New LM Unit Root Tests in the Presence of a Possible Break of Unknown Date and Size

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Abstract: Lagrange Multiplier (LM) unit root tests of Schmidt-Phillips (1992) and Schmidt-Lee (1991) offer improved power over the more popular Dickey-Fuller (1979) and Phillips-Perron (1988) tests. These tests have recently been extended by Amsler-Lee (1995) to allow for a structural break at a known date, and by Lee-Strazicich (LS) (2003) who use the minimum-τ criterion of Zivot-Andrews (1992) to identify unknown break dates. We find that the min-τ criterion often performs poorly when used in LM unit root tests and propose an alternative procedure that uses the supF tests or Chow-Quandt-Andrews (CQA) tests. The supF test is applied to a univariate specification in first-differenced series to obtain a detrending procedure that is optimal under the null hypothesis. The detrended series is then used in Schmidt-Phillips and Schmidt-Lee style LM unit root tests. Critical values for these two variants of the LM test with an unknown break date are reported. Our power analysis suggests that the new tests dominate those in the existing literature.

Keywords: unit root tests, structural change, break points, trend stationarity, macroeconomic time series

JEL classifications: C15 (Statistical Simulation Methods; Monte Carlo Methods; Bootstrap Methods), C32 (Time-Series Models), C53 (Forecasting and Other Model Applications)

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I. Introduction

The LM unit root tests of Schmidt and Phillips (1992) and Schmidt-Lee (1991) offer improved power over the more popular Dickey-Fuller (1979) and Phillips-Perron (1988) tests because they use detrending procedures that are optimal under the null hypothesis of a unit root. As Vougas (2003) explains: “LM unit root testing imposes the unit root to obtain optimal trend parameters and an asymptotically free-of-trend residual series to be used for unit root testing in a second step. It has the benefit of avoiding inconsistent (if error dynamics is not taken into account) LS estimation in the first step, and produces invariant (and efficient) unit root tests, which can be more powerful than usual DF tests in region [0.8,1) in many cases.”

Since Perron (1989), there has been widespread interest in allowing for structural breaks when carrying out unit root tests [See, e.g., Perron (1997), Zivot-Andrews (1992), Banerjee, Lumsdaine and Stock (1992), among many others]. This complicates unit root testing because: (i) ignoring breaks greatly reduces the power of standard unit root tests, (ii) the break date may or may not been known *a priori* and ‘data mining’ to choose the break date biases test results, and (iii) some unit root test procedures (e.g. Zivot-Andrews) allow for a break only under the alternative hypothesis. That is, they test the null hypothesis of a unit root with no break against the alternative hypothesis of (mean or trend) stationarity with a break. This approach proves awkward for analysts who are often equally interested in questions about the presence unit roots and structural breaks.

Given their higher power (when the largest root is near unity), it is natural to extend LM unit root tests to allow for breaks. Work on this topic was initiated by Amsler-Lee (1995), who consider a single break at a known date, and Lee-Strazicich (LS) (2003) who consider two breaks at *unknown* dates. As these authors stress, the LM approach to unit root testing has the major advantage of allowing for a break(s) under *both* the null and the alternative hypotheses.

To choose the unknown break dates, LS follow Zivot-Andrews (and Lumsdaine-Papell's (1997) extension to the two break case) by using the so-called minimum-τ procedure. This procedure estimates the chosen unit root testing equation considering all possible break dates in the (trimmed) sample and chooses the break date(s) so that the t statistic for the unit root hypothesis (called the τ statistic in this context) is minimized.[1]

Unfortunately, the min-τ criterion for choosing break points does not always work well. Lee-Strazicich (2001) use the min-τ criterion to select the break date in a ZA testing equation that has been extended to include an extra dummy(ies) to allow for a break under the unit root null hypothesis. They find it tends to select a break date that is one period *before* the true break point. Our analysis below suggests that min-τ break selection does much worse when used in LM unit root test equations than it does in the ZA test.[2] Therefore, there should be efficiency gains from improving the break date selection method used in LM unit root tests in the presence of unknown break dates.

This paper uses the supF tests developed in the 'new economics of structural breaks' [e.g., Quandt (1960), Andrews (1993), Bai (1997) and Hansen (2001)] to determine the most likely break date in a univariate time series model *in first-differences*.[3],[4] This is key because the first-difference specification is appropriate under the null hypothesis of a unit root. The break point chosen from this formulation is then used to obtain an optimal demeaning or 'detrending' procedure for the series when carrying out LM-style unit root tests.[5]

Our test has several advantages over those in the literature. First, it is much more likely to choose the correct break date under either the null or alternative hypothesis. Second, our test is invariant with respect to the date (λT) and magnitude (d_1, d_2) of the break, as well as invariant to the presence or absence of a break *(i.e.* $\lambda \neq 0$ *vs.* $\lambda = 0$*)*. Third, it is straightforward to test for the presence of a unit root in the presence of a *possible* break.[6]

The paper reports critical values for our new LM-style unit root tests where the possible break date is selected using the supF criterion. Our invariance and power analyses suggest that the new tests generally dominate the Zivot-Andrews (1992) test – an ADF equation with break selected by the min-τ criterion – and the Lee-Strazicich (2003) test – a LM specification with break selected by the min-τ criterion.

II. Alternative Unit Root Tests in the Presence of a Break

Perron (1989) first highlighted the critical importance of modeling structural breaks when carrying out unit root tests. These breaks might take the form of level shifts ('Model A'), or growth rate shifts ('Model B') or both ('Model C') in the underlying data generating process for a time series y_t. That is:

$$\begin{aligned} & y_t = \mu + \delta * t + d_1 * DUM_{\lambda T} + d_2 * DUM_{\lambda T} * t + e_t \\ & where \\ & e_t = \rho * e_{t-1} + v_t \end{aligned} \qquad (1)$$

T is the total number of observations and λ is the portion of the sample before the break occurs. The dummy variable $DUM_{\lambda T}$ is zero for the first λT observations and one thereafter. The coefficient d_1 determines the size of the level shift. The coefficient d_2 on the shift dummy-time trend interaction term ($DUM_{\lambda T} * t$) determines the size of the growth rate shift from δ to $\delta+d_2$ after period λT. In what follows, we consider both 'big' and 'small' breaks of type A ($d_1\neq 0, d_2=0$) and C ($d_1\neq 0, d_2\neq 0$), as summarized in Table 1. Note that in all of Perron's break models, a trend is included (i.e., δ may or may not equal zero).

Perron's unit root testing equation for a known break date augments the Dickey-Fuller specification by including a 'spike' dummy $D(DUM_{\lambda T})$ to capture the effect of a level shift break under the null hypothesis of a unit root, as well as level shift $DUM_{\lambda T}$ and level shift-trend interaction $DUM_{\lambda T}*t$ dummies:

$$\Delta y_t = (\rho-1)y_{t-1} + \beta_0 + \beta_1 * t + \beta_2 * D(DUM_{\lambda T}) + \beta_3 * DUM_{\lambda T} + \beta_4 * DUM_{\lambda T} * t + \upsilon_t \qquad (2)$$

This specification allows for type A and C break specifications under both the null and alternative hypotheses. Model A is nested within Model C (with the growth rate shift coefficient, β_4, equal to zero).

