Elliptical craters and basins on the terrestrial planets

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Abstract. The four largest well-preserved impact basins in the solar system, Borealis, Hellas and Utopia on Mars, and South Pole-Aitken on the Moon, are all significantly elongated, with aspect ratios >1.2. This population stands in contrast to experimental studies of impact cratering that predict <1% of craters should be elliptical, and the observation that ~5% of the small crater population on the terrestrial planets is elliptical. We develop a simple geometric model to represent elliptical crater formation, and apply it to understanding the observed population of elliptical craters and basins. A projectile impacting the surface at an oblique angle leaves an elongated impact footprint. We assume that the crater expands equally in all directions from the scaled footprint until it reaches the mean diameter predicted by scaling relationships, allowing an estimate of the aspect ratio of the final crater. For projectiles that are large relative to the size of the target planet, the curvature of the planetary surface increases the elongation of the projectile footprint for even moderate impact angles, thus increasing the likelihood of elliptical basin formation. The results suggest that Hellas, Utopia, and South Pole-Aitken were formed by impacts inclined at angles less than ~45° from horizontal, with a probability of occurrence of ~0.5. For the Borealis basin on Mars, the projectile would likely have been decapitated, with the topmost portion of the projectile on a trajectory that does not intersect with the surface of the planet.

INTRODUCTION

While the vast majority of impact craters are roughly circular in planform, a small fraction of craters produced both experimentally and observed on planetary surfaces have significantly elongated shapes. Experimental work suggests that these elliptical craters form in
only the most oblique impacts, with a critical impact angle for elliptical crater formation ($\theta_c$, representing the angle between the projectile trajectory and the horizontal) of 4.7° (Gault and Wedekind 1978). For a population of projectiles with random trajectories, the probability of an impact occurring at an angle of less than a given angle, $\theta$, is $\sin^2(\theta)$ (Gilbert 1893), irrespective of the gravity of the target planet (Shoemaker 1962). Thus, the experimentally determined threshold angle for elliptical crater formation would suggest that ~0.7% of craters should be elliptical.

However, surveys of the observed population of elliptical craters on Mars, Venus, and the Moon found that roughly 5% of the population of small (<150 km diameter) craters are elliptical (Schultz and Lutz-Garihan 1982; Barlow 1988; Bottke et al. 2000). Schultz and Lutz-Garihan (1982) proposed that for Mars this paradox could be resolved if the planet possessed a population of small moons that spiraled inwards to strike the surface at low angles. However, Bottke et al. (2000) found that such a scenario is not necessary, using a simple conceptual model of elliptical crater formation to reconcile the frequency of elliptical craters produced experimentally with that observed on the terrestrial planets. They assumed that the tendency of an impact to produce an elliptical crater is related to the ratio between the final crater diameter ($D_c$) and the projectile diameter ($d_p$). They consider results from two different experimental studies, investigating impacts into sand (Gault and Wedekind 1978) and aluminum (Christiansen et al. 1993), with different $D_c/d_p$ ratios and $\theta_c$ values. Bottke et al. (2000) used the results from these two studies to generalize an empirical power-law relationship between $\theta_c$ and $D_c/d_p$:

$$\theta_c = \theta_0 (D_c/d_p)^m.$$  \[1\]
where $\theta_0 = 68.1^\circ$ and $m = -0.648$. Utilizing the $D_c/d_p$ ratio indicated by $\pi$-scaling relationships (Holsapple and Schmidt 1982; Melosh 1989), this power law successfully explained the 5% abundance of impact craters on the terrestrial planets, and suggested that $\theta_c$ will also depend on projectile diameter.

At the other end of the size spectrum, the small population of the largest impact basins reveals a further discrepancy between the expected and observed population of elliptical impact craters. We here show that four of the six largest well-preserved impact basins in the solar system (Borealis, Hellas and Utopia on Mars, and South Pole-Aitken on the Moon) all exhibit pronounced elliptical shapes. Of the basins we classify as “giant impact basins”, with a basin diameter greater than half the planetary radius, only Caloris on Mercury and Imbrium on the Moon fall short of our assumed criterion for classification as an elliptical basin. Adopting the 5% abundance of elliptical craters observed in the small crater population, the probability that four of the six giant basins would be elliptical by random chance is $8.5 \times 10^{-5}$, and thus can be effectively ruled out.

We suggest that this paradox can be resolved by considering the effect of the curvature of the planetary surface on the resulting basin shape. For the largest impacts, the surface of the planet curves away from the projectile path, leading to more elongated projectile footprints (the projection of the projectile onto the surface of the planet). This increased elongation of the projectile footprint for a given impact angle increases the probability of elliptical basin formation. We develop a simple geometrical model for elliptical crater formation, based on the calculated projectile footprint aspect ratio and the final crater diameter from $\pi$-scaling relationships. The model can explain both the critical angle in small-scale laboratory experiments and the observed fraction of elliptical craters in the 1-150 km diameter range, and
further offers the expectation that a Hellas-sized impact basin would have a probability of being elliptical of ~0.4.

**OBSERVED ELLIPTICAL BASINS ON MARS AND THE MOON**

**Known impact basins**

While several studies have considered the statistics of small crater shapes (Schultz and Lutz-Garihan 1982; Bottke et al. 2000), there has been less attention paid to the shapes of the giant impact basins. Since we are interested in the effect of the curvature of the surface on the resulting basin shape, we impose an arbitrary cutoff on what is considered a “giant” impact basin, considering here those basins whose diameters are >50% of the planetary radius.

In order to compare the observed basin dimensions with the predictions of scaling models, we focus on the diameter of the primary topographic rim of the basin \((D_c)\), which we equate with the post-modification excavation cavity. Since the non-mascon basins are largely isostatic today, the topographic rim should also correspond with the basin rim in the lower resolution crustal thickness data that was used as a proxy for the excavation cavity by Wieczorek and Phillips (1999). We describe this as the primary topographic basin since the observed topography and crustal thickness will reflect the post-impact modification of the transient excavation cavity. The exterior and interior rings associated with many basins are generally characterized by a weaker expression in both topography and crustal thickness models, and are likely a product of the multi-ring basin modification process (Wieczorek and Phillips 1999).
The primary topographic basin rim is traced just above the steep slope leading down into the main basin cavity, making use of both color topography (Figure 1) and contour maps (not shown). The best-fit ellipse matching the basin rim is calculated by iterating the ellipse orientation, and major and minor axes in order to minimize the RMS misfit. Basin dimensions are here reported to 10 km precision, though it should be recognized that the uncertainty in identifying the main topographic rim exceeds this in some cases. These basin dimensions differ from previously reported measurements in some cases. However, for the purpose of this study, it is preferable to maintain a consistent methodology throughout, rather than to adopt the measurements of previous studies that have employed different criteria in basin rim identification.

