$y'' + 6y' + 12y = \delta_\pi(t) + f(t) \qquad f(t) = t + u_2(t)(t-2) \qquad y'(0) = 14$

$s^2Y(s) - s + 14 + 6sY(s) - 6 + 12Y(s) = e^{-\pi s} + \frac{1}{s^2} + e^{-2s}\mathcal{L}\{3-(t+2)\}$

$+\frac{1}{s} - \frac{1}{s^2}$

$Y(s)(s^2+6s+12) = e^{-\pi s} + \frac{1}{s^2} + e^{-2s}\left(\frac{1}{s} - \frac{1}{s^2}\right) + s - 8$

$$Y(s) = \frac{e^{-\pi s}}{s^2+6s+12} + \frac{1}{s^2(s^2+6s+12)} + e^{-2s}\left(\frac{1}{s(s^2+6s+12)} + \frac{1}{s^2(s^2+6s+12)}\right) + \frac{s-8}{s^2+6s+12}$$

$$\frac{1}{s^2(s^2+6s+12)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+6s+12}$$

$1 = As^3 + 6As^2 + 12As + Bs^2 + 6Bs + 12B + Cs^3 + Ds^2$

$0 = A + C \qquad D = -6A - B = -6\left(-\frac{1}{24}\right) - \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$

$0 = 6A + B + D$

$0 = 12A + 6B \qquad 12A = -6B$

$1 = 12B \qquad B = \frac{1}{12} \qquad A = -\frac{1}{24} \qquad C = \frac{1}{24}$

$$\frac{1}{s(s^2+6s+12)} = \frac{A}{s} + \frac{Bs+C}{s^2+6s+12}$$

$1 = As^2 + 6As + 12A + Bs^2 + Cs$

$0 = A + B \qquad B = -1/12$

$0 = 6A + C \qquad C = -\frac{1}{2}$

$1 = 12A \qquad A = 1/12$

$$Y(s) = e^{-\pi s}\left(\frac{1}{(s+3)^2+3}\right) - \frac{1}{24}\frac{1}{s} + \frac{1}{12}\frac{1}{s^2} + \frac{1}{24}\left[\frac{s+4}{(s+3)^2+3}\right]$$

$$+ e^{-2s}\left[\frac{1}{12}\left(\frac{1}{s}\right) - \frac{1}{12}\left(\frac{s+6}{(s+3)^2+3}\right) + \frac{1}{24}\frac{1}{s} - \frac{1}{12}\frac{1}{s^2} - \frac{1}{24}\left(\frac{s+4}{(s+3)^2+3}\right)\right] + \frac{s-8}{(s+3)^2+3}$$

$$Y(s) = e^{-\pi s}\left(\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{(s+3)^2+3}\right) - \frac{1}{24}\frac{1}{s} + \frac{1}{12}\frac{1}{s^2} + \frac{1}{24}\left[\frac{s+3}{(s+3)^2+3} + \frac{1(\sqrt{3})}{(\sqrt{3})(s+3)^2+3}\right]$$

$$+ e^{-2s}\left\{\frac{1}{12}\left(\frac{1}{s}\right) - \frac{1}{12}\left[\frac{s+3}{(s+3)^2+3} + \frac{3(\sqrt{3})}{(\sqrt{3})(s+3)^2+3}\right] + \frac{1}{24}\frac{1}{s} - \frac{1}{12}\frac{1}{s^2} - \frac{1}{24}\left[\frac{s+3}{(s+3)^2+3} + \frac{\sqrt{3}}{\sqrt{3}\,(s+3)^2+3}\right]\right\}$$

$$+ \frac{s+3}{(s+3)^2+3} - \frac{11(\sqrt{3})}{(\sqrt{3})(s+3)^2+3}$$

$$y(t) = u_\pi(t)\left(\frac{1}{\sqrt{3}}e^{-3(t-\pi)}\sin\sqrt{3}(t-\pi)\right) - \frac{1}{24} + \frac{1}{12}t + \frac{1}{24}\left[e^{-3t}\cos\sqrt{3}t + \frac{1}{\sqrt{3}}e^{-3t}\sin\sqrt{3}t\right]$$