Modifying the Perron's test to deal with an unknown break date, Zivot and Andrews use a test equation that excludes the spike dummy:

$$\Delta y_t = (\rho-1)y_{t-1} + \beta_0 + \beta_1 * t + \beta_3 * DUM_{\lambda T} + \beta_4 * DUM_{\lambda T} * t + \upsilon_t \qquad (3)$$

They search over all possible break dates in the (trimmed) sample [2, T-1],[7] using the min-τ criterion to select the break date λT where τ is the t statistic for the null hypothesis of a unit root: ρ-1=0. The ZA tests considers a null hypothesis of a unit root with no break against an alternative of mean or trend stationarity with a type A or C break at an unknown date λT. They show that an invarinant null distribution is obtained when the 'spike' dummy is excluded from Perron's test equation, but not otherwise. Hence, they estimate (3) instead of (2). Because of the asymmetric treatment of the break under the null and alternative hypotheses, however, the ZA test is poorly suited to 'testing' for the presence of a structural break, although it has often been (mis)used for this purpose in the applied time series literature.

There are two variants of the LM unit root test: Schmidt-Phillips (1992) and Schmidt-Lee (1991), which we denote as LM1 and LM2 respectively. The Schmidt-Phillips test has been generalized to the case of a single break at a known date by Amsler-Lee (1995). This test is denoted 'LM1-known break' in what follows. We provide a corresponding generalization of the Schmidt-Lee test when the break date is known ("LM2-known break"), then turn to alternative methods for selecting a break point when its location is unknown *a priori*.

Vougas' (2003) discussion of optimal detrending in LM models is straightforward to generalize to allow for a structural break(s). Under the null hypothesis ρ=1, the specification in (1) reduces to:

$$\Delta y_t = \delta + d_1 * D(DUM_{\lambda T}) + d_2 * DUM_{\lambda T} + \nu_t \tag{4}$$

To carry out a LM-known break unit root test, first obtain an optimal detrending function under the null hypothesis by estimating null equation (4). The resulting residuals $\hat{\nu}_t = \Delta y_t - \hat{\delta} - \hat{d}_1 D(DUM_{\lambda T}) - \hat{d}_2 DUM_{\lambda T}$ are estimates of the first-difference of the detrended series. To get the *level* of the detrended series denoted x_t, use the initial condition $x_1 = 0$ combined with the identity $x_t = x_{t-1} + \hat{\nu}_t$ for $t > 1$.[8]

The Amsler-Lee (1995) LM unit root test in the presence of a known break (LM1-known break) inserts the detrended series x_t as well as spike and level-shift dummies at the known break date into the Schmidt-Phillips (1992) test equation:

$$\Delta y_t = \mu + \alpha^{AL} x_{t-1} + d_1^{AL} D(DUM)_{\lambda T} + d_2^{AL} DUM_{\lambda T} + e_t^{AL} \tag{5}$$

Simulated critical values is then used to test $\alpha^{AL} \equiv \rho - 1 = 0$.[9]

An alternative approach, LM2-known break, would follow Lee-Schmidt (1991) by simply performing a (no-intercept) unit root test on the detrended series (here generalized to allow for a break):

$$\Delta x_t = \alpha^V x_{t-1} + e_t^V \qquad (6)$$

where V denotes 'variant.'

Vougas (2003) compared the SP (LM1) and SL (LM2) specifications in the absence of breaks and concluded that the SP test is inefficient and has lower power when the true data generating process has a root near unity ($0.9 < \rho < 1$). In such situations, the SL test dominates. We show below that Vougas' conclusions generalize to LM test specifications allowing for breaks at known or unknown dates.

Turning now to situations where the break date, if any, is unknown, we consider two criteria for identifying the unknown break date. The first is the min-τ criterion proposed by ZA. The second is the supF criterion. The LM1-min-τ test has been applied in a two-break generalization of equation (5) by Lee-Strazicich (2003). We investigate the one-break variant of their testing procedure here, and also consider a LM2-min-τ test using equation (6). The one-break min-τ LM unit root test statistics are denoted as follows:

$$\tau(\lambda_{AL}) = \inf_{\lambda \in \Lambda} \tau_{\alpha^{AL}}(\lambda) \qquad \ldots(7)$$

$$\tau(\lambda_V) = \inf_{\lambda \in \Lambda} \tau_{\alpha^V}(\lambda) \qquad \ldots(8)$$

where $\tau_{\alpha^{AL}}$ is the t statistics on x_{t-1} in equation (5), τ_{α^V} is the corresponding t statistic in (6), and Λ is the 15% trimmed sample. The min-τ criterion is used to estimate the break date in each specification, λ_{AL} and λ_V respectively.

The second method for choosing the break date λT applies the supF procedure to equation (4). Specifically, we calculate the value of the F statistics test d_1=0 in the case of Model A (and d_1=d_2=0 for Model C) for all possible break dates in the trimmed sample. The largest value of the F statistics, supF, indicates the candidate break point. Next, the supF statistic is compared to the simulated 5% significance level under the null hypothesis of no break to decide whether or not the break is statistically significant. If significant, we take λT as the date chosen by supF. If not, we conclude there is no break. It is in this sense that our test is an LM test for a unit root in the presence of a *possible* break, unlike other tests in the literature. After obtaining optimal break date, if any, based on the supF statistic, we apply LM1-supF and LM2-supF unit root tests using (5) and (6). Note that in the event that no break is detected, (5) and (6) collapse to the SP (LM1) and SL (LM2) tests, respectively. The Monte Carlo analysis in Section III compares the five unit root tests, which are summarized in Table 2 for ease of reference.

III. Monte Carlo Simulations

A. Critical Values for Unit Root Tests in the Presence of a Known Break

Before comparing the various unit root tests when the break date is unknown, it is useful to consider the case where the break date, if any, is known. Table 3 reports critical values for three unit root tests in the presence of known break at λ=0.0, 0.2, 0.4, 0.5, 0.6, and 0.8 based on sample size of 100 and 50,000 iterations. Two break sizes are considered, as shown in Table 1. The first test 'Perron ADF' is based on (3), which collapses to the DF test with a constant and trend when λ=0 (i.e. no break). The second is the Amsler-Lee LM test (LM1-known break based on (4)-(5)); it becomes the Schmidt-

Phillips test when λ=0. The third is LM2-known break test (based on (4)-(6)), which is the Schmidt-Lee test when λ=0. Critical values for the LM2-known break test are not, to our knowledge, available in the literature.

First, consider the situation where there is known to be no break. The λ=0 column in the Table 3 closely reproduces the critical value reported in Dickey-Fuller for a break of type A or C (-3.45 is the DF critical value when the test equation contains a constant and trend). The LM1 test with λ=0 replicates the SP critical value (-3.04, -3.05). The LM1 critical values for various values of $\lambda \neq 0$ converge to the SP critical values for model A, confirming the λ-invariant property discussed by Amsler and Lee. They did not report critical values for model C, but pointed out that the invariance property fails. Our critical values in the lower panel of Table 3 highlight the failure of the invariance property.

The LM2 test with λ=0 replicates the SL critical value (-2.62, -2.63). There are no results in the literature for the SL with break specification (LM2 with $\lambda \neq 0$ cases for models A or C). We simulate the critical values for the LM2-known break test and confirm that these critical values cluster tightly around the SL critical value for type A breaks. Therefore, invariant property of AL holds for our SL variant with type A breaks as well ('LM2-known A breaks'). Interestingly, our LM2-known break critical values for type C breaks exhibit much less variation than do those for the Perron or LM1-known C break tests. The failure of the λ invariance property is relatively mild for the 'LM2-known C break' test.

An ideal unit root test would have critical values that are invariant with respect to the presence or absence of a break, its location (λ invariance), and its size (d invariance). A quick summary statistic for (λ, d) invariance is the standard deviation of the critical values across the 12 break date/size combinations for a given test.[10] If this standard deviation is zero, the test is (λ, d) invariant. Table 3 clearly shows that the (λ, d)-invariance property holds reasonably well in Model A (level shifts) for all three tests with LM2 showing the lowest variability in critical values across alternative λ values. The standard deviation is the largest for the Perron ADF test and smallest for the LM2-variant, with the LM1 test lying in between.

