) and ignoring air drag/lift, the trajectory of the center-of-mass of the jumper is the classic parabola:

$$y(x) = x \tan \theta_T - x^2 \frac{g}{2v_0^2 \cos^2 \theta_T}. \tag{7}$$

where the direction of the initial velocity for the assumed inert rider is the same angle as the takeoff.

To find the horizontal range of the jump, one must include the details of the geometry of the jump. For the standard tabletop shown in Fig. 1, there will be two cases to consider: (1) landing on the deck and (2) landing on the sloped landing surface (beyond the knuckle). The critical initial speed, $v_c$, required to just reach the end of the deck, i.e. land on the knuckle, is a special case given by

$$v_0 \rightarrow v_c = \frac{D}{\cos \theta_T} \sqrt{\frac{g}{2(h + D \tan \theta_T)}}. \tag{8}$$

Thus, for initial speeds greater than $v_c$, the jumper will land on the sloped landing surface and for initial speeds less than this, the jumper will land on the deck.

The equation for the landing surface is:

$$y_L(x) = \begin{cases} 
-\tan \theta_L(x - D) - h, & x \geq D \\
-h, & x < D.
\end{cases} \tag{9}$$

The horizontal range, $x_L$, is the intersection of this line with the trajectory, Eq. 7. After selecting the correct root of the resulting quadratic equation, one finds:

$$x_L = \frac{R_f}{2} \left\{ \begin{array} {l}
1 + \sqrt{1 + \frac{2g}{v_0^2 \sin^2 \theta_T}}, \quad v_0 < v_c \\
1 + \frac{\tan \theta_L}{\tan \theta_T} + \sqrt{\left(1 + \frac{\tan \theta_L}{\tan \theta_T}\right)^2 + \frac{2g(h - D \tan \theta_L)}{v_0^2 \sin^2 \theta_T}}, \quad v_0 \geq v_c
\end{array} \right\}, \tag{10}$$

where a convenient length scale is given by the flat-surface range.

$$R_f = \sin 2\theta_T \frac{v_0^2}{g}. \tag{11}$$

As described above, if the rider jumps or “pops” at takeoff, a component of velocity, $\vec{v}_p$, normal to the takeoff surface is added. As this affects only the initial velocity (direction and magnitude), the equation for the horizontal landing distance is the same as Eq. 10 but with the substitutions given in Eqs. 4-5.
TABLE IV: Values for the constants for the total distance traveled to be used in the SSSH equation (Eq. 13) for landing before the knuckle (i.e., on the deck), $v_0 < v_c$, and for landing beyond the knuckle, $v_0 \geq v_c$. The first value is taken from SSSH, Ref. [2], and compared with the value (in parentheses) using Eq. 12 with the jump geometrical parameters as given in Table II. Note the large discrepancy for the $b_0$ coefficient for jump B.

<table>
<thead>
<tr>
<th>Jump (km/h)</th>
<th>$v_c$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$ (m-h/km)</td>
<td>$a_2$ (h/km)</td>
<td>$a_3$</td>
<td>$a_4$ (h/km)$^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>28.5</td>
<td>0.025 (0.0255)</td>
<td>0.122 (0.122)</td>
<td>11.96 (11.77)</td>
<td>0.015 (0.0148)</td>
</tr>
<tr>
<td>B</td>
<td>40.5</td>
<td>0.026 (0.0264)</td>
<td>0.100 (0.0995)</td>
<td>38.871 (39.24)</td>
<td>0.010 (0.00991)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jump (km/h)</th>
<th>$v_c$</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landing beyond the knuckle</td>
<td>$b_0$ (m)</td>
<td>$b_1$ (m-h$^2$/km$^2$)</td>
<td>$b_2$</td>
<td>$b_3$</td>
<td>$b_4$ (km/h)$^2$</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>28.5</td>
<td>-0.686 (-0.631)</td>
<td>0.0071 (0.00701)</td>
<td>0.954 (0.954)</td>
<td>0.910 (0.910)</td>
<td>702.9 (706.5)</td>
</tr>
<tr>
<td>B</td>
<td>40.5</td>
<td>-0.875 (-1.903)</td>
<td>0.0073 (0.00792)</td>
<td>0.961 (0.961)</td>
<td>0.924 (0.924)</td>
<td>1492.7 (1488.3)</td>
</tr>
</tbody>
</table>