$$+ u_2(t)\left[\frac{1}{12} - \frac{1}{12}\left(e^{-3(t-2)}\cos\sqrt{3}(t-2) + \frac{3}{\sqrt{3}}e^{-3(t-2)}\sin\sqrt{3}(t-2)\right)\right.$$

$$\left. + \frac{1}{24} - \frac{1}{12}(t-2) - \frac{1}{24}\left(e^{-3(t-2)}\cos\sqrt{3}(t-2) + \frac{1}{\sqrt{3}}e^{-3(t-2)}\sin\sqrt{3}(t-2)\right)\right]$$

$$+ e^{-3t}\cos\sqrt{3}t - \frac{11}{\sqrt{3}}e^{-3t}\sin\sqrt{3}t$$

Solve

$$y'' + 6y + 12y = \delta(t-\pi) + t + f(t) + g(t) \qquad \begin{array}{l} y(0) = 1 \\ y'(0) = -14 \end{array}$$

$$f(t) = \begin{cases} t & 0 \le t \le 2 \\ 3 & 2 \le t \end{cases}$$

$$g(t) = 1 + \int_0^t e^{-\tau} g(t-\tau)\,d\tau$$

Ans:

$$y(t) = \frac{1}{\sqrt{3}} U(t-\pi)\left(e^{-3(t-\pi)} \sin\sqrt{3}(t-\pi)\right)$$
$$+ U(t+2)\left\{\frac{1}{8} - \frac{1}{12}(t-2) - \frac{1}{8}e^{-3(t-2)}\cos\sqrt{3}(t-2) - \frac{7}{24\sqrt{3}} e^{-3(t-2)} \sin\sqrt{3}(t-2)\right\}$$
$$+ \frac{1}{6}t + e^{-3t}\cos\sqrt{3}t - \frac{67}{6\sqrt{3}} e^{-3t} \sin\sqrt{3}t$$

$$y'' + 6y + 12y = \delta_\pi(t) + t + u_2(t)(3-t) + 1 + t$$

$$s^2Y(s) - s + 14 + 6s - 6 + 12Y(s) = e^{-\pi s} + \frac{1}{s^2} + e^{-2s}\left(\frac{1}{s} - \frac{1}{s^2}\right) + \frac{1}{s} + \frac{1}{s^2}$$

$$Y(s)(s^2+6s+12) = e^{-\pi s} + e^{-2s}\left(\frac{1}{s} - \frac{1}{s^2}\right) + \frac{1}{s} + \frac{2}{s^2} + s - 8$$

$$Y(s) = \frac{e^{-\pi s}}{s^2+6s+12} + e^{-2s}\left(\frac{1}{s(s^2+6s+12)} - \frac{1}{s^2(s^2+6s+12)}\right) + \frac{1}{s(s^2+6s+12)} + \frac{2}{s^2(s^2+6s+12)} + \frac{s-8}{s^2+6s+12}$$

$$Y(s) = \frac{e^{-\pi s}}{(s+3)^2+3} + e^{-2s}\left(\frac{s-1}{s^2(s^2+6s+12)}\right) + \frac{s+2}{s^2(s^2+6s^2+12)} + \frac{s-8}{(s+3)^2+3}$$

1

$$Y(s) = e^{-\pi s}\left(\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{(s+3)^2+3}\right) + e^{-2s}\left(\frac{1}{8}\frac{1}{s} - \frac{1}{12}\frac{1}{s^2} - \frac{1}{8}\frac{s+3}{(s+3)^2+3} - \frac{7}{24\sqrt{3}}\frac{\sqrt{3}}{(s+3)^2+3}\right)$$

$$+ \frac{1}{6}\left(\frac{1}{s^2}\right) + \left(\frac{s+3}{(s+3)^2+3} - \frac{\sqrt{3}}{(s+3)^2+3}\right)$$