For Model C (growth rate shifts), in contrast, λ-invariance fails. The relative performance of the three tests, however, remains unchanged: the standard deviation is largest for the Perron ADF test and smallest for the LM2 test.

Table 3 establishes the 'best that one can expect' scenarios as we go on to consider situations where the break point is unknown. The ideal break selection procedure would (among other things) virtually always pick the correct break point, resulting in a test where the break date was in effect 'known.'

B. Alternative Break Date Selection Criteria with Unknown Break Date

This section reports Monte Carlo simulation results for the five unit root tests in Table 2 and the four data generating processes in Table 1 assuming the break date is unknown *a priori*.[11] To get a sense of the accuracy of each break date selection method, Fig.1 shows the histograms of the break date chosen by each for the case of a 'small' level break (model A) at observation $\lambda T=0.5*100=50$ when $\rho=1$. The supF criterion clearly dominates min-τ for selecting the break date for the case shown in Fig. 1.

How general is this finding? Table 4 shows fraction of total replications (50,000) where each method chooses the correct break date depending on: (i) the underlying data generating model (model A-big break, Model A-small break, Model C-big, Model C-small), (ii) the size of the root in the error process (ρ=1.0, 0.9, 0.8, 0.7, 0.6, and 0.5), and (iii) the unknown break point (λ= 0.2, 0.5, and 0.8). Several conclusions emerge. First, the supF method clearly dominates the others in choosing the correct break date, regardless of the size of the break, the type of break (A or C), and the presence or absence of a unit root. Note also that the LM1-min-τ and LM2-min-τ methods fare much worse than ZA-min-τ (except in the Model A- small break scenario). Finally, all of the methods using the min-τ criterion for choosing the break date deteriorate as ρ moves from 0.5 to 1.0.

Note that our results on the efficacy of the ZA method for selecting the break point differ from those reported in LS (2001). They conclude that the ZA-min-τ method typically selects a break point that is one period before the true break date. This finding, however, is caused by their inclusion of a spike dummy $D(DUM_{\lambda T})$ to the ZA test equation. That is, they use (2) rather than (3). Table 4 shows that the bias they identify does not occur when the $D(DUM_{\lambda T})$ term is omitted, as in the original ZA equation.

C. Unit Root Tests: Critical Values

Table 5 reports simulated 5% critical values for our five unit root tests in the presence of big and small level-shift breaks for Model A. (See Appendix Table 1 for 1%, 5%, and 10% critical values.) As mentioned above, the ideal unit root test would have critical values that are (λ, d) invariant. Note that ZA critical values have a standard deviation of 1.43 across the 12 (λ, d) combinations, whereas the corresponding statistics for the two variants of the LM-supF tests are virtually zero (.01). The ZA test's lack of λ

invariance is well known.[12] In the presence of a break, if one uses the ZA critical value of -4.82, severe over-rejection of the unit root hypothesis occurs. These results confirm those of Nunes, Newbold, and Kuan (1997): spurious rejections can occur as the magnitude of a break increases under the null when the ZA test is used. (The ZA null hypothesis is really the presence of a unit root *and* the absence of a break, as emphasized in the Introduction.)

The LM1-min-τ test appears to be λ invariant except perhaps when a break does not exist (λ=0).[13] To see this, look across row 4 or 5 in Table 5. Interestingly, however, the test is not *d*-invariant; i.e. the critical values in row 4 (big break) differ from the corresponding ones in row 5 (small break). The LM2-min-τ test has similar properties, but the critical values are uniformly closer to zero, indicating that its distribution is more compact.

In contrast to the tests that use the min-τ criterion to select the break point, the LM1-supF and LM2-supF tests for type A breaks exhibit strong λ and *d* invariance. The critical values for the two variants of the LM-supF test are -3.05 and -2.63, respectively, regardless of the presence or absence of a break, or its size or location. It is striking that these critical values are virtually identical to those in Table 3 where the break point is assumed to be known *a priori*. In contrast, the critical values for the LM1-min-τ and LM2-min-τ tests are much larger when the break date is unknown (Table 5) than when it is known (Table 3).

Table 6 reports analogous 5% critical values and summary statistics for Model C (both level and growth rate shifts). (See Appendix Table 2 for 1%, 5%, and 10% critical values.) As the shading in the table indicates, the critical values are (must be) insensitive to the size of the break where there is, in fact, no break! Sampling variation explains the slight difference in critical values for the $\lambda=0$ column values for any given test.

These results show that the ZA test again exhibits wide variation in critical values depending on the break date/size. Clearly, the test is unusable when one is concerned about the possibility of a break and a unit root. The LM1-min-τ and LM2-min-τ tests also exhibit considerable variation in critical values depending on break location/size, although the standard deviation (0.21) is much lower than that for the ZA test (4.49).[14] The LM1-supF and LM2-supF tests clearly dominate the others. The critical values are very similar to those for the corresponding λ values in Table 3 when $\lambda \neq 0$. For the $\lambda=0$ case, the critical values further from zero. This presumably reflects the inefficiency caused by including redundant dummy variables in the LM test equations. If one knew that $\lambda=0$, the dummies should be excluded.

IV. Power Analysis

Table 7 examines the power of the five unit root tests for both Models A and C when $\rho=0.8$ and $\rho=0.9$. Employing the critical values for the unknown break cases for each estimated value of λ taken from Table 5 and 6, we report the percentage of the time that each unit root test correctly rejects the null hypothesis. The higher the percentage of the rejections the more powerful is the test. The actual power should match our chosen nominal power of 5% under the null hypothesis ($\rho=1$) and should increase as the ρ specified in the data generating process decreases.

There are several interesting features of the results in table 7A for type A breaks. First, generalizing the findings in Vougas (2003), we find that the LM2-supF test is more powerful than the LM1-supF test when $\rho=0.9$ but not when $\rho=0.8$. Second, when there is no break and ρ=0.9, the LM1-min τ and LM2-min τ tests have power comparable to the LM1-supF and LM2-supF tests. Third, in the presence of a break, however, the tests using the min-τ criteria are less powerful than those using the supF critierion for break point selection.

The power results for type C breaks are less decisive, as Table 7B shows. For big C breaks when ρ=0.9, the LM2-supF test is most powerful (as it was for type A breaks). When ρ=0.8, however, the power of LM1-supF and LM2-supF tests are comparable. LM2-min-τ is undominated in cases where there is no break (λ=0). The LM1-min-τ test is most powerful in the case of small C breaks when the actual break does not occur near the ends of the sample (i.e. $0.2 < \lambda < 0.6$).

V. Empirical Application to the Nelson-Plosser Dataset

The Nelson-Plosser dataset of U.S. macroeconomic variables has been widely used to illustrate the differences among alternative unit root testing procedures and alternative break point selection methods [see, e.g., Perron (1989), Amsler-Lee (1995), Lee- Strazicich (2003) among others].[15] Table 8 summarizes the stationarity and break conclusions that emerge from the use of the ZA test, the LM1-min-τ test, and our two variants of the LM-supF test. Recall that the ZA test does not allow for a break under the I(1) null hypothesis, while the LM1 test assumes that there is a single break under either the null or alternative hypotheses. Hence, the 'NA' entries in the Table. Our LM1-supF and LM2-supF tests allow for the presence or absence of a break under both the null hypothesis of a unit root and the alternative of trend stationarity.