The Hellas basin on Mars is perhaps the best preserved of the giant impact basins, and has been recognized as an elliptical structure produced by an oblique impact (Tanaka and Leonard 1995). We reproject the MOLA topography (Smith et al. 2001) of Hellas in a basin-centered polar coordinate system (Figure 1a), preserving both the distance and angle between all points and the basin center. The dimensions of the primary topographic rim of the basin are found to be 2280 km by 1590 km (Table 1), with the ratio between the major and minor axes \((A/B)\) of 1.43. The ratio of the mean basin diameter to the planetary radius \((D_c/r_p)\) is 0.57.

The shape of the Utopia basin on Mars (McGill 1989) is not as immediately apparent. While Utopia likely started out as an isostatically compensated basin with a depth comparable to that of Hellas, it was subsequently filled with volcanic, sedimentary, and aeolian material resulting in a pronounced positive gravity anomaly (Smith et al. 1999; Searls et al. 2006). Rather than examining the considerably muted topographic signature (Figure 1b) or the distribution of tectonic features within and surrounding the basin, we focus on the crustal structure in order to
estimate the original basin dimensions. We use a spherical harmonic membrane-flexural model (Banerdt 1986; Banerdt and Golombek 2000) to invert the gravity (Konopliv 2008) and topography (Smith et al. 2001) of Mars in order to isolate the isostatic crustal roots representing the pre-fill basin topography (Andrews-Hanna et al. 2008). In short, this method divides the crust into surface loads and isostatic roots that, together with the resulting membrane and flexural displacement, reproduce the observed gravity and topography. Applying this technique to Mars, and plotting the pre-fill topography in a Utopia-centered polar projection (Figure 1c), we find the dimensions of the primary pre-fill topographic basin to be 2400 km by 2000 km (accounting for the lower resolution of the gravity data), corresponding to an aspect ratio of 1.2, and $D_c/r_p$ of 0.65.

The observed basin size in the isostatic roots is substantially smaller than the 3200-km diameter circular basin found by Thomson and Head (2001) and the 3380-km diameter basin found by Frey (2008a). This discrepancy may be explained by noting the apparent ring structure that surrounds Utopia in the isostatic root map, leading to a surface expression in the topography and tectonic features that was interpreted as the basin rim in those studies. The relief in the isostatic roots across the basin rim identified here is a factor of 3-5 times greater than that across the outer ring structure. Even in well-preserved lunar basins, discrimination between the primary basin cavity and surrounding ring structures requires careful consideration of the topography and/or crustal thickness (Wieczorek and Phillips 1999). For the case of Utopia, the topographic signature is highly muted and the crustal thickness models also include the voluminous fill within and surrounding the basin (Searls et al. 2006). The gravity-topography inversion utilized here distinguishes between the pre-fill isostatically-compensated basin and the later flexurally supported fill, thereby providing a more accurate characterization of the original basin. We find
similar basin dimensions utilizing crustal thickness models (Neumann et al. 2008), though the relief across the primary basin rim is diminished relative to that across the outer ring.

The Borealis basin on Mars, encompassing the northern lowlands, is the largest proposed impact structure in the solar system. Wilhelms and Squyres (1984) first suggested that the northern lowlands of Mars may be the expression of a single giant impact basin. Subsequent studies often dismissed the possibility of an impact origin for the Martian crustal dichotomy, primarily due to the fact that the present-day lowlands cannot be fit with a single circular impact basin (McGill and Squyres 1991), with the proposed circular basin leaving ~40% of the lowlands unaccounted for. However, Andrews-Hanna et al. (2008) used an inversion of the gravity and topography to trace the dichotomy boundary beneath Tharsis, and found that the resulting globally-continuous dichotomy boundary is accurately fit by an ellipse measuring 10,600 km by 8,500 km. This elliptical basin is characterized by a ratio between the major and minor axes \(A/B\) of 1.25, and a ratio of the mean basin diameter to the planetary radius \(D_{c}/r_{p}\) of 2.81. Projections of the Borealis basin in a basin-centered polar coordinate system (Figure 1d,e) bear a striking resemblance to the much smaller Hellas, Utopia, and South Pole-Aitken basins (Figure 1a,c,f), though it is a factor of four greater in size. The elliptical shape and bimodal crustal thickness distribution of this Borealis basin strongly support an origin through an oblique impact. An impact origin for the dichotomy is further supported by simulations demonstrating the feasibility of such a giant impact (Marinova et al. 2008; Nimmo et al. 2008), and the tentative interpretation of Arabia Terra as a partial multi-ring structure around the basin (Andrews-Hanna et al. 2008).

The largest recognized impact basin on the Earth’s Moon is the South Pole-Aitken basin, which also exhibits an elliptical shape (Garrick-Bethell 2004) in both topography and elemental
abundance data (Smith et al. 1997; Lawrence et al. 2002; Lawrence et al. 2003). Plotting the
topography (Smith et al., 2009a, 2009b; Zuber et al., 2009) in a basin-centered polar coordinate
system (Figure 1f), we find the primary topographic basin dimensions to be 2330 km by 1780
km, corresponding to an aspect ratio of 1.31 and $D_c/r_p$ of 1.18. This basin aspect ratio is similar
to the main topographic rim found by Garrick-Bethel (2004), though the basin dimensions are
slightly larger due to the application of different criteria in identifying the rim. While slightly
smaller than Hellas and Utopia in absolute size, South Pole-Aitken is substantially larger relative
to the planetary radius. The Imbrium basin on the Moon also falls into the category of a giant
impact basin. The primary basin diameter in crustal thickness models has been estimated as 895
km (Hikida and Wieczorek 2007). In order to maintain a consistent methodology, we re-analyze
the basin in Lunar Orbiter Laser Altimeter (LOLA) topography (Smith et al., 2009a, 2009b;
Zuber et al., 2009) and find the diameter of the main topographic rim to be 1120 km ($D_c/r_p =
0.64$; Fig 1g), consistent with the results of Spudis (1993). This discrepancy is a result of use of
different conventions and datasets in identifying the rim. The basin is nearly circular, with $A/B =
1.05$.