SSSH provide theoretical expressions for the total distance traveled which they define as the total distance along the deck and then, if the rider’s initial speed is greater than $v_c$, along the sloped landing surface as well. In terms of the horizontal distance given in Eq. 10 above, the SSSH defined total distance traveled, $D_t$, is

$$D_t = \begin{cases} 
\frac{R_f^2}{2} \left(1 + \sqrt{1 + \frac{3gh}{v_0^2 \sin^2 \theta_T}}\right), & v_0 < v_c \\
D \left(1 - \frac{1}{\cos \theta_L}\right) + \frac{R_f}{2 \cos \theta_L} \left(1 + \tan \theta_L \tan \theta_T + \sqrt{1 + \frac{\tan \theta_L \tan \theta_T}{\tan \theta_T}} \right)^2 + \frac{2g(h-D \tan \theta_L)}{v_0^2 \sin^2 \theta_T}, & v_0 \geq v_c.
\end{cases}$$

(12)

However, SSSH only provided numerical functions of the takeoff speed, $v_0$, in the form:

$$D_t = \begin{cases} 
a_1 v_0 \left(a_2 v_0 + \sqrt{a_3 + a_4 v_0^2}\right), & v_0 < v_c \\
b_0 + b_1 v_0^2 \left(b_2 + \sqrt{b_3 - \frac{b_4}{v_0^2}}\right), & v_0 \geq v_c,
\end{cases}$$

(13)

where $\{a_i, b_i\}$ are (dimensioned) constants that depend on the geometrical parameters of the jump and $g$, the acceleration of gravity. Table IV gives values for the constants as reported in SSSH for the A and B jump parameters of Table II compared with the values using Eq. 12. Most of the constants are reproduced correctly to three digits; however a few, and $b_0$ in particular, are significantly different. For the A jump the effect of this correction is to change the theoretically predicted total distance at the average takeoff speed ($31.7 ± 0.52$ km/h) slightly from the SSSH theoretical value of 9.4 m to 9.30 ± 0.50 m (compared to the SSSH measured value of 8.47 ± 0.25 m). For the B jump at its average takeoff speed ($41.3 ± 0.21$ km/h) the theoretical total distance changes from the SSSH theoretical value of 13.8 m to 14.14 ± 0.42 m (compared to the SSSH measured value of 14.4 ± 0.12 m). (The errors on my theoretical calculations are those propagated from the errors in the average takeoff speed given in Table 2 of SSSH.) The corrected values for the theoretical calculations of the total distance now lie closer to the experimental values. In the case of the B jump, they agree to within errors and in the case of the A jump the errors now almost overlap. However, before any useful conclusions can be drawn, a more careful analysis is needed that would examine film data to monitor the orientation of the jumper and include any additional shifts in the center-of-mass due to rotations of the jumper. This data is not available to the author.
FIG. 5: Example calculations of the “pop” effect for the three sample jumps. The “no-pop” speeds for each jump are the same as used in the earlier drag calculations. The “pop” speeds for the A and B jumps were chosen to recover the extremes of Fig. 5 in SSSH giving “pop” speeds of \{-2.48 \text{ m/s}, 0, +0.83 \text{ m/s}\} for the A jump and \{-0.65 \text{ m/s}, 0, +1.12 \text{ m/s}\} for the B jump. The “pop” speeds for the C jump were arbitrarily chosen to be: \{-1.0 \text{ m/s}, 0, +1.0 \text{ m/s}\}. (color online)

B. Equivalent Fall Height

One important parameter that quantifies the landing impact is the equivalent fall height, \( h \), which is defined as the vertical free fall distance that would result in the same impact on a horizontal surface as that experienced by a jumper landing on a sloped surface. Specifically, if \( v_\perp \) is the component of the jumper’s landing velocity normal to the landing surface, then

\[
h = \frac{v_\perp^2}{2g}. \tag{14}
\]

