MATH225, Fall 2010 **Name:**
Worksheet 3 (2.1, 2.2, 2.3) **Section:**

For full credit, you must show all work and box answers.

1. (a) Match the following differential equations to their direction fields. (All the direction fields are graphed for $-3 \leq t \leq 3$, $-3 \leq y \leq 3$.)

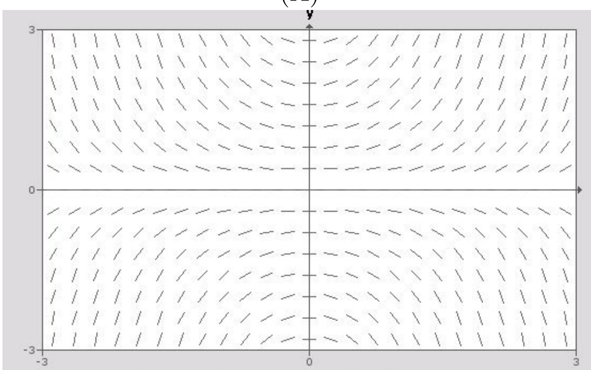
   i. $\dfrac{dy}{dt} = t$ ____________

   ii. $\dfrac{dy}{dt} = y$ ____________

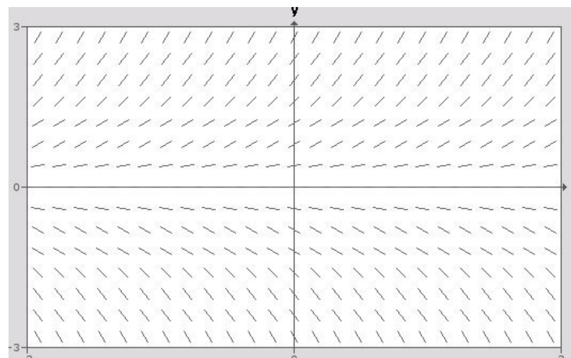
   iii. $\dfrac{dy}{dt} = ty$ ____________

   iv. $\dfrac{dy}{dt} = (t-2)y$ ____________

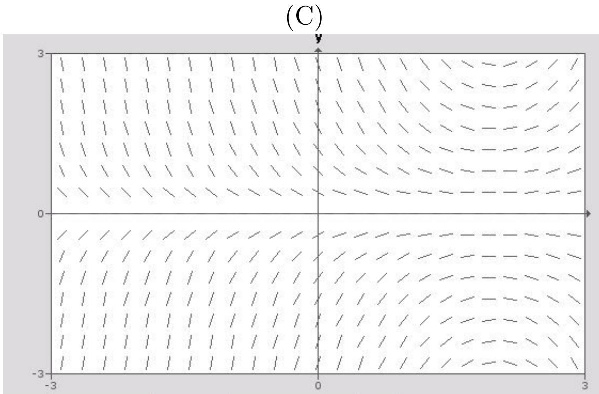
(A)



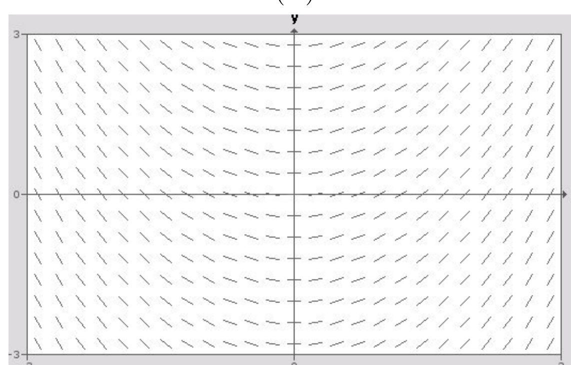
(B)



(C)



(D)



(b) On graph (A), sketch an approximate solution curve passing through the point $y(0) = 1$.

2. The following differential equation

$$\frac{dy}{dt} = y\left(1 - \frac{y}{5}\right)$$

is called a logistic growth model. If y represents the size of a population at time t, $N = 5$ is the carrying capacity, or maximum amount the environment can sustain.

(a) Sketch the phase line (portrait) and classify all of the critical (equilibrium) points.

(b) Next to your phase line, sketch the graph of solutions satisfying the initial conditions: $y(0) = 2, \quad y(0) = -1, \quad y(0) = 5 \quad y(1) = 11.$

(c) Find $\lim_{t\to\infty} y(t)$ for the solution satisfying the inital condition $y(0) = 2$.

3. Given $\frac{dy}{dt} = -1 + \cos(y)$,

(a) Sketch the phase line (portrait) and classify all of the critical (equilibrium) points for $-3\pi \leq y \leq 3\pi$.

(b) Next to your phase line, sketch the graph of solutions satisfying the initial conditions: $y(0) = 0, \quad y(0) = \frac{\pi}{2}, \quad y(0) = \pi.$

4. Solve the following differential equations or initial-value problems by separation of variables. Put your solutions in explicit form.

   (a) $\dfrac{dy}{dx} = x^4 y$

   (b) $\dfrac{dx}{dt} = \dfrac{t^2}{x + t^3 x}, \quad x(0) = -2$

   (c) $\dfrac{dy}{dt} - \dfrac{y^2 t}{t^2 + 1} = 0, \quad y(0) = -2$

5. Solve the following differential equations or initial-value problems using the Method of Integrating Factors. Put your solutions in explicit form.

(a) $\dfrac{dy}{dt} - 5y = 3e^{-5t}$

(b) $\dfrac{dy}{dt} = \dfrac{3}{t}y + 2t^4 e^{2t}, \quad t > 0$

(c) $(x+1)\dfrac{dy}{dx} + y = \ln(x), \quad y(1) = 10, \quad x > 0$