On Mercury, only the Caloris basin, with an estimated diameter of 1550 km ($D_c/r_p =
0.64$) fits the adopted criterion of a giant basin (Murchie et al. 2008). The first full-basin images
obtained by MESSENGER revealed an approximately circular basin (Murchie et al. 2008).
Recent mapping of the basin-related units and sculpture has suggested a slightly elliptical basin
with dimensions of 1525 km by 1315 km and an aspect ratio of 1.16 (Fassett 2009), though this
still falls short of our minimum criterion for consideration as an elliptical basin. While Caloris
and Imbrium are the smallest of the giant basins considered here, their sizes relative to the
planetary radius ($D_c/r_p \sim 0.6$) are similar to Hellas and Utopia on Mars. Of the six basins we
classify as giant impact basins, only Caloris and Imbrium fall short of the criterion we adopt for elliptical basins of an aspect ratio of 1.2. However, even these two basins exhibit significant departures from circularity. At the scale of the largest impact basins, a circular outline appears to be the exception rather than the rule.

Decreasing the requirement for consideration as a “giant” impact basin to a simple diameter cutoff of 500 km would include the Isidis ($D_c = 1500$ km; $A/B < 1.2$) and Argyre (780 km; $A/B = 870$ km/690 km = 1.26) basins on Mars (Figures 1h,i). Several additional basins with diameters greater than 500 km have been identified on Mars (Barlow crater database, available online at http://webgis.wr.usgs.gov/). However, we find that the primary topographic basins exhibit diameters less than 500 km in MOLA topography, or have experienced significant erosion and relaxation (Mohit and Phillips 2007) which prevents an accurate determination of the basin size and shape. Nevertheless, these basins are all approximately circular in outline. Many possible buried impact basins in various states of degradation have been identified on Mars using MOLA topography and crustal thickness models (Frey et al. 2002; Frey 2006; Frey 2008a), but we do not include these in the present discussion due to the difficulties in identifying the primary topographic basin rim and shape of these poorly expressed basins. These “quasi-circular depressions” are characterized by extremely shallow depth and/or weak expression in crustal thickness models, suggesting that they are either in an extreme state of relaxation or are substantially infilled. As a result, unambiguous identification of the primary basin rim, as opposed to a larger outer ring structure, is not possible. Furthermore, inclusion of quasi-circularity in the criteria for basin identification might bias the results towards circular rather than elliptical structures. We have limited this study to those basins with either a clear and well-
preserved topographic expression or, in the case of Utopia, a clear gravitational expression that enables us to reconstruct the pre-fill topography.

A large number of confirmed and possible impact basins greater than 500 km in diameter have been identified on the Moon (Wilhelms 1987; Frey 2008b), though the partial preservation of many of the basins often makes determination of the basin size and shape difficult. We again limit ourselves to the well-preserved basins whose primary topographic rim corresponding to the modified excavation cavity can be identified. We analyzed all “definite” lunar basins (Wilhelms 1987) larger than 500 km in diameter using LOLA topography (Smith et al., 2009a, 2009b; Zuber et al., 2009). We exclude basins whose topographic signature is degraded such that the basin rim cannot be clearly identified around the majority of the circumference (e.g., Fecunditatis, Tranquilitatis). For basins with a pronounced multi-ring structure, the innermost significant topographic ring was chosen as the main basin rim, as suggested for Orientale (Wieczorek and Phillips 1999; Hikida and Wieczorek 2007). Thus, some basins previously identified as being greater than 500 km in diameter were found to be substantially smaller, and were excluded (e.g., Hertzprung). We also find a slightly smaller diameter of ~450 km in the topography for the basin Smythii than found in crustal thickness models (Hikida and Wieczorek 2007). While the basin diameter from the crustal thickness model may be more accurate, we exclude this basin for the sake of consistency. The remaining basins greater than 500 km in diameter include the Nubium \( (D_e = 860 \text{ km}, A/B = 900 \text{ km}/810 \text{ km} = 1.11) \), Serenitatis \( (D_e = 690 \text{ km}, A/B = 710 \text{ km}/670 \text{ km} = 1.06) \), and Crisium \( (D_e = 510 \text{ km}, A/B = 600 \text{ km}/420 \text{ km} = 1.43) \) basins (Figures 1j-l). Of these, only Crisium is significantly elongated. There is significant uncertainty in these basin diameters and dimensions due to the difficulty in assigning the primary
topographic rim. Venus possesses no confirmed basins that can be considered giant impact basins (Phillips et al. 1991).

Unconfirmed possible giant impact basins

While Borealis, Hellas, Utopia, South Pole-Aitken, Imbrium, and Caloris are the largest clearly expressed impact basins in the Solar System, several other mega-basins have been proposed. It has been suggested that a single impact basin can be circumscribed around the majority of the nearside maria, termed either the “Gargantuan” (Cadogan 1974) or “Procellarum” (Whitaker 1980) basin. An impact origin for this region is highly uncertain, and the location of the rim of the proposed basin is poorly defined and cannot be fit by an ellipse with a reasonable degree of accuracy. Much of the basin “rim” can be ascribed to other impact features, no clear rim can be identified around much (~40%) of the basin, and there exist large areas of unexplained high topography within the putative basin.