The conclusions based on the LM-supF test indicate that structural breaks are less prevalent than the other tests would suggest. Only three of the series exhibit statistically significant breaks (at the 5% level). Unit root processes are, on the other hand, more prevalent than the ZA analysis would suggest. Eight of the eleven series are I(1) processes, and one of these also has a break, according to the LM 2-supF test results.

Table 9 reports the details for each of the eleven series. The estimated break dates differ from test to test. The ZA test estimates most of the breaks around 1930, a year after the Great Crash. Compared to the results from min-τ criteria (ZA test and LM2-min-τ test), supF criteria suggest that only three series have a statistically significant break: nominal GNP (λT=1921), GNP deflator (λT=1917), and money stock (λT=1932). Among these, we can only reject the null hypothesis of the unit root for the money stock and possibly nominal GNP. While LM1-supF test cannot reject the null hypothesis of stationarity for nominal GNP series, the LM2-supF test indicates rejection.

VI. Conclusion

The LM unit root tests of Schmidt-Phillips (1992) and Schmidt-Lee (1991) offer improved power over the more popular Dickey-Fuller (1979) test. Moreover, this power gain carries over to tests in the presence of breaks, with the LM tests of Amsler-Lee and Lee-Starzicich dominating earlier tests by Perron and Zivot and Andrews. In this paper, we develop a detrending method allowing for breaks to be detected using the supF criterion. The results of two LM unit root tests in the presence of possible breaks dominate those in the literature. Our tests are invariant with respect to the date (λT) and magnitude *(d_1, d_2)* of the break, as well as invariant to the presence *($\lambda \neq 0$)* or absence *($\lambda = 0$)* of a break. They also have high power relative to most other tests.

The LM1-supF and LM2-supF tests should be very useful for applied time series work where the investigator is interested in simultaneously testing for the presence of a unit root and the presence and dating of possible structural breaks. Our application to the Nelson-Plosser macroeconomic time series suggests that unit roots are more prevalent and structural breaks are less prevalent than the existing literature would suggest. This seems consistent with received wisdom following the unit root revolution of the last twenty five years.

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Table & Figures

Table 1: Alternative Structural Break Models Considered

	Model Parameters	‘Big’ break	‘Small’ break
Model A: Break in Mean; No growth	μ	0.0	0.0
	δ	0.0	0.0
	d_1	**10.0**	**5.0**
	d_2	0.0	0.0
Model C: Break in Mean & Growth Rate	μ	0.0	0.0
	δ	**0.2**	**0.2**
	d_1	**10.0**	**5.0**
	d_2	**0.2**	**0.1**

Note 1: The error distribution is v_t~N(0,1) throughout. This is a much larger standard deviation than most macro series [Nelson and Plosser (1982)], but only the ratio of the mean to the standard deviation is relevant so we specify the mean of our growth rate to be around 0.2 of the standard deviation.
Note 2: The level shift magnitudes of 5 and 10 were also used in Lee and Strazicich (2003).

Table 2: Alternative Unit Root Tests in the Presence of Structural Breaks

Unit Root Tests	Test Equation	Break Date Selection Method
1. ZA-Min-τ (Zivot-Andrews)	(3)	Min-τ in (3)
2. LM1-Min-τ (Amsler-Lee/Schmidt-Phillips)	(4)-(5)	Min-τ in (5)
3. LM2-Min-τ (Schmidt-Lee variant)	(4)-(6)	Min-τ in (6)
4. LM1-supF	(4)-(5)	supF in (4), provided supF > critical value
5. LM2-supF	(4)-(6)	supF in (4), provided supF > critical value

Table 3

5% Critical Values When Model A Break Date is Known (Iterations: 50,000)

Unit Root Test	Model\ Break Point	0	20	40	50	60	80	Row Avg.	Std.Deviation	
1-Perron ADF	1a_big	-3.45	-3.79	-3.77	-3.77	-3.79	-3.81	-3.79	0.02	0.02
	1a_small	-3.45	-3.79	-3.79	-3.75	-3.78	-3.79	-3.78	0.02	
	Col. Avg.	-3.45	-3.79	-3.78	-3.76	-3.79	-3.80	-3.78		
2-LM1-known A break (Amsler-Lee test)	1a_big	-3.04	-3.05	-3.06	-3.05	-3.06	-3.06	-3.06	0.01	0.01
	1a_small	-3.04	-3.06	-3.06	-3.05	-3.06	-3.05	-3.05	0.01	
	Col. Avg.	-3.04	-3.05	-3.06	-3.05	-3.06	-3.06	-3.06		
3-LM2-known A break	1a_big	-2.62	-2.63	-2.63	-2.64	-2.62	-2.63	-2.63	0.01	0.00
	1a_small	-2.62	-2.63	-2.64	-2.63	-2.63	-2.63	-2.63	0.00	
	Col. Avg.	-2.62	-2.63	-2.63	-2.64	-2.63	-2.63	-2.63		

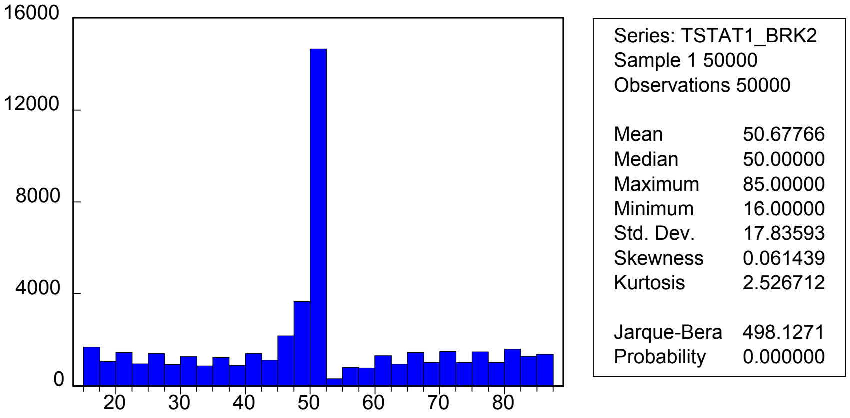
5% Critical Values When Model C Break Date is Known (Iterations: 50,000)

Unit Root Test	Model\ Break Point	0	20	40	50	60	80	Row Avg.	Std.Deviation	
1-Perron ADF	1a_big	-3.45	-3.97	-4.23	-4.25	-4.22	-3.97	-4.13	0.15	0.15
	1a_small	-3.45	-3.96	-4.22	-4.25	-4.22	-3.95	-4.12	0.15	
	Col. Avg.	-3.45	-3.96	-4.22	-4.25	-4.22	-3.96	-4.12		
2-LM1-known C break (Amsler-Lee test)	1a_big	-3.05	-3.52	-3.71	-3.71	-3.69	-3.53	-3.63	0.10	0.09
	1a_small	-3.05	-3.54	-3.69	-3.71	-3.70	-3.53	-3.63	0.09	
	Col. Avg.	-3.05	-3.53	-3.70	-3.71	-3.69	-3.53	-3.63		
3-LM2-known C break	1a_big	-2.63	-2.97	-3.06	-3.07	-3.06	-2.98	-3.03	0.05	0.05
	1a_small	-2.63	-2.99	-3.07	-3.08	-3.06	-2.98	-3.03	0.05	
	Col. Avg.	-2.63	-2.98	-3.07	-3.07	-3.06	-2.98	-3.03		

Figure 1:

Histogram of Estimated Break Data under Null (ρ=1) in Model A with Small Break at λ=0.5