Including drag/lift, one would solve the general equations of motion as described in Section II and use the resulting \( v_\perp \) to calculate the EFH. Ignoring air drag/lift, by algebraically solving for the landing point, the component of the landing velocity normal to the surface for the tabletop jump geometry can be calculated analytically,

\[
v_\perp = v_0 \left\{ \sin \theta_T \sqrt{1 + \frac{2gh}{v_0^2 \sin^2 \theta_T}}, \quad v_0 < v_c \right. \]
\[
\left. - \sin(\theta_T + \theta_L) + \cos \theta_L \sin \theta_T \left( 1 + \frac{\tan \theta_L}{\tan \theta_T} \right) + \sqrt{\left( 1 + \frac{\tan \theta_L}{\tan \theta_T} \right)^2 + \frac{2g(H - D \tan \theta_L)}{v_0^2 \sin^2 \theta_T}} \right\} \quad v_0 \geq v_c. \tag{15}
\]

Combining Eqs 14 and 15 leads to expressions for the EFH for tabletop jumps essentially equivalent to those derived by Swedberg and Hubbard [20]. As described in Section II above, including “pop” requires only that the substitutions of Eq. 4 be used. Table III gives the EFH values for the three example jumps using the same parameters as used earlier. As one can see from Eq. 15, the EFH scales as the takeoff speed squared and is roughly proportional to the sine of the takeoff angle. Thus “pop” can influence the EFH though both these quantities.

The EFH measure can be used to understand two of the principal impact hazards posed by the basic table-top design arising from landing short of the knuckle or landing beyond the sloped landing surface. In either case the angle of the landing surface decreases dramatically from the design landing angle, \( \theta_L \), to zero, in the case of landing on the deck, or to the background slope angle, \( \theta_B < \theta_L \), in the case of landing too “deep”, i.e. beyond the sloped landing surface. The increase in EFH for landing just short of the knuckle relative to landing just beyond the knuckle on the landing surface can be dramatic. For the jump B example considered here, using the critical speed and an inert jumper assumption, the EFH for landing just beyond the knuckle is a mere 0.073 m while the EFH for landing just short of the knuckle is 2.83 m. SSSH provided no information regarding the length of the landing surfaces for the jumps they studied, but for the purposes of comparison suppose that the landing surface (as measured along the slope) is twice the deck length before transitioning to a flat, “bucket”, region. The EFH for landing just before the end of the sloped surface is 3.37 m compared to a disastrous 15.8 m for landing just beyond the end of the sloped surface, i.e. in the “bucket”. As discussed at length by Swedberg [20], these two cases constitute perhaps the greatest injury risk to table-top jumpers, even when they land on their feet. The first can be mitigated by alternative maneuver area surface designs, e.g. “turtle back” shapes, and the second can be mitigated by making the landing surface long enough to accommodate the maximum takeoff speeds likely to be experienced (including “pop”) as discussed in detail in Ref. [11].

The total energy absorbed by the jumper upon impact is related to the change in the jumper’s \( v \) velocity which includes a component tangent to the landing surface as well. Assuming that the jumper lands normally (upright) and does not significantly disrupt the landing surface, the tangential component of the change in velocity can be related to the normal component through the coefficient of kinetic friction. Consider the normal component of the impulse delivered by the ground to the jumper over the short time interval of the landing, \( \tau \):

\[
\Delta p_\perp = mv_\perp = \int_0^\tau (F_\perp(t) - mg \cos \theta_L)dt, \tag{16}
\]

where \( v_\perp = \sqrt{2gh} \) is the normal component of the landing velocity, \( F_\perp(t) \) is the time-dependent normal force of the ground on the jumper during the impact, and \( \tau \) is the time interval during which the normal force acts such that for
\( F(t > \tau) \rightarrow mg \cos \theta_L \). So after time, \( \tau \), following the landing, there is no further change in the normal component of the jumper’s momentum. Assuming no significant damage to the landing surface, the tangential component of the impulse will be related to the normal component by the coefficient of kinetic friction, \( \mu \):

\[
\Delta p_\parallel = \int_0^\tau \mu F_\perp(t) \, dt \approx \mu \Delta p_\perp,
\]