A much larger basin has been suggested to explain the lunar crustal and topographic asymmetry, with the basin cavity explaining the thinner crust and lower topography of the lunar nearside (Byrne 2007). However, unlike the Martian crustal dichotomy, the lunar crustal distribution is best described as an asymmetry and is accurately represented by a degree 1 variation in the crustal thickness. The resulting unimodal crustal thickness histogram argues against an impact origin (e.g., Andrews-Hanna et al. 2008), unless perhaps the basin has reached an advanced stage of horizontal relaxation. There exist several alternative and more likely explanations for the lunar asymmetry, including thickening of the farside crust by ejecta from the South Pole-Aitken basin (Zuber et al. 1994), degree-1 mantle processes (Zhong et al. 2000), or a
degree-1 Rayleigh-Taylor instability developing in an over-dense ilmenite-rich cumulate layer in the upper mantle (Parmentier et al. 2002).

At this point, there is no compelling evidence in favor of either a Procellarum or nearside-encompassing basin on the Moon. However, the extensive geophysical data analysis required to isolate the Borealis basin on Mars dictates that such large structures that date from earliest planetary evolution can be difficult to identify. That said, the existence of Borealis demonstrates the possibility of hemisphere-scale mega-basins. The excellent preservation of the Borealis rim suggests a relatively late-stage impact (Andrews-Hanna and Zuber 2008). Earlier mega-impacts on all of the terrestrial planets during the late stages of accretion were likely, though such basins may be poorly preserved, if preserved at all, and difficult to recognize. While future work may confirm or deny the possible existence of these and other mega-basins, we do not include them in this study.

**Statistical implications**

The assumption that small crater statistics apply to these basins, with a 0.05 probability of a particular crater or basin having an elliptical shape, leads to a likelihood that four of the six confirmed giant impact basins would be elliptical by random chance of only $8.5 \times 10^{-5}$. Expanding our definition of a giant impact basin to include all eleven well-characterized basins greater than 500 km diameter (six of which are elliptical) decreases the probability of finding six elliptical basins to $5.6 \times 10^{-6}$.

Clearly, the probability of elliptical crater formation observed for small craters cannot hold for the largest basins. If we assume that the probability of elliptical crater formation is
independent of crater diameter (which we will later show to be false), we can calculate the probability of elliptical crater formation and the critical angle from the observed basins. The four out of six elliptical giant basins leads to a probability of elliptical basin formation of 0.67, corresponding to a critical angle of 54°. For the eleven well-characterized impact basins greater than 500 km in diameter, the six elliptical basins suggest that the probability that a given basin will be elliptical is ~0.55, corresponding to a critical angle of 48°. In the next section we develop a simple geometrical model for the critical angle of elliptical crater and basin formation, which will then be compared to the observed population of elliptical craters and basins.

MODEL

The basis of our conceptual model is similar to that of Bottke et al. (2000). Oblique impacts onto the surface of a planet result in an elongated projectile footprint, with an aspect ratio of \( a/b = 1/\sin(\theta) \) for the case of a projectile that is small relative to the size of the planet. The excavation cavity then expands outwards from this projectile footprint, and the final post-modification crater aspect ratio \( (A/B) \) will be a function of both the aspect ratio of the projectile footprint and the ratio between the final crater diameter and the projectile diameter \( (D_c/d_p) \). However, while Bottke et al. (2000) used this conceptual model to constrain a semi-empirical power-law relationship between \( \theta_c \) and \( D_c/d_p \) based on experimental results, we use this geometric approach to estimate the crater aspect ratio as a function of impact angle and size for both the experimental results and the observed crater and basin populations on the terrestrial planets. This model also takes into account the effect of the curvature of the planet’s surface on the resulting crater shape, allowing its application to the formation of giant impact basins.
For the case of a curved planetary surface, the projectile footprint aspect ratio depends on both the impact angle and the sizes of the projectile and target planet. We calculate this aspect ratio geometrically, as a function of the impact angle at the point of first contact between the projectile and target (defining the impact angle as such preserves the $\sin^2\theta$ dependence of the probability). It can be shown that the $x$-coordinate of the point of intersection of the top/bottom of the projectile (defined as the two points on the projectile surface tangential with the direction of motion and aligned in the same vertical plane as the point of first impact) with the surface of the planet is given by:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = \tan^2 \theta + 1$$

$$b = \mp 2r \sin \theta \tan^2 \theta - 2 \tan \theta (R + r \pm r \cos \theta)$$

$$c = r^2 \sin^2 \theta \tan^2 \theta \pm 2r \sin \theta \tan \theta (R + r \pm r \cos \theta) + (R + r \pm r \cos \theta)^2 - R^2$$

where $r$ is the radius of the projectile, $R$ is the planetary radius, and the top sign in each sign pair applies to the projectile top. From this, the projectile footprint dimensions can be calculated as:

$$a = R \cdot \left[ \arcsin \left( \frac{x}{R} \right) + \arcsin \left( \frac{x}{R} \right) \right]$$

$$b = 2R \cdot \arcsin \left( \frac{r}{R} \right)$$

For the special case of $R = \infty$, the aspect ratio from [2] and [3] reduces to $1/\sin(\theta)$, as predicted for a flat planet.
The projection of a spherical projectile onto the surface of a planet results in a highly centralized mass/energy distribution within the projectile footprint, with the projectile mass decreasing to zero at the edges (Figure 2). We scale the projectile footprint down by a factor $\gamma$ to account for this non-uniform mass distribution:

\[
(a', b') = \gamma \cdot (a, b)
\]

\[
\gamma = \frac{\int_0^R \rho(r) r \, dr}{\int_0^R \rho(r) \, dr} = \frac{\int_0^R \left( R^2 - r^2 \right)^{1/2} r \, dr}{\int_0^R \left( R^2 - r^2 \right)^{1/2} \, dr}
\]

\[
= \frac{4}{3\pi}
\]

where $\rho(r)$ is the surface mass density within a circular projectile footprint as a function of the radial distance $r$ from the center of the footprint. While this scaling is an ad hoc correction, some justification is garnered from the success of the model in reproducing both the experimental results and the observed fraction of elliptical craters, as will be seen in the next section. This scaling factor is formulated such that if all of the mass of the projectile were concentrated as a delta function at $r=R$, $\gamma$ would equal 1. Using a total mass weighting rather than density weighting (replacing $\rho(r)$ with $2\pi r \rho(r)$ in [4]), overestimates the final crater ellipticity for the small crater population. Alternatively, we can numerically integrate the energy radiating outwards from each point in the footprint and assume that the final crater rim is an iso-energy surface, though this underestimates the final crater aspect ratio.