$$y(t) = \frac{1}{\sqrt{3}}\,U(t-\pi)\left(e^{-3(t-\pi)}\sin\sqrt{3}(t-\pi)\right)$$

$$+ U(t-2)\left\{\frac{1}{8} - \frac{1}{12}(t-2) - \frac{1}{8}e^{-3(t-2)}\cos\sqrt{3}(t-2) - \frac{7}{24\sqrt{3}}e^{-3(t-2)}\sin\sqrt{3}(t-2)\right\}$$

$$\frac{1}{6}t + e^{-3t}\cos\sqrt{3}t - \frac{67}{6\sqrt{3}}e^{-3t}\sin\sqrt{3}t$$

$$G(s) = \frac{1}{s} + \frac{1}{s+1} G(s)$$

$$\left(1 - \frac{1}{s+1}\right) G(s) = \frac{1}{s}$$

$$\frac{s+1-1}{s+1} G(s) = \frac{1}{s}$$

$$G(s) = \frac{s+1}{s^2} = \frac{1}{s} + \frac{1}{s^2}$$

$$g(t) = 1 + t$$

$$\mathcal{L}\{u_2(t)\{3-t\}\}$$

$$e^{-2s}\mathcal{L}\{3-(t+2)\}$$

$$e^{-2s}\left(\frac{1}{s} - \frac{1}{s^2}\right)$$

1

$$\frac{s-1}{s^2(s^2+6s+12)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+6s+12}$$

$$s-1 = As^3 + 6As^2 + 12As + Bs^2 + 6Bs + 12B + Cs^3 + Ds^2$$

s^3: $A + C = 0$ $\quad C = -\frac{1}{8}$

s^2: $6A + B + D = 0$ $\quad D = -\frac{9}{12}\left(\frac{1}{8}\right) + \frac{1}{12} = -\frac{8}{12} = -\frac{2}{3}$

s: $12A + 6B = 1$ $\quad 12A = 1 + \frac{1}{2}$ $\quad A = \frac{3}{2(12)} = \frac{1}{8}$

$\#$: $12B = -1$ $\quad B = -\frac{1}{12}$

$$= \frac{1}{8}\frac{1}{s} - \frac{1}{12}\frac{1}{s^2} - \frac{1}{8}\left(\frac{s + \frac{16}{3}}{s^2+6s+12}\right)$$

$$= \frac{1}{8}\frac{1}{s} - \frac{1}{12}\frac{1}{s^2} - \frac{1}{8}\left(\frac{s+3}{(s+3)^2+3} + \frac{7}{3}\frac{1}{(s+3)^2+3}\right)$$

$$\frac{s+2}{s^2(s^2+6s+12)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+6s+12}$$

$$2s+1 = As^3 + 6As^2 + 12As + Bs^2 + 6Bs + 12B + Cs^3 + Ds^2$$

s^3: $A + C = 0$ $\quad C = 0$

s^2: $6A + B + D = 0$ $\quad D = -\frac{1}{6}$

s: $12A + 6B = 1$ $\quad 12A = 1 - 1 = 0$ $\quad A = 0$

$\#$: $12B = 2$ $\quad B = \frac{1}{6}$

$$\frac{1}{6}\left(\frac{1}{s^2}\right) - \frac{1}{6\sqrt{3}}\left(\frac{\sqrt{3}}{(s+3)^2+3}\right)$$

$$\frac{s-8}{(s+3)^2+3} = \frac{s+3}{(s+3)^2+3} - \frac{11}{\sqrt{3}}\frac{\sqrt{3}}{(s+3)^2+3}$$

$$-\frac{66}{6\sqrt{3}} - \frac{1}{6\sqrt{3}} = -\frac{67}{6\sqrt{3}}$$

$$\frac{1}{6}\left(\frac{1}{s^2}\right) + \frac{s+3}{(s+3)^2+3} - \frac{67}{6\sqrt{3}}\left(\frac{\sqrt{3}}{(s+3)^2+3}\right)$$