(17)

where, since \( F_\perp(t < \tau) \gg mg \), to a good approximation one can neglect the \( mg \cos \theta_L \) term in Eq. 16. Let \( \vec{p}_1 \) (\( \vec{p}_f \)) be the momentum of the jumper just before (after) landing, then the total change in energy, \( \Delta U \), is still proportional to \( h \):

\[
\Delta U = \frac{(\vec{p}_f^2 - \vec{p}_1^2)}{2m} = \frac{p_{f\parallel}^2 + p_{f\perp}^2 - (p_{\perp} - \Delta p_{\perp})^2 - (p_{\parallel} - \Delta p_{\parallel})^2}{2m}
\]

\[
\approx \frac{\Delta p_{\perp}^2 + 2\mu p_{\parallel}\Delta p_{\perp} - \mu^2 \Delta p_{\parallel}^2}{2m} = [1 - \mu^2 + 2\mu \cot(\theta_j - \theta_L)] \frac{\Delta p_{\perp}^2}{2m}
\]

\[
\approx [1 - \mu^2 + 2\mu \cot(\theta_j - \theta_L)] mgh,
\]

(18)

where \( \theta_j \) is the angle of the jumper’s trajectory with respect to the horizontal at impact, \( \Delta p_{\perp} = p_{\perp} \), and \( p_{\parallel} = \cot(\theta_j - \theta_L) p_{\perp} \). (Note that there is no singularity in \( \Delta U \) when \( \theta_j = \theta_L \) since \( h \) vanishes.) To get an upper bound on the absorbed energy, note that the cross term is proportional to the product \( p_{\parallel} \cdot \Delta p_{\perp} \) which is maximal when they are equal. Thus, the maximum energy change is given by:

\[
\Delta U \leq [2 - (1 - \mu)^2] mgh.
\]

(19)

This represents the maximum energy absorbed by the jumper assuming the landing surface absorbs none. Of course, in all realistic cases the surface will experience some distortion which will lower the amount of energy absorbed by the jumper. This effect is carried to the extreme with landing air bags that absorb most of the energy. For snowboard/ski surfaces \( 0.04 < \mu < 0.12 \) [26, 27]. Thus the effect of including the tangential component of the change in landing velocity on the absorbed energy is generally small and the absorbed energy is well described by the EFH.

To estimate a recommended maximum EFH, Minetti, et al. have studied the role of leg muscles as shock absorbers in falls and found that for athletic males the maximum fall height at which control can be maintained is \( 1.5 \) m [29], the same value as recommended by the US Terrain Park Council based on a study (unpublished) of leg compression in falls.

The above analysis establishes the EFH as a useful measure of the impact hazard presented by a design and therefore can be used as a parameter to characterize a jump feature or to optimize in a jump design process. While the standard tabletop design is not ideal from the point of view of minimizing the EFH, it is relatively easy to construct and has become the most common terrain park jump design. McNeil and McNeil [10] have proposed a parabolic shape which minimizes the EFH for a wide range of takeoff speeds, and Hubbard [8] has proposed an alternative landing shape that has a fixed EFH regardless of the takeoff speed. The so-called curved landing shape designs incorporate some of these nontraditional shapes and are tentative steps toward landing surfaces that reduce the EFH. Nevertheless, with the widespread use of the tabletop design it would be helpful to use the kinematic results derived above to inform a design decisions that minimize the EFH for the tabletop design. From the basic observation that for a linear landing surface the EFH grows with jump distance, the minimum EFH for tabletop (i.e. straight slope) landings is one which starts at zero at the knuckle. The optimization criterion is to match the slope of the jumpers trajectory to the slope of the landing at the knuckle. Typically, this means the takeoff angle will be less than the landing angle.