We assume that the scaled projectile footprint expands uniformly in all directions by a distance $\Delta$ in order to reach the mean crater diameter predicted by the scaling relationships
Most studies adopt the $\pi$-scaling relationships, in which the final mean crater diameter ($D_c$) is calculated (Melosh 1989; Bottke et al. 2000) as:

$$D_c = 1.25 \cdot C_D \rho_p^{1-\beta} \left( \frac{\rho_p}{\rho_t} \sin \theta \right)^{1/3} \left( \frac{1.61g}{V^2} \right)^{-\beta},$$  \hspace{1cm} [5]$$

where $C_D$ is a drag coefficient, $\rho_p$ is the projectile density, $\rho_t$ is the target density, $\theta$ is the impact angle measured from horizontal, $g$ is the gravitational acceleration of the target planet, $V$ is the impact velocity, and $\beta$ is a constant. The factor of 1.25 accounts for the enlargement of complex craters relative to the transient cavity (Melosh 1989), though the applicability of this scaling to basins is uncertain. The $\sin \theta$ dependence has been added to account for the effect of impact angle on mean crater diameter (Gault and Wedekind 1978).

Alternative scaling relationships are often used. In a study of the formation of the Borealis basin on Mars, Marinova et al. (2008) found that the results of their smooth particle hydrodynamics (SPH) models agreed better with the energy scaling relationship used by Wilhelms and Squyres (1984):

$$D_c = K \cdot (E \cdot \sin \theta)^{\alpha} g^{-1/6},$$  \hspace{1cm} [6]$$

where $E$ is the impact energy, $K$ equals 0.0348 (mks units), $\alpha$ equals 0.29 (Housen et al. 1979), and we have introduced the $\sin \theta$ dependence into the energy term (Gault and Wedekind 1978). This scaling relationship diverges from the $\pi$-scaling relationship for large basins, resulting in a difference in predicted projectile diameter for a given crater of up to a factor of 3 for the largest
hemisphere-scale basins. While the \( \pi \)-scaling relationship is preferred for small craters, it may greatly underestimate the expansion of the basin from the projectile footprint for the largest impacts. The uncertainty in the relationship between projectile diameter/energy and basin size also extends to numerical simulations. Nimmo et al. (2008) found that the projectile diameter required to produce a Borealis-sized basin in SPH simulations was twice that required by their finite element model. There is clearly still much that remains to be learned regarding the dynamics of giant basin excavation.

We here consider both the \( \pi \)-scaling relationship and the energy scaling relationship, though we will focus primarily on the former in the discussion that follows. While the uncertain applicability of these scaling relationships to basin-forming impacts introduces significant uncertainty in the specific numerical results of this study, this approach allows us to characterize the effect of planetary curvature on basin shape and to efficiently investigate a wide range of parameter space. The general conclusions of this work are not sensitive to this uncertainty.

Given the projectile footprint dimensions and the predicted final mean basin diameter based on the scaling relationships, we can calculate the aspect ratio of the crater \( \varepsilon_{\text{crater}} \) as:

\[
\varepsilon_{\text{crater}} = \frac{a' + \Delta}{b' + \Delta} \\
\Delta = D_c - \frac{a' + b'}{2}
\]

Note that both the aspect ratio of the impact footprint and the mean final crater diameter depend on the impact angle. Previous inventories of craters generally assumed a threshold aspect ratio of 1.2 for classification as an elliptical crater (e.g., Bottke et al. 2000), and we adopt this value here as well.
While this first-order geometrical model is greatly oversimplified, it will be shown in the next section that it successfully predicts the onset of crater ellipticity both in laboratory experiments and in the observed population of small craters on the terrestrial planets, thus justifying its application to the larger impact basins.

RESULTS

Experimentally-Produced Elliptical Craters

We first apply this model to the experimental study of oblique impact cratering of Gault and Wedekind (1978). That study fired 1.6-12.5 mm diameter aluminum and Pyrex spheres at a target of unconsolidated quartz sand at velocities ranging from 3.6 to 7.2 km/s. In our analysis, we assume the mean impact velocity of 5.4 km/s for 1- and 10-mm spheres, with projectile (2700 kg/m³) and target (1700 kg/m³) densities appropriate for aluminum and quartz sand, and values of $\beta$ (0.17) and $C_d$ (1.68) appropriate for quartz sand (Melosh 1989) (Table 2). We find that our simple geometrical model accurately predicts the crater aspect ratio over the full range of impact angles (Figure 3).

Elliptical Crater and Basin Predictions for Mars

In this and the following sections, we will discuss the results from the $\pi$-scaling calculations, though results from the alternative energy scaling are also presented in Table 1. For all planetary crater scaling calculations, we assume values of $\beta$ (0.22) and $C_d$ (1.6) appropriate
for wet sand or rock, as in Melosh (1989) and Bottke et al. (2000). Choosing instead target
parameters determined from impacts into dry quartz sand, which may be more appropriate for
gravity-regime craters, does not significantly change the results. We assume projectile (2200
kg/m³) and target (2700 kg/m³) densities appropriate for chondritic and crustal materials,
respectively. Projectile velocities are assumed to be 12 km/s, comparable to estimated velocities
of asteroids striking the Moon and Mars (Bottke et al. 1994; Bottke et al. 2000).

In order to highlight the effect of the curvature of the planetary surface, we first consider
the case of a “flat Mars” in which the curvature of the planet is neglected. The projectile
footprint aspect ratio is independent of projectile size, and follows the simple 1/sinθ dependence
(Figure 4a). For this case, the critical angle for elliptical crater formation still increases with
increasing crater size, as pointed out by Bottke et al. (2000). Larger projectiles result in a lower
$D_c/d_p$ ratio, decreasing the enlargement of the crater beyond the impact footprint ($\Delta$) relative to
the projectile diameter ($d_p$). For small impactors ($d_p \sim 1$ km; $D_c \sim 10$ km), the critical angle for
elliptical crater formation is predicted to be 13°, corresponding to a probability of $P=0.05$
(Figure 4c), in agreement with observed 5% abundance on the terrestrial planets. As the
projectile and crater sizes increase, however, the critical angle for elliptical crater formation
increases to 30° for a 1000-km-diameter projectile (producing a basin roughly 50% larger than
Hellas or Utopia), corresponding to a 0.25 probability of ellipticity.

