1. For the following, state the order of the differential equation and determine whether it is linear or nonlinear.

(a) \((2 - t)y'' - 3ty' + 5t = \sin(t)\)

(b) \(x \left( \frac{d^3y}{dx^3} \right) - \frac{d^5y}{dx^5} + y = 0\)

(c) \(y' = y\)

(d) \(\frac{dy}{dt} = 2y \left( 1 - \frac{y}{5} \right)\)

2. Verify that the indicated function (or family of functions) is a solution of the given differential equation. Assume an appropriate interval \(I\) of definition for each solution.

(a) \(y' = 2y; \quad y(t) = e^{2t}\)

(b) \((y - x)y' = y - x + 8; \quad y(x) = x + 4\sqrt{x + 2}\)

(c) \(\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0; \quad y = c_1e^{2x} + c_2xe^{2x}\)

(d) \((x^2 - y)\frac{dy}{dx} + 2xy = 0, \quad -2x^2y + y^2 = 1\)
3. Given \( y = c_1 e^{2x} + c_2 x e^{2x} \) is a family of solutions of \( y'' - 4y' + 4y = 0 \). Find a solution of the second-order IVP (initial-value problem) consisting of this differential equation and the following initial conditions.

(a) \( y(0) = 0, \quad y'(0) = 0 \)

(b) \( y(0) = 1, \quad y'(0) = 2 \)

(c) \( y(1) = 0, \quad y'(0) = -1 \)

4. Determine if the following statement is true or false for each initial value.

There is guaranteed to exist a unique solution to the initial value problem, where \( \frac{dy}{dt} = \sqrt{4 - y^2} \) and the initial condition is given below.

(a) \( y(0) = 0 \) ____________

(b) \( y(1) = 3 \) ____________

(c) \( y(3) = 1 \) ____________

(d) \( y(-3) = -2 \) ____________
5. Given the differential equation \( y' = y^2 \),
(a) Verify that \( y = \frac{-1}{x + c} \) is a one-parameter family of solutions.

(b) Determine whether the differential equation is guaranteed to have a unique solution through the point (0,1).

(c) Find a solution of the first-order IVP (initial-value problem) consisting of this differential equation and the initial condition \( y(0) = 1 \).

(d) Determine the largest interval \( I \) of definition for the solution of the initial-value problem in part (c).

6. A bacteria culture grows at a rate proportional to its size.
(a) Write the differential equation that models this situation. Let \( P \) be the number of bacteria at time, \( t \). Is this equation linear?

(b) Find an explicit solution to your differential equation from part (a). (Hint: What function do you know from calculus is such it is its own derivative? What function has a derivative that is a constant times itself?)

(c) Now assume that the bacteria are exposed to a poison. The rate at which the bacteria is being killed by the poison is \( .5P^2 \). Change the differential equation from part (a) to take into account this new factor, but do not solve this new ODE.
7. (a) Suppose a student carrying a flu virus comes to an isolated college campus of 5000 students. Determine a differential equation for the number of people $x(t)$ who have contracted the flu if the rate at which the disease spreads is proportional to the product of the number of students who have the flu and the number of students who have not been exposed to it. (Hint: The number of students who have not been exposed is $5000 + 1 - x$. We are assuming everyone exposed contracts the flu.)

(b) What would be the initial condition for this situation?

8. A cup of coffee is initially 165°F and is left in a room with an ambient temperature of 75°F. Suppose that at time $t = 0$ it is cooling at a rate of 20°F per minute. Assume that Newton's Law of Cooling applies: The rate of cooling is proportional to the difference between the current temperature and the ambient temperature.

(a) Write an initial-value problem that models the temperature of the coffee. Let $T$ represent the current temperature of the coffee at time, $t$, in minutes.

(b) Find the proportionality constant from your model in part (a).