A possible design decision process employing this criterion might work as follows. The terrain park jump design problem is under-constrained so some of the design parameters must first be determined somewhat arbitrarily. In the construction of terrain park jumps one important practical design constraint is the maximum slope, about \( 30^\circ \), that a snow grooming machine can create in practice. Suppose one takes this to be the landing angle, \( \theta_L = 30^\circ \). The designers then decide on the deck length, \( D \), (assumed horizontal) and a takeoff lip height, \( H \), above the deck. Then using the Reinke condition that the slope of the jumper’s trajectory at the knuckle equals the landing angle [28], an optimal takeoff angle, \( \theta_T^* \), is determined. As this will be less than \( 30^\circ \), this takeoff will always be practical to construct. Having fixed three of the needed four tabletop jump parameters, \( \{\theta_L, H, D\} \), the optimal takeoff angle, \( \theta_T^* \), that meets the slope-matching condition is given by:

\[
\tan \theta_T^* = \tan \theta_L - \frac{2H}{D}.
\]

(20)

Applying this result to the three example jumps gives \( \theta_T^* = 15.1^\circ \) for the A jump, \( \theta_T^* = 14.2^\circ \) for the B jump, and \( \theta_T^* = 15.5^\circ \) for the C jump. Assuming a takeoff speed (without “pop”) that will result in landing 2 m beyond the
Extensions to arbitrary landing shapes is straight-forward [11].

FIG. 6: “Pop” speeds for the B jump extracted from SSSH, Fig.5b, along with a straight line fit (slope = -0.395 (m/s)/(km/h)). (color online)

V. SUMMARY AND CONCLUSIONS

Epidemiological studies of injuries at ski resorts have found that jumping poses a significantly greater risk of spine and head injuries to patrons [3, 5]. At ski resorts today jumping activities are largely focussed in terrain parks; so it is not surprising that Henrie, et al. recently found that the rate of spine and head injuries inside the terrain park was double that outside terrain parks [4]. Currently, there is no significant quantifiable engineering design involved in the construction of commercial terrain park jumps and, indeed, the position of the National Ski Areas Association is that such efforts would be futile due to snow and rider variability. Snow variability was not treated in this work, but it is noted that there are known physical bounds on snow properties that can usefully inform designers. Regarding the question of rider variability, two recent papers by Shealy and collaborators appear to support the NSAA’s position with field studies of terrain park jumpers. The first of these contained significant errors [7]. In the second the authors reported no statistically significant relationship between the takeoff speed and the distance traveled’ [2]. If this were true, then engineering design would indeed be impractical. However, I have shown that there are inconsistencies in at least some of their experimental data as well as errors in their theoretical analysis and that a Newtonian analysis using the right initial conditions can account for their data quite well. I also show that below the 10% level one can neglect drag/lift effects for jumps of less than 20 m which allows one to use simple analytic expressions for several quantities useful to terrain park designers, such as the total distance traveled, Eq. 10, and the equivalent fall height at impact, Eq. 14. The EFH measure is useful in quantifying the already well-known special hazards posed to tabletop jumpers of landing too short or too long where the EFH undergoes a discontinuity as the landing angle changes dramatically. Although the tabletop design is not to be endorsed or encouraged, as a design exercise the EFH measure was used to optimize the takeoff angle of a standard tabletop design, Eq. 20, which resulted in dramatically lower EFH values.

Aside from the takeoff speed, one of the most important sources of rider variability is that of “pop” whereby the rider jumps up (or drops down) just before takeoff thereby changing his initial velocity vector. The rider “pop” effect was included through a simple modification of the initial conditions, Eq. 4, and the effect of “pop” was illustrated for three example tabletop jumps, the first two of which were the same as those studied in SSSH. The SSSH data was reanalyzed using the full “pop” model (with drag/lift) to provide a range of realistic “pop” speeds which were found to vary from -2.48 m/s to +1.12 m/s. When the “pop” speed was plotted versus the tangential takeoff speed. Fig. 6, a strong negative correlation was found whereby riders with low tangential takeoff speed would apparently try to increase their distance with positive “pop” while riders with high tangential speed would try to decrease their jump distance with negative “pop”. The take-home messages are that while rider “pop” can influence the total distance traveled, the range of possible “pop” speeds is bounded, and that a simple Newtonian analysis can provide bounds on the range of trajectories, landing locations, landing velocities, and equivalent fall heights, all of which will be useful to winter terrain park designers.
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