Upon including the effect of the curvature of the planet’s surface, the projectile footprint
aspect ratio becomes dependent upon both the impact angle and the projectile and planet
diameters (Figure 4d). For Mars, this effect begins to be significant for projectile diameters
greater than ~100 km. A projectile diameter of 1000 km now leads to a critical angle of 43°,
with a corresponding probability of 0.47. For large impactors on highly oblique trajectories, the
curvature of the planet also increases the likelihood of a glancing or decapitating impact, in
which a portion of the projectile misses the surface altogether (Schultz and Crawford 2008). The
critical angle for a decapitating impact of a 1000-km-diameter projectile on Mars is 42° (P=0.45). These decapitating impacts would likely affect the resulting crater morphology and encourage
elliptical crater formation. While both scaling analyses and the geometric method in this study
are inapplicable to decapitating impacts, the formation of an elliptical basin would be a likely
outcome, since the projectile would be traveling parallel to the surface at the down-range edge of
the projectile footprint, leading to a pronounced down-range focusing of the impact energy and
basin excavation.

We now consider the observed population of giant impact basins on Mars (Table 1). The
critical angle for producing an elliptical basin for the Isidis impact ($D_c=1500$ km) is $\theta_c=34°$, with
a probability of $\theta<\theta_c$ of 0.31. For Argyre ($D_c=780$ km) the critical angle and probability are
reduced to 29° and 0.24, respectively. The larger Hellas and Utopia ($D_c=1940-2220$ km) impacts
would have had a critical angle of 38-40°, occurring with a probability of 0.38-0.41. However, the critical angle for projectile decapitation would have been 35-37°, with a probability of 0.34-
0.36. Given that these basins are known to be elliptical, projectile decapitation was likely. For
the Borealis basin, the critical angle for projectile decapitation of 78° (P=0.96) occurs before the
onset of elliptical crater formation. If the energy scaling relationship is adopted, the probabilities
of elliptical basin formation for Hellas, Utopia, and Borealis are decreased to 0.15, 0.17, and
0.62, respectively.

Elliptical Crater and Basin Predictions for the Moon and Mercury
The smaller size of the Moon results in an increased likelihood that a basin of a given diameter will be elliptical upon taking into consideration the curvature of the surface. The critical angle for a 100-km-diameter projectile is 24.5° (P=0.17). For the South Pole-Aitken basin, the critical angle for elliptical crater formation is roughly the same as that for projectile decapitation of 44° (P=0.48), suggesting that projectile decapitation would have likely occurred in the SPA-forming impact. The smaller Crisium, Serenitatis, Nubium, and Imbrium basins have \( \theta_c = 25°-33° \) (\( P = 0.18-0.30 \)). Applying the model to Mercury, we find that \( \theta_c \) for the Caloris-forming impact would have been 33° (P=0.30).

**Statistical Comparison With Observed Population of Elliptical Basins**

The probabilities of elliptical crater formation for the individual basins in Table 1 can be used to calculate the most probable number of elliptical basins given the size distribution in the population of giant basins using a simple Monte Carlo approach. In this analysis, we are not concerned with which particular basins are elliptical, though the larger basins are significantly more likely to be elongated. For the six largest confirmed impact basins (Borealis, Hellas, Utopia, South Pole-Aitken, Imbrium, and Caloris), the probabilities in Table 1 suggest that 2.8±1.1 (1σ uncertainty) of these basins would be expected to be elliptical, less than the observed four elliptical basins but within the 2σ uncertainty. Expanding the definition of a giant impact basin to include all eleven well-characterized basins greater than 500 km in diameter on the Moon, Mercury, and Mars (Borealis, Hellas, Utopia, Isidis, Argyre, South Pole-Aitken, Imbrium, Serenitatis, Nubium, Crisium, and Caloris) yields an expected outcome of 4.0±1.4 elliptical basins, again less than the observed six elliptical basins, but within the 2σ uncertainty. Given the
small sample sizes, these results are generally consistent with the observed population of elliptical basins, though they suggest that this work may underestimate the probability of elliptical basin formation for the largest impacts.

The effect of the curvature of the planet’s surface is most evident when comparing the probability of elliptical crater formation as a function of the crater diameter for the equivalent vertical impact (Figure 5). The results for a flat Mars compare well with those of Bottke et al. (2000), and the critical angle for elliptical crater formation follows a power-law of the form in equation [1], with $\theta_0$ equal to 54.6° and $m$ equal to −0.543, similar to the values derived in that study (Figure 5a). For the case of a flat planet, the probability of elliptical crater formation increases only modestly for basin diameters greater than about 1000 km (Figure 5b) as a result of the power-law form of the relationship. In contrast, for the curved surfaces of Mars and the Moon the probability continues to rise steadily with increasing basin diameters, leading to a significant departure from the power-law of equation [1]. On the Moon, the probability of a South Pole-Aitken-sized basin being elliptical is more than twice what it would be neglecting the curvature of the surface (P=0.43 and 0.18, respectively), while the probability of a Hellas-sized basin on Mars being elliptical is nearly doubled when the curvature of the planet is included (P=0.40 vs. 0.24).

**DISCUSSION**

This work has shown that there is a clear tendency for the largest impact basins the solar system to have an elliptical rather than circular shape. The Borealis, Hellas, Utopia, and South Pole-Aitken basins on Mars and the Moon have aspect ratios ranging from 1.2 to 1.4. Of the
basins we classify as giant impact basins, only Caloris and Imbrium fall short of the assumed threshold aspect ratio of 1.2 for classification as elliptical, though even these basins display clear departures from circularity.