ZA-min τ criterion

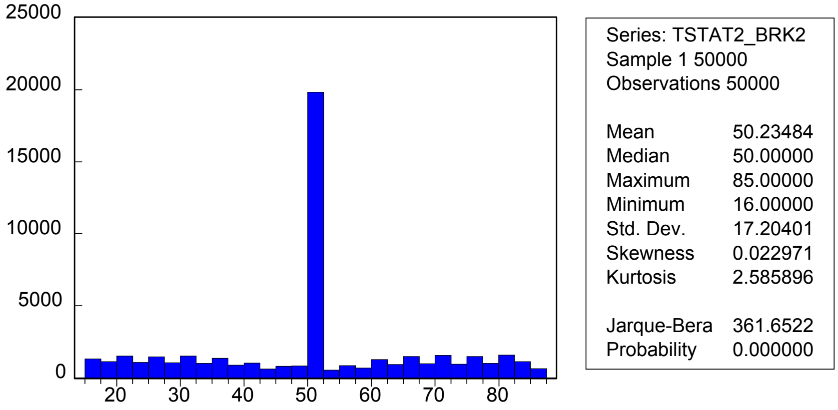


```
Series: TSTAT1_BRK2
Sample 1 50000
Observations 50000

Mean          50.67766
Median        50.00000
Maximum       85.00000
Minimum       16.00000
Std. Dev.     17.83593
Skewness      0.061439
Kurtosis      2.526712

Jarque-Bera   498.1271
Probability   0.000000
```

LM1-min τ criterion (Amsler-Lee)

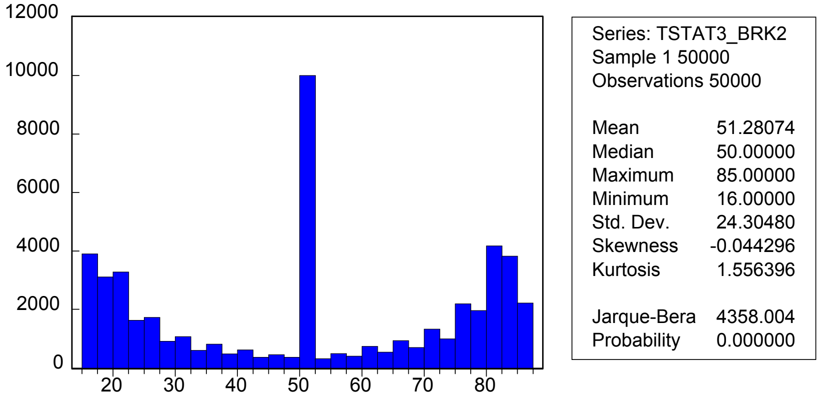


```
Series: TSTAT2_BRK2
Sample 1 50000
Observations 50000

Mean          50.23484
Median        50.00000
Maximum       85.00000
Minimum       16.00000
Std. Dev.     17.20401
Skewness      0.022971
Kurtosis      2.585896

Jarque-Bera   361.6522
Probability   0.000000
```

LM2--min τ criterion

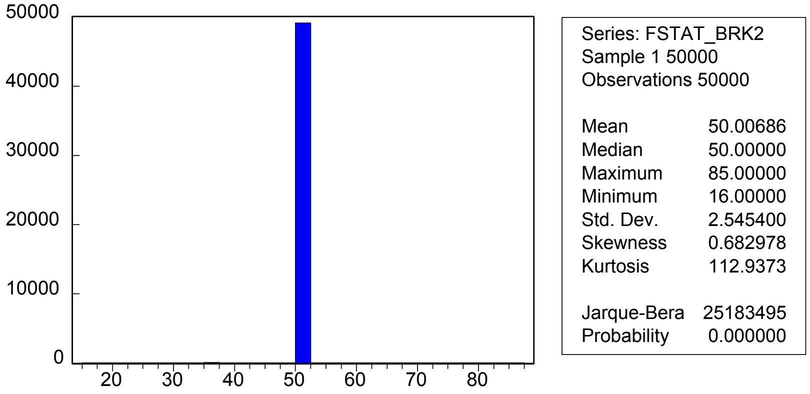


```
Series: TSTAT3_BRK2
Sample 1 50000
Observations 50000

Mean          51.28074
Median        50.00000
Maximum       85.00000
Minimum       16.00000
Std. Dev.     24.30480
Skewness     -0.044296
Kurtosis      1.556396

Jarque-Bera   4358.004
Probability   0.000000
```

Sup F procedure in equation (4)



```
Series: FSTAT_BRK2
Sample 1 50000
Observations 50000

Mean         50.00686
Median       50.00000
Maximum      85.00000
Minimum      16.00000
Std. Dev.    2.545400
Skewness     0.682978
Kurtosis     112.9373

Jarque-Bera  25183495
Probability  0.000000
```

Table 4

Accuracy of Break Date Estimation:

Proportion of Total Simulations Where Specified Method Selects True Break Date

Model A ('crash' or mean-shift model)

	True Break Date	Big Break					Small Break				
ρ	λT	1-ZA-min t	2-LM1-min t	3-LM2-min t	supF(4,5)	supF(5%)	1-ZA-min t	2-LM1-min t	3-LM2-min t	supF(4,5)	supF(5%)
1.0	20	0.808	0.477	0.579	1.000	1.000	0.318	0.370	0.491	0.984	0.926
1.0	50	0.749	0.483	0.255	1.000	1.000	0.291	0.381	0.193	0.984	0.924
1.0	80	0.810	0.477	0.576	1.000	1.000	0.317	0.370	0.495	0.985	0.925
0.9	20	0.947	0.741	0.798	1.000	1.000	0.491	0.488	0.593	0.977	0.903
0.9	50	0.926	0.732	0.579	1.000	1.000	0.475	0.525	0.374	0.978	0.901
0.9	80	0.943	0.741	0.791	1.000	1.000	0.483	0.496	0.596	0.978	0.904
0.8	20	0.984	0.938	0.913	1.000	1.000	0.705	0.675	0.701	0.972	0.879
0.8	50	0.981	0.907	0.824	1.000	1.000	0.691	0.669	0.538	0.970	0.878
0.8	80	0.983	0.933	0.907	1.000	1.000	0.702	0.677	0.699	0.971	0.881
0.7	20	0.994	0.987	0.953	1.000	1.000	0.828	0.808	0.765	0.962	0.962
0.7	50	0.993	0.969	0.928	1.000	1.000	0.822	0.775	0.659	0.962	0.962
0.7	80	0.994	0.987	0.949	1.000	1.000	0.821	0.805	0.763	0.963	0.963
0.6	20	0.998	0.998	0.969	1.000	1.000	0.889	0.886	0.799	0.950	0.950
0.6	50	0.998	0.987	0.966	1.000	1.000	0.893	0.837	0.737	0.952	0.952
0.6	80	0.998	0.997	0.967	1.000	1.000	0.888	0.881	0.797	0.952	0.952
0.5	20	0.999	0.999	0.976	1.000	1.000	0.926	0.920	0.815	0.937	0.937
0.5	50	0.999	0.993	0.981	1.000	1.000	0.927	0.870	0.788	0.938	0.938
0.5	80	0.999	0.999	0.976	1.000	1.000	0.928	0.919	0.812	0.936	0.936

Model C ('Combined Model' with level and growth rate breaks)

