The Metal Detector and Faraday's Law

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Electromagnetism has proven a notoriously difficult subject for beginning physics and engineering students. When simultaneously exercising their newly acquired calculus skills on abstract electromagnetism concepts, these students often lose the physics in the mathematical formalism. One way we have addressed this problem is by including engineering-style design and fabrication activities in the laboratory portion of the course. In addition to providing concrete examples of the electromagnetic concepts under study, the integration of mathematics and physics concepts in the design activities can significantly raise the level of student interest, increase the ownership of their learning, and ultimately improve learning and retention.

Design is a central goal of engineering education. In addition to functional literacy in the physical sciences, the Accrediting Board for Engineering and Technology (ABET) has identified design as an essential outcome for engineering education.[1] Physics departments that serve engineering programs have a primary responsibility in meeting the literacy goal, but by introducing design and fabrication activities in the introductory physics sequence, they can also serve the ABET design goal while improving physics learning. At our institution we have developed an introductory electromagnetism laboratory that incorporates basic design and fabrication activities. The pedagogic approaches used in the design of this course, including the integration with the lecture and laboratories, is given in Ref. 2. The essential point is

that new knowledge is constructed from prior knowledge in some socially relevant context.[3] We use the laboratories to connect physics lecture concepts to a laboratory experience in an engineering context. This paper describes one of these laboratories in which the students design and build one of the components of a metal detector while learning about Faraday's law.

Including design and fabrication activities is not new. J. Pine, J. King, P. Morrison, and Ph. Morrison created an introductory electronics course for engineers called "ZAP!" which includes these activities extensively.[4] Indeed, the metal detector laboratory described here is a modification of a ZAP! experiment on mutual inductance. This laboratory occurs in the tenth week of the semester following treatment of the Biot-Savart law and is concurrent with treatment of Faraday's law. Among the learning objectives for this laboratory are:

1. To illustrate and reinforce the concept of Faraday's law in an applied context,
2. To use the Biot-Savart and Faraday laws to design the coils used in the metal detector,
3. To learn about the magnetic properties of materials, and
4. To develop observation, interpretation, and evaluation skills.

Design is the creative application of science and mathematics to the solution of practical problems. True design is open ended and iterative, constrained

by physical, financial, and possibly, social and legal conditions. While a full design activity is not practical in a closed fixed-time laboratory, it is possible, even desirable, to incorporate some of the elements of design into the laboratory experiments. Elements of design include establishing objectives and criteria, concept synthesis, analysis, fabrication, testing, and evaluation. What follows is an example of how we have included some design elements in building a metal detector to illustrate Faraday's law.

Metal Detector Basics

An introductory description of how metal detectors work can be found online,[5] and a general description of metal detectors and their uses is given in C.L. Garrett's *Modern Metal Detectors*.[6] Commercial metal detectors use advanced signal processing and phase information to improve detection sensitivity, but the fundamental physics principle is Faraday's law. Our "bare-bones" metal detector also operates on the principle of Faraday's law, but uses just the amplitude of the response. It consists of two coplanar concentric circular coils of wire, an oscillator, and a voltmeter. The oscillator drives current in one coil (the field coil), which creates an oscillating magnetic field, which in turn induces an electromotive force (emf) in the other coil (the pick-up coil). The presence of a magnetic material (ferromagnetic or diamagnetic) in the vicinity of the pick-up coil alters the emf. As shown in Fig. 1, the location of the sample determines the response. In the case of ferromagnetic materials, such as iron, when the sample is located in the center of the pick-up coil, due to the ferromagnetic enhancement of the field, the magnetic flux in the pick-up coil increases, thereby increasing the induced emf. However, if the sample is located in the region between the pick-up coil and the field coil, the magnetic flux in the pick-up coil is diminished, thereby decreasing the emf. For nonferrous conductors (e.g., copper, aluminum, gold, and silver) eddy currents induced in the sample will give rise to a diamagnetic behavior, which will lead to a response opposite to that of the ferromagnetic sample.