We explain this predominance of elliptical giant impact basins as resulting from the effects of the curvature of the planets’ surfaces. The simple geometric approach taken here, which builds on the work of Bottke et al. (2000), can to first order explain the results of both small-scale cratering experiments and the observed population of small elliptical craters on the terrestrial planets. Applying this approach to the giant basin-forming impacts demonstrates that curvature of the planetary surfaces leads to a significant increase in the likelihood of formation of basins with an elliptical shape by increasing the aspect ratio of the projectile footprint, thereby increasing the effective obliqueness of the entry angle of the impact. This work predicts that the probability that any one of the three largest confirmed impact basins (Hellas, Utopia, and South Pole-Aitken) would be elliptical is ~0.4, while the much larger Borealis basin on Mars would have had a probability of being elliptical approaching unity.

In the population of eleven well-characterized impact basins greater than 500 km in diameter considered here, this analysis suggests that between two and six elliptical basins would be expected, consistent with the observed six elliptical basins. If one considers only the six largest giant impact basins, this analysis would predict that between two and four basins should be elliptical, again consistent with the observed four elliptical basins, though this calculation is clearly compromised by the statistics of small numbers. While generally consistent with the occurrence of elliptical giant impact basins, these results suggest that this work may underestimate the predicted ellipticity of the largest basins. A possible explanation may lie in the simplistic assumption that the crater is excavated uniformly in all directions from the projectile
footprint. In oblique impacts there is a pronounced down-range focusing of the impact energy, with the isobaric core elongating and remaining close to the surface (Pierazzo and Melosh 2000a). For very large and oblique impacts, this isobaric core would likely re-intersect the planetary surface. The excavation process in these cases would be strongly focused downrange, likely favoring crater elongation and decreasing the critical angle for elliptical basin formation (Garrick-Bethell 2004). Alternatively, it is possible that at the scale of the largest basins, the larger melt volume produced by less oblique impacts may prevent basin preservation (Marinova et al. 2008), thus biasing the population of preserved basins towards higher aspect ratios.

Another factor that can contribute to crater elongation is the projectile density, through its effect on the ratio between the crater and projectile diameters. It has been suggested that the late heavy bombardment may have resulted from an influx of both asteroids and icy bodies from the outer solar system that were destabilized by the outward migration of the giant planets (Gomes et al. 2005; Strom et al. 2005). Low-density icy projectiles would lead to a lower $D_c/D_p$ ratio, increasing the probability of elliptical basin formation, though the higher impact velocities would counter this effect. Alternately, the largest projectiles would be expected to be more compacted and thus denser than the assumed uncompressed chondritic density, thereby decreasing the resulting crater ellipticity. However, basin-forming impacts may also excavate into the dense mantle, tending to decrease the size and increase the ellipticity of the resulting basin. Impact velocity also plays a key role, with slower projectiles predicting more elliptical craters due to both the decreased $D_c/D_p$ ratio and the greater influence of the planetary curvature for the larger projectiles required to produce a basin of a given diameter.

Despite the success of this simple approach in explaining the elliptical crater population observed both experimentally and on the surfaces of the planets, it cannot capture the full
complexity of the impact and excavation process. This work suffers the limitation that it relies on extrapolating scaling relationships by many orders of magnitude to apply them to basin-forming impacts. The validity of this extrapolation is questionable even for the limit of a flat planet, and the curvature of the surface would likely further affect the relationship between mean crater diameter and impact energy. Nevertheless, this work underscores the importance of both the size of the impact and the curvature of the surface in determining the resulting morphology of giant impact basins. Much work remains to be done to understand the process of giant impact basin formation and modification.

This work has focused on the basin shape, demonstrating that the curvature of the planetary surfaces increases the likelihood of elliptical crater formation at the scale of giant impact basins, essentially increasing the effective obliqueness of the impact. Many aspects of crater morphology are dependent upon the angle of impact, including crater ellipticity, ejecta distribution, and the amount of impact melt (Pierazzo and Melosh 2000b). Small-scale laboratory experiments and numerical simulations of impacts into generally flat targets reveal a progression of crater and ejecta morphology with decreasing impact angle. Impacts at angles less than \(~30^\circ\) produce asymmetric ejecta; less than 15-30° produce dramatically less impact melt relative to the crater volume; less than 10-15° produce elliptical craters; less than 5-10° produce “butterfly” ejecta blankets displaced downrange with large forbidden zones (Pierazzo and Melosh 2000b). This work on crater elongation suggests that at least one step in this progression is not universal, but rather depends on both the ratio of the crater to projectile size and the radius of curvature of the planet. For the case of the large basin-forming impacts, a large fraction (and possibly the majority) of impacts must be considered to be “highly oblique” on account of the curvature of the planetary surface.
Acknowledgements.

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References.


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Table 1. OBSERVED BASIN DIMENSIONS AND MODEL PREDICTIONS.

<table>
<thead>
<tr>
<th>Basin</th>
<th>$D_1$ (km)</th>
<th>$A_2$ (km)</th>
<th>$B_3$ (km)</th>
<th>$A/B_3$ (km)</th>
<th>$d_p\sin(\theta/3)$,5 (km)</th>
<th>$0_4$ (km)</th>
<th>$d_p'\sin(\theta/3)$ (km)</th>
<th>$P(\theta&lt;0.5)_8$ (W&amp;S)</th>
<th>$0_9$ (W&amp;S)</th>
<th>$d_p''$ (W&amp;S)</th>
<th>$P(\theta&lt;0.5)$ (W&amp;S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borealis</td>
<td>9550</td>
<td>10,600</td>
<td>8,500</td>
<td>1.25</td>
<td>4369 (78°)</td>
<td>4401</td>
<td>0.96</td>
<td>1480 (51°)</td>
<td>1612</td>
<td>0.62</td>
<td></td>
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<tr>
<td>Utopia</td>
<td>2200</td>
<td>2400</td>
<td>2000</td>
<td>1.20</td>
<td>669 (40°)</td>
<td>775</td>
<td>0.41</td>
<td>275 (24°)</td>
<td>372</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>Hellas</td>
<td>1940</td>
<td>2280</td>
<td>1590</td>
<td>1.43</td>
<td>561 (38°)</td>
<td>659</td>
<td>0.38</td>
<td>235 (23°)</td>
<td>322</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>SP-A</td>
<td>2060</td>
<td>2330</td>
<td>1780</td>
<td>1.31</td>
<td>481 (44°)</td>
<td>550</td>
<td>0.48</td>
<td>215 (32°)</td>
<td>266</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Caloris</td>
<td>1550</td>
<td>1525</td>
<td>1315</td>
<td>&lt;1.2</td>
<td>337 (33.4°)</td>
<td>--</td>
<td>0.30</td>
<td>149 (24°)</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Imbrium</td>
<td>1120</td>
<td>1140</td>
<td>1090</td>
<td>&lt;1.2</td>
<td>221 (33°)</td>
<td>--</td>
<td>0.30</td>
<td>107 (24°)</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Idaeus</td>
<td>1500</td>
<td>1570</td>
<td>1430</td>
<td>&lt;1.2</td>
<td>405 (34°)</td>
<td>--</td>
<td>0.31</td>
<td>175 (21°)</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Nubium</td>
<td>860</td>
<td>900</td>
<td>810</td>
<td>&lt;1.2</td>
<td>156 (30°)</td>
<td>--</td>
<td>0.25</td>
<td>79 (22°)</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Serenitatis</td>
<td>690</td>
<td>710</td>
<td>670</td>
<td>&lt;1.2</td>
<td>119 (27°)</td>
<td>--</td>
<td>0.21</td>
<td>61 (20°)</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Cruisium</td>
<td>510</td>
<td>600</td>
<td>420</td>
<td>1.43</td>
<td>81 (25°)</td>
<td>108</td>
<td>0.18</td>
<td>43 (18°)</td>
<td>64</td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>