	True Break Date	Big Break					Small Break				
ρ	λT	1-ZA-min t	2-LM1-min t	3-LM2-min t	supF(4,5)	supF(5%)	1-ZA-min t	2-LM1-min t	3-LM2-min t	supF(4,5)	supF(5%)
1.0	20	0.949	0.023	0.102	1.000	1.000	0.512	0.027	0.086	1.000	1.000
1.0	50	1.000	0.397	0.393	1.000	1.000	0.913	0.269	0.309	1.000	1.000
1.0	80	1.000	0.016	0.049	1.000	1.000	0.951	0.021	0.058	1.000	1.000
0.9	20	0.997	0.085	0.254	1.000	1.000	0.710	0.082	0.176	1.000	1.000
0.9	50	1.000	0.658	0.687	1.000	1.000	0.968	0.458	0.545	1.000	1.000
0.9	80	1.000	0.074	0.197	1.000	1.000	0.996	0.081	0.195	1.000	1.000
0.8	20	1.000	0.294	0.528	1.000	1.000	0.890	0.226	0.331	1.000	1.000
0.8	50	1.000	0.903	0.906	1.000	1.000	0.991	0.703	0.762	1.000	1.000
0.8	80	1.000	0.311	0.523	1.000	1.000	1.000	0.284	0.445	1.000	1.000
0.7	20	1.000	0.588	0.742	1.000	1.000	0.953	0.416	0.495	1.000	1.000
0.7	50	1.000	0.980	0.975	1.000	1.000	0.997	0.861	0.890	1.000	1.000
0.7	80	1.000	0.669	0.768	1.000	1.000	1.000	0.567	0.655	1.000	1.000
0.6	20	1.000	0.800	0.861	1.000	1.000	0.976	0.583	0.620	0.999	0.999
0.6	50	1.000	0.996	0.993	1.000	1.000	0.999	0.933	0.945	1.000	1.000
0.6	80	1.000	0.887	0.887	1.000	1.000	1.000	0.775	0.784	1.000	1.000
0.5	20	1.000	0.905	0.924	1.000	1.000	0.985	0.697	0.712	0.998	0.998
0.5	50	1.000	0.999	0.998	1.000	1.000	1.000	0.968	0.972	1.000	1.000
0.5	80	1.000	0.962	0.942	1.000	1.000	1.000	0.888	0.865	1.000	1.000

Note: The number of iteration is 50,000

Table 5

5% Critical Values for Model A for Five Unit Root Tests with Different Unknown Break Dates and Magnitudes (50,000 iterations)

	A	B	C	D	E	F	G	H	I	J	K
	Unit Root Test	Model	Lambda (Break Point)						Row Avg.	Std.Deviation	
			0	20	40	50	60	80			
1	1-ZA-min τ	1a_big	-4.82	-8.00	-7.85	-7.91	-7.92	-7.93	-7.40	1.27	1.43
2		1a_small	-4.81	-5.33	-5.29	-5.29	-5.33	-5.31	-5.22	0.21	
3		Col. Avg.	-4.81	-6.67	-6.57	-6.60	-6.63	-6.62	-6.31		
4	2-LM1-min τ	1a_big	-3.50	-3.39	-3.39	-3.41	-3.39	-3.41	-3.42	0.04	0.06
5		1a_small	-3.47	-3.51	-3.52	-3.52	-3.52	-3.51	-3.51	0.02	
6		Col. Avg.	-3.48	-3.45	-3.46	-3.46	-3.45	-3.46	-3.46		
7	3-LM2-min τ	1a_big	-3.08	-3.03	-3.03	-3.04	-3.03	-3.04	-3.04	0.02	0.04
8		1a_small	-3.06	-3.12	-3.09	-3.10	-3.11	-3.12	-3.10	0.02	
9		Col. Avg.	-3.07	-3.07	-3.06	-3.07	-3.07	-3.08	-3.07		
10	4-LM1-supF	1a_big	-3.07	-3.05	-3.05	-3.07	-3.04	-3.06	-3.05	0.01	0.01
11		1a_small	-3.05	-3.06	-3.04	-3.06	-3.06	-3.06	-3.05	0.01	
12		Col. Avg.	-3.06	-3.05	-3.04	-3.06	-3.05	-3.06	-3.05		
13	5-LM2-supF	1a_big	-2.65	-2.63	-2.63	-2.64	-2.62	-2.64	-2.64	0.01	0.01
14		1a_small	-2.63	-2.62	-2.63	-2.64	-2.64	-2.64	-2.63	0.01	
15		Col. Avg.	-2.64	-2.63	-2.63	-2.64	-2.63	-2.64	-2.63		

Table 6

5% Critical Values for Model C for Five Unit Root Tests with Different Unknown Break Dates and Magnitudes (50,000 iterations)

	A	B	C	D	E	F	G	H	I	J	K
	Unit Root Test	Model	Lambda (Break Point)						Row Avg.	Std.Deviation	
			0	20	40	50	60	80			
1		1c_big	-5.12	-11.0	-14.4	-15.8	-17.0	-17.8	-13.51	4.75	4.49
2	1-ZA-min τ	1c_small	-5.13	-6.58	-8.14	-8.76	-9.26	-9.51	-7.90	1.71	
3		Col. Avg.	-5.12	-8.78	-11.3	-12.3	-13.1	-13.6	-10.70		
4		1c_big	-4.40	-4.17	-3.85	-3.81	-3.80	-4.19	-4.04	0.25	0.21
5	2-LM1-min τ	1c_small	-4.39	-4.33	-4.24	-4.19	-4.15	-4.21	-4.25	0.09	
6		Col. Avg.	-4.39	-4.25	-4.04	-4.00	-3.97	-4.20	-4.14		
7		1c_big	-3.82	-3.60	-3.30	-3.26	-3.26	-3.51	-3.46	0.23	0.21
8	3-LM2-min τ	1c_small	-3.81	-3.80	-3.68	-3.64	-3.62	-3.71	-3.71	0.08	
9		Col. Avg.	-3.81	-3.70	-3.49	-3.45	-3.44	-3.61	-3.58		
10		1c_big	-3.61	-3.53	-3.70	-3.71	-3.70	-3.52	-3.63	0.09	0.08
11	4-LM1-supF	1c_small	-3.60	-3.53	-3.69	-3.72	-3.69	-3.52	-3.62	0.09	
12		Col. Avg.	-3.60	-3.53	-3.69	-3.71	-3.70	-3.52	-3.63		
13		1c_big	-3.03	-2.99	-3.07	-3.07	-3.07	-2.97	-3.04	0.05	0.04
14	5-LM2-supF	1c_small	-3.02	-2.98	-3.07	-3.08	-3.08	-2.98	-3.03	0.05	
15		Col. Avg.	-3.03	-2.98	-3.07	-3.08	-3.07	-2.98	-3.03		

Table 7A

Power Analysis for Break Type A

DGP	ρ	λ	ZA-min-τ	LM1-min-τ	LM2-min-τ	LM1-supF	LM2-supF
A-big break	0.9	0.0	0.12	0.25	0.27	0.26	0.27
A-big break	0.9	0.2	0.12	0.15	0.14	0.26	0.28
A-big break	0.9	0.4	0.14	0.17	0.18	0.26	0.28
A-big break	0.9	0.5	0.13	0.16	0.17	0.25	0.28
A-big break	0.9	0.6	0.14	0.16	0.17	0.26	0.29
A-big break	0.9	0.8	0.13	0.14	0.14	0.25	0.27
A-small break	0.9	0.0	0.12	0.25	0.28	0.26	0.28
A-small break	0.9	0.2	0.00	0.23	0.23	0.25	0.27
A-small break	0.9	0.4	0.00	0.25	0.25	0.25	0.28
A-small break	0.9	0.5	0.00	0.24	0.24	0.24	0.27
A-small break	0.9	0.6	0.00	0.24	0.24	0.26	0.28
A-small break	0.9	0.8	0.00	0.22	0.23	0.24	0.26
A-big break	0.8	0.0	0.35	0.73	0.72	0.74	0.70
A-big break	0.8	0.2	0.35	0.53	0.47	0.74	0.69
A-big break	0.8	0.4	0.38	0.54	0.51	0.74	0.70
A-big break	0.8	0.5	0.37	0.53	0.50	0.72	0.70
A-big break	0.8	0.6	0.37	0.54	0.50	0.74	0.71
A-big break	0.8	0.8	0.37	0.51	0.46	0.72	0.68
A-small break	0.8	0.0	0.35	0.73	0.72	0.74	0.70
A-small break	0.8	0.2	0.00	0.61	0.59	0.69	0.64
A-small break	0.8	0.4	0.00	0.64	0.60	0.69	0.67
A-small break	0.8	0.5	0.00	0.63	0.60	0.68	0.67
A-small break	0.8	0.6	0.00	0.64	0.59	0.70	0.68
A-small break	0.8	0.8	0.00	0.60	0.59	0.68	0.63