According to Faraday's law, the emf induced in the pick-up coil is

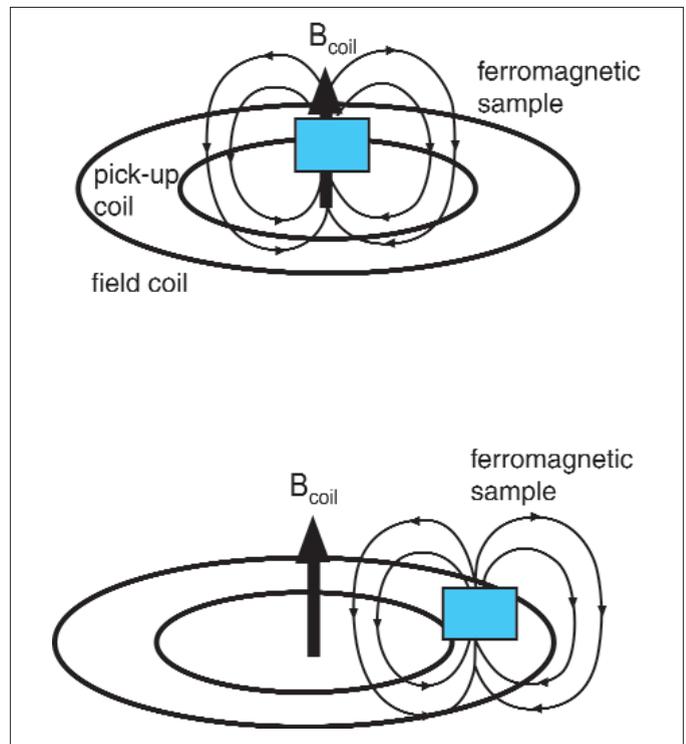$$\varepsilon = - N_p \, d\Phi/dt, \tag{1}$$

Fig. 1. Sketch of the effect of a ferromagnetic sample on the magnetic flux when placed in the vicinity of the pick-up coil.

where Φ is the magnetic flux in the pick-up coil due to the magnetic field produced by the field coil, and N_p is the number of turns in the pick-up coil. For our case, the radius of the field coil is about twice that of the pick-up coil. In this instance, for design purposes, the magnetic field inside the smaller pick-up coil may be treated as uniform and equal to the magnetic field at its center (the accuracy of this approximation is discussed later).

Suppose $I(t)$ is the current in the field coil. The Biot-Savart law gives the magnitude of the magnetic field at the center as

$$B = N_f \, \mu_0 \, I(t)/(2R_f), \tag{2}$$

where μ_0 ($4\pi \times 10^{-7}$ T m/A) is the vacuum permeability constant, N_f is the number of turns in the field coil, and R_f is the radius of the field coil.[7] Approximating the magnetic field to be constant throughout the interior of the smaller pick-up coil, the magnetic flux in the pick-up coil is

$$\Phi = (\pi R_p{}^2) \, N_f \, \mu_0 \, I(t)/(2R_f). \tag{3}$$

The exact flux can be written in terms of the complete elliptic integrals, but this is beyond the capabilities of the beginning engineer.[8] For the geometry treated here the approximate expression underestimates the exact flux by about 10%. Greater accuracy can be obtained by using a power series expansion in the ratio of the radii,[8] but such refinements are unnecessary for our purposes.

Since the current in the field coil is the only time-dependent quantity, the emf induced in the pick-up coil is

$$\mathcal{E} = \mu_0 \ N_p N_f \ \pi \ R_p^2/(2R_f) \ dI(t)/dt. \tag{4}$$

With this background the students are ready to design one of the components of their metal detectors.