1Mean basin diameter
2Basin semi-major axis
3Basin semi-minor axis
4Basin aspect ratio (if less than 1.2, the basin is not considered elliptical)
5Projectile diameter as a function of impact angle $\theta$ from the $\pi$-scaling relationship
6Critical angle for elliptical crater formation from the $\pi$-scaling relationship. Value in parentheses gives critical angle for projectile decapitation if this occurs prior to onset of crater ellipticity.
7Projectile diameter at the onset of basin ellipticity (if observed basin is elliptical)
8Probability of an impact occurring at or below the critical angle for basin ellipticity
9As before, but for the energy scaling relationship of Wilhelms and Squyres (1984).
10The Caloris basin dimensions were taken from Fassett et al. (2009) and thus may not compare directly with measurements made in this study.
Table 2. SUMMARY OF PARAMETERS USED IN MODELS

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Planetary</th>
</tr>
</thead>
<tbody>
<tr>
<td>target material</td>
<td>quartz sand</td>
<td>rock/wet sand</td>
</tr>
<tr>
<td>drag coefficient ($C_d$)</td>
<td>1.4</td>
<td>1.6</td>
</tr>
<tr>
<td>$\beta^1$</td>
<td>0.16</td>
<td>0.22</td>
</tr>
<tr>
<td>target density ($\rho_t$)</td>
<td>1700 kg/m$^3$</td>
<td>2700 kg/m$^3$</td>
</tr>
<tr>
<td>projectile material</td>
<td>aluminum/pyrex</td>
<td>chondrite</td>
</tr>
<tr>
<td>projectile density ($\rho_p$)</td>
<td>2700 kg/m$^3$</td>
<td>2200 kg/m$^3$</td>
</tr>
<tr>
<td>Impact velocity (V)</td>
<td>5.4 km/s</td>
<td>12 km/s</td>
</tr>
</tbody>
</table>

$^1$Constants governing the $\pi$-scaling relationship in equation (5)
Figure 1. Basin-centered polar projection topographic maps of the largest known and proposed impact basins on Mars and the Moon (units on the color bars are km). The best-fit ellipses to the preserved rims are shown. The basins qualifying as giant impact basins \( (D_c > 0.5 \ R_p) \) include Hellas (A), Utopia (B, in present day topography; C, in modeled pre-fill topography), and Borealis (D, in present day topography; E, in reconstructed pre-Tharsis topography) on Mars; and South Pole-Aitken (F) and Imbrium (G) on the Moon. Expanding the definition to include all basins greater than 500 km in diameter includes Isidis (H) and Argyre (I) on Mars; and Nubium (J), Serenitatis (K), and Crisium (L) on the Moon. Global polar projection maps (D-E) are circular, and introduce large distortions in features not centered on the origin (e.g., the Hellas
basin in D). Points along the rims for ellipse fitting were selected in regions that have escaped subsequent modification by impact, volcanic, and fluvial processes.
Figure 2. Diagram of the conceptual model, in which a projectile of diameter $d_p$ strikes a planet of radius $R$ at velocity $v$ and impact angle $\theta$, producing a projectile footprint of aspect ratio $a/b$, characterized by a centrally condensed mass distribution. The basin cavity is assumed to expand outward from the scaled projectile footprint, $a'/b'$, by a uniform amount $\Delta$ in the radial direction to create a crater of aspect ratio $A/B$. 
Figure 3. Comparison of predicted crater aspect ratio to the experimental results of Gault and Wedekind (1978). Gault and Wedekind (1978) used aluminum and Pyrex spheres with diameters between 1.6-12.5 mm, fired into quartz sand at 3.6 – 7.2 km/s. Calculations used \( \pi \)-scaling relationships assuming projectile diameters of 1 and 10 mm, fired into quartz sand at 5.4 km/s.
Figure 4. Predicted projectile footprint aspect ratio, crater diameter, and crater aspect ratio for craters on Mars neglecting surface curvature (A-C), Mars including effects of surface curvature (D-F), and the Moon including effects of surface curvature (G-I). The final crater diameters here are based on the $\pi$-scaling relationship. Projectiles range in diameter from 1 to 1000 km. Projectile and target properties are as given in Table 1. The vertical and horizontal gray lines in the bottom panels indicate the critical angle corresponding to the observed 5% abundance of elliptical craters on the terrestrial planets, and the assumed threshold aspect ratio of 1.2 for consideration as elliptical, respectively.
Figure 5. Relationship between $\theta_c$ and $D_c/d_p$ (based on the $\pi$-scaling relationship), and between the probability of elliptical crater formation and the crater diameter for Mars, Mars with neglecting effects of planetary curvature (“flat Mars”), and the Moon. Power-law relationships from Bottke et al. (2000) for the Moon and Mars are shown for comparison (the Mars and Moon curves are coincident in the top panel).