Table 7B

Power Analysis for Break Type C

DGP	ρ	λ	ZA-min-τ	LM1-min-τ	LM2-min-τ	LM1-supF	LM2-supF
C-big break	0.9	0.0	0.11	0.14	0.15	0.12	0.15
C-big break	0.9	0.2	0.10	0.12	0.09	0.17	0.18
C-big break	0.9	0.4	0.08	0.12	0.10	0.15	0.16
C-big break	0.9	0.5	0.08	0.12	0.11	0.15	0.16
C-big break	0.9	0.6	0.09	0.13	0.12	0.15	0.17
C-big break	0.9	0.8	0.13	0.13	0.12	0.19	0.22
C-small break	0.9	0.0	0.11	0.14	0.16	0.12	0.15
C-small break	0.9	0.2	0.00	0.16	0.18	0.17	0.18
C-small break	0.9	0.4	0.00	0.22	0.18	0.15	0.16
C-small break	0.9	0.5	0.00	0.23	0.19	0.15	0.16
C-small break	0.9	0.6	0.00	0.24	0.21	0.15	0.17
C-small break	0.9	0.8	0.00	0.14	0.20	0.19	0.21
C-big break	0.8	0.0	0.32	0.46	0.53	0.44	0.45
C-big break	0.8	0.2	0.29	0.36	0.26	0.55	0.51
C-big break	0.8	0.4	0.22	0.40	0.36	0.49	0.48
C-big break	0.8	0.5	0.22	0.43	0.38	0.49	0.49
C-big break	0.8	0.6	0.24	0.44	0.40	0.50	0.51
C-big break	0.8	0.8	0.35	0.36	0.34	0.59	0.59
C-small break	0.8	0.0	0.31	0.45	0.53	0.43	0.45
C-small break	0.8	0.2	0.00	0.43	0.42	0.55	0.51
C-small break	0.8	0.4	0.00	0.52	0.41	0.49	0.48
C-small break	0.8	0.5	0.00	0.52	0.44	0.48	0.49
C-small break	0.8	0.6	0.00	0.53	0.46	0.50	0.51
C-small break	0.8	0.8	0.00	0.40	0.46	0.59	0.59

Table 8

The Nelson-Plosser Dataset: Roots and/or Breaks - Where do We Stand?

Conclusion: each series	ZA test (5%)	LM with Min t (5%)	LM1-supF (5%)	LM2-supF (5%)
No. of I(0) Processes without Break	NA	NA	3	1
No. of I(0) Processes with Break	5	4	1	2
No. of I(1) Processes without Break	6	NA	5	7
No. of I(1) Processes with Break	NA	7	2	1
Subtotal: All Processes with Breaks	5	11	3	3
Subtotal: All I(1) Processes	6	7	7	8

Table 9

Unit Root Tests in the Presence of Type A Breaks for the Nelson-Plosser Dataset

	Zivot and Andrews (1992)			LM1-min t		
SERIES	Break Date (λT)	unit root test stat (τ)	conclusion (5% sig. level)	Break Date (λT)	unit root test stat (τ)	conclusion (5% sig. level)
Real GNP	1930	-5.576 ***	I(0,B=1930)	1921	-3.256 *	I(1,B=1921)
Nominal GNP	1930	-5.824 ***	I(0,B=1930)	1922	-2.959	I(1,B=1922)
Real per capita GNP	1930	-4.606 *	I(1)	1921	-3.189 *	I(1,B=1921)
Industrial Production	1930	-5.946 ***	I(0,B=1930)	1938	-3.664 **	I(0,B=1938)
Employment	1930	-4.947 **	I(0,B=1930)	1932	-3.272 *	I(1,B=1932)
GNP deflator	1930	-4.122	I(1)	1922	-2.632	I(1,B=1922)
Consumer prices	1941	-3.100	I(1)	1917	-3.791 **	I(0,B=1917)
Nominal wages	1930	-5.302 **	I(0,B=1930)	1921	-3.462 **	I(0,B=1921)
Money stock	1930	-4.344	I(1)	1932	-3.973 ***	I(0,B=1932)
Velocity	1948	-3.353	I(1)	1894	-2.193	I(1,B=1894)
Interest rate	1933	-0.983	I(1)	1954	-1.364	I(1,B=1954)

	LM with sup F break selection						
	Break Date (λT)	F-stat for break significance	No. of significant breaks (at 5% level)	unit root test stat (τ) from LM1-supF	conclusion (5% sig. level)	unit root test stat (τ) from LM2-supF	conclusion (5% sig. level)
Real GNP	1932	10.160	0	-3.502 **	I(0)	-2.105	I(1)
Nominal GNP	1921	13.862 **	1	-2.671	I(1,B=1921)	-2.719 **	I(0,B=1921)
Real per capita GNP	1932	9.122	0	-2.664	I(1)	-2.105	I(1)
Industrial Production	1921	9.883	0	-1.873	I(1)	-1.801	I(1)
Employment	1946	8.602	0	-2.808 *	I(0)	-2.650 **	I(0)
GNP deflator	1917	15.673 ***	1	-1.919	I(1,B=1921)	-1.033	I(1,B=1921)
Consumer prices	1918	7.057	0	-2.832 *	I(0)	-1.165	I(1)
Nominal wages	1932	11.638 *	0	-2.710	I(1)	-2.142	I(1)
Money stock	1932	16.235 ***	1	-3.973 ***	I(0,B=1932)	-3.370 ***	I(0,B=1932)
Velocity	1946	5.394	0	-2.106	I(1)	-0.873	I(1)
Interest rate	1918	4.820	0	-1.460	I(1)	-1.006	I(1)

1) We define break point (TB) as the first point of new regime. Although Perron, Zivot and Andrews (1992) and others define the break point as the last period of the initial regime, our table reports results in a consistent way using our definition.

2) Significance levels are indicated as *** 1%, **5%, *10% .