The Design Activity

Due to time limitations, the design task for any fixed-time laboratory must be highly constrained. Here, the size of the field and pick-up coils, the frequency and amplitude of the current available from the function generator, and the desired output voltage of the pick-up coil are all constrained to given values. The students are required to use the Biot-Savart and Faraday laws along with basic calculus concepts to estimate the number of turns needed in each coil for the metal detector to produce the desired output while using the least amount of magnet wire. Using coil-winding forms provided, they then fabricate the coils and construct the metal detector.

Here is our example set of constraints. The function generator provides a sinusoidal current in the field coil with an amplitude of about 500 mA at a frequency of about 5 kHz. The geometry of the coils is fixed by two PVC tubes with radii of 2 in (~5 cm) and 1 in (~2.5 cm), which are used as forms for winding the coils. A digital multimeter reads the output of the pick-up coil. Most digital multimeters are not accurate for ac signals above about 100 Hz; so, while they are acceptable for detecting relative responses, common voltmeters should not be used for quantitative measurements. For that purpose we recommend using an oscilloscope.

To have a reasonable range of response, the pick-up coil should have an output amplitude of around 250 mV. Of course, these values are arbitrary and equipment dependent. The design task given the students is to estimate the number of turns needed for each of the coils to meet the desired output emf subject to the above constraints using the smallest total length of magnet wire.

Here is an example analysis. For sinusoidal field coil current, the amplitude of the emf induced in the pick-up coil is also sinusoidal with an amplitude obtained from Eq. (4):

$$\mathcal{E}_0 = \mu_0 \ N_p N_f \ \pi \ R_p^2 \ I_0 \ \omega/(2R_f), \tag{5}$$

where ω is the (angular) frequency and I_0 is the current amplitude provided by the function generator. For our example constraints, the parameters have the following values:

$$R_p = 0.025 \text{ m}$$
$$R_f = 0.05 \text{ m}$$
$$I_0 = 0.5 \text{ A}$$
$$\omega = (2\pi) \ 5000 \text{ rad/s}$$
$$\mathcal{E}_0 = 0.25 \text{ V} \tag{6}$$

Thus, the product of the number of turns in each coil is determined:

$$N_p N_f = 2 \ R_f E_0/(\mu_0 \ \pi \ R_p^2 \ I_0 \ \omega) \sim 645. \tag{7}$$

The optimization problem is to use the least amount of wire while meeting this constraint, a classic calculus problem. The total length of wire in both coils is

$$L = N_f (2\pi R_f) + N_p (2\pi R_p)$$
$$= 2\pi (R_f N_f + R_p \ 645/N_f), \tag{8}$$

where Eq. (7) has been used to eliminate N_p. The minimum length is found by taking the derivative of this expression with respect to N_f and setting the result equal to 0. The result of this calculation yields

$$N_f = (645 \ R_p/R_f)^{1/2} \sim 18 \text{ turns, and}$$
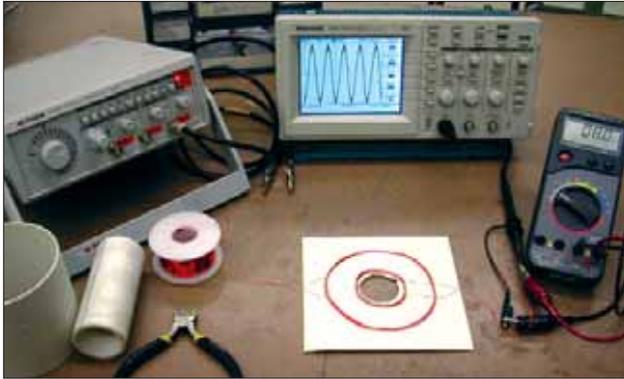
$$N_p = 645/N_f \sim 36 \text{ turns.} \tag{9}$$

Fig. 2. Setup for the "The Metal Detector" laboratory. The students design, construct, install, and test the field and pick-up coils (center). The equipment shown (from left to right) consists of the function generator, oscilloscope, amplifier, and digital multimeter. The PVC forms for winding the coils are also shown.

Fig. 3. Students using their metal detector to locate hidden metallic samples.

The students are required to perform this analysis prior to coming to the laboratory. During lab, they fabricate the metal detector coils and assemble the detector. They wind the two coils on the PVC forms provided, remove them from the forms, and secure them with tape to a piece of cardboard. Next they connect the field coil to the function generator and the pick-up coil to the digital multimeter and an oscilloscope. (In our implementation we also include a simple amplifier, but this component is not central to the Faraday's law lesson and can be omitted.) Figure 2 shows the experimental setup.

When set to ac volts (rms), one can use the digital multimeter to determine *qualitatively* the presence of magnetically active metals. However, since most common digital multimeters have limited frequency response (for example, our Metex ME-11 multimeters are not accurate at frequencies above 100 Hz), a different approach is necessary if an accurate quantitative measurement is needed. In one example experiment with the apparatus constructed as described above, we obtained the following results using an oscilloscope. With a field coil current amplitude of 485 mA at a frequency of 4,600 Hz, we measured an emf amplitude in the pick-up coil of 245 mV. Under these conditions, Eq. (5) predicts 227 mV, which underestimates the experimental value by about 7%, close to the expected underestimate of 10% arising from the constant field approximation. Under the same conditions, the Metex ME-11 digital multimeter reads only 18.2 mV (rms). Placing a copper (diamagnetic) sam-

ple (2 cm x 2 cm x 0.7 cm) in the pick-up coil causes the meter reading to drop to 16.7 mV (-9%). A rolled-steel sample (2 cm x 2 cm x 1.3 cm), the other hand, causes the meter reading to increase to 19.6 mV (+ 8%). Thus, while the meter's quantitative results are unreliable, the meter is sufficient to determine the presence of these metal samples.

After constructing their metal detectors, the students then test them with a variety of ferromagnetic and diamagnetic samples. They study the response of their detector to both ferromagnetic and diamagnetic samples in preparation for their laboratory quiz that will test their powers of observation and interpretation. The quiz consists of using their metal detector to locate metal samples hidden between two pieces of cardboard with a location grid drawn on it. Figure 3 shows one student group trying to locate the metal samples hidden in the cardboard holder. The students generally enjoy this activity, respond well to the challenge of finding the hidden samples, and seem to retain the knowledge of Faraday's law better by having connected it to something concrete and relevant. Three years after taking the introductory electromagnetism course with this laboratory, one mechanical engineering student reported being asked in a job interview to describe something technical. Of all the material this young mechanical engineer had learned in four years, she chose to describe her metal detector. (She was offered the job!)

Summary

In an attempt to improve learning of electromag-

netic concepts by engineering students, we have created an introductory electromagnetism laboratory that incorporates elements of engineering design. This paper describes one of these laboratories in which the concept of Faraday's law is illustrated and reinforced through the design and fabrication of one of the components of a metal detector. Anecdotal evidence suggests that engineering students retain a better understanding when the concepts are connected to something both concretely experienced and relevant to the practice of engineering.

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5. http://www.howstuffworks/metal-detector.htm.

6. Charles L. Garrett, *Modern Metal Detectors* (Ram Publishing Co., Dallas, TX, 2002).

7. See for example Eq. (29-5) in Paul. A. Tipler, *Physics for Scientists and Engineers*, 4th ed. (W.H. Freeman, 1999).

8. Problem 6.4 in J.D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1974). Note however that

the argument of the elliptic integrals should be k^2, not k. While a treatment of the exact flux integral is a graduate-level problem, it is relatively simple to execute an expansion in the ratio of the radii, R_p/R_f, which gives
$$\Phi = \Phi_0 \left(1 + 3/8 \, (R_p/R_f)^2 + 15/64 \, (R_p/R_f)^4 + ...\right),$$
where Φ_0 is the flux in the constant magnetic field approximation. For $R_p/R_f = 1/2$, including these terms gives the correct result to within 0.3%.

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