Appendix I:
Summary of 1%, 5%, and 10% Critical Values
for Five Unit Root with Break Tests

Appendix Table 1

Critical Values for the Model A (Iteration: 50,000)

1% critical values

	Model A: DGP with y=y(-1)+10*d(dum_lamdba)					Model A: DGP with y=y(-1)+5*d(dum_lamdba)				
	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF
Lambda=0	-5.37	-4.15	-3.74	-3.68	-3.26	-5.37	-4.11	-3.73	-3.64	-3.22
Lambda=0.2T	-9.34	-3.96	-3.60	-3.64	-3.23	-6.24	-4.12	-3.72	-3.63	-3.21
Lambda=0.4T	-9.28	-3.92	-3.57	-3.63	-3.22	-6.22	-4.11	-3.72	-3.64	-3.21
Lambda=0.5T	-9.27	-3.95	-3.59	-3.64	-3.21	-6.21	-4.11	-3.73	-3.63	-3.22
Lambda=0.6T	-9.23	-3.92	-3.58	-3.58	-3.20	-6.22	-4.15	-3.74	-3.64	-3.26
Lambda=0.8T	-9.32	-3.94	-3.58	-3.64	-3.21	-6.21	-4.13	-3.72	-3.63	-3.22
average	-8.64	-3.97	-3.61	-3.63	-3.22	-6.08	-4.12	-3.72	-3.63	-3.22
standard deviation	1.599	0.089	0.066	0.031	0.022	0.349	0.015	0.009	0.005	0.019

5% critical values

	Model A: DGP with y=y(-1)+10*d(dum_lamdba)					Model A: DGP with y=y(-1)+5*d(dum_lamdba)				
	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF
Lambda=0	-4.82	-3.50	-3.08	-3.07	-2.65	-4.81	-3.47	-3.06	-3.05	-2.63
Lambda=0.2T	-8.00	-3.39	-3.03	-3.05	-2.63	-5.33	-3.51	-3.12	-3.06	-2.62
Lambda=0.4T	-7.85	-3.39	-3.03	-3.05	-2.63	-5.29	-3.52	-3.09	-3.04	-2.63
Lambda=0.5T	-7.91	-3.41	-3.04	-3.07	-2.64	-5.29	-3.52	-3.10	-3.06	-2.64
Lambda=0.6T	-7.92	-3.39	-3.03	-3.04	-2.62	-5.33	-3.52	-3.11	-3.06	-2.64
Lambda=0.8T	-7.93	-3.41	-3.04	-3.06	-2.64	-5.31	-3.51	-3.12	-3.06	-2.64
average	-7.40	-3.42	-3.04	-3.05	-2.64	-5.22	-3.51	-3.10	-3.05	-2.63
standard deviation	1.268	0.041	0.018	0.013	0.011	0.206	0.017	0.023	0.007	0.008

10% critical values

	Model A: DGP with y=y(-1)+10*d(dum_lamdba)					Model A: DGP with y=y(-1)+5*d(dum_lamdba)				
	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF
Lambda=0	-4.52	-3.17	-2.73	-2.78	-2.35	-4.52	-3.16	-2.72	-2.76	-2.34
Lambda=0.2T	-7.29	-3.13	-2.76	-2.76	-2.34	-4.92	-3.20	-2.80	-2.77	-2.34
Lambda=0.4T	-7.12	-3.12	-2.75	-2.77	-2.34	-4.87	-3.20	-2.77	-2.76	-2.33
Lambda=0.5T	-7.17	-3.14	-2.76	-2.78	-2.35	-4.88	-3.21	-2.77	-2.77	-2.35
Lambda=0.6T	-7.21	-3.13	-2.75	-2.76	-2.33	-4.92	-3.21	-2.78	-2.77	-2.35
Lambda=0.8T	-7.24	-3.14	-2.77	-2.78	-2.35	-4.90	-3.20	-2.80	-2.77	-2.35
average	-6.76	-3.14	-2.76	-2.77	-2.34	-4.83	-3.19	-2.77	-2.77	-2.34
standard deviation	1.099	0.019	0.014	0.009	0.010	0.155	0.019	0.028	0.005	0.006

Appendix Table 2

Critical Values for the Model C (Iteration: 50,000)

1% critical values

	y=0.2+0.2*tdum_lamdba+10*dum_lamdba+e, e=e(-1)+nrnd					y=0.2+0.1*tdum_lamdba+5*dum_lamdba+e, e=e(-1)+nrnd				
	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF
Lambda=0	-5.69	-4.98	-4.38	-4.19	-3.61	-5.70	-4.97	-4.38	-4.17	-3.62
Lambda=0.2T	-12.77	-4.63	-4.08	-4.13	-3.57	-7.76	-4.91	-4.38	-4.14	-3.60
Lambda=0.4T	-16.38	-4.32	-3.77	-4.25	-3.65	-9.41	-4.77	-4.24	-4.27	-3.64
Lambda=0.5T	-17.98	-4.29	-3.74	-4.26	-3.65	-10.07	-4.70	-4.19	-4.28	-3.66
Lambda=0.6T	-19.32	-4.29	-3.73	-4.28	-3.66	-10.65	-4.63	-4.14	-4.26	-3.64
Lambda=0.8T	-20.39	-4.48	-3.86	-4.11	-3.57	-11.12	-4.67	-4.19	-4.13	-3.57
average	-15.42	-4.50	-3.93	-4.21	-3.62	-9.12	-4.77	-4.25	-4.21	-3.62
standard deviation	5.459	0.270	0.258	0.070	0.039	2.044	0.136	0.102	0.070	0.035

5% critical values

	y=0.2+0.2*tdum_lamdba+10*dum_lamdba+e, e=e(-1)+nrnd					y=0.2+0.1*tdum_lamdba+5*dum_lamdba+e, e=e(-1)+nrnd				
	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF
Lambda=0	-5.12	-4.40	-3.82	-3.61	-3.03	-5.13	-4.39	-3.81	-3.60	-3.02
Lambda=0.2T	-10.98	-4.17	-3.60	-3.53	-2.99	-6.58	-4.33	-3.80	-3.53	-2.98
Lambda=0.4T	-14.40	-3.85	-3.30	-3.70	-3.07	-8.14	-4.24	-3.68	-3.69	-3.07
Lambda=0.5T	-15.79	-3.81	-3.26	-3.71	-3.07	-8.76	-4.19	-3.64	-3.72	-3.08
Lambda=0.6T	-17.01	-3.80	-3.26	-3.70	-3.07	-9.26	-4.15	-3.62	-3.69	-3.08
Lambda=0.8T	-17.75	-4.19	-3.51	-3.52	-2.97	-9.51	-4.21	-3.71	-3.52	-2.98
average	-13.51	-4.04	-3.46	-3.63	-3.04	-7.90	-4.25	-3.71	-3.62	-3.03
standard deviation	4.753	0.250	0.226	0.088	0.046	1.715	0.091	0.080	0.086	0.047

10% critical values

	y=0.2+0.2*tdum_lamdba+10*dum_lamdba+e, e=e(-1)+nrnd					y=0.2+0.1*tdum_lamdba+5*dum_lamdba+e, e=e(-1)+nrnd				
	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF	1-ZA-min t	2-LM1-min t	3-LM2-min t	4-LM1-supF	5-LM2-supF
Lambda=0	-4.83	-4.11	-3.52	-3.31	-2.73	-4.83	-4.10	-3.51	-3.31	-2.73
Lambda=0.2T	-10.05	-3.95	-3.37	-3.23	-2.69	-5.97	-4.05	-3.51	-3.23	-2.69
Lambda=0.4T	-13.33	-3.63	-3.07	-3.42	-2.78	-7.51	-3.97	-3.40	-3.41	-2.78
Lambda=0.5T	-14.65	-3.59	-3.04	-3.43	-2.78	-8.10	-3.94	-3.37	-3.44	-2.79
Lambda=0.6T	-15.77	-3.58	-3.05	-3.42	-2.79	-8.55	-3.91	-3.36	-3.41	-2.77
Lambda=0.8T	-16.35	-4.04	-3.36	-3.22	-2.67	-8.69	-3.99	-3.46	-3.23	-2.68
average	-12.50	-3.82	-3.23	-3.34	-2.74	-7.28	-3.99	-3.43	-3.34	-2.74
standard deviation	4.372	0.244	0.207	0.097	0.050	1.552	0.072	0.066	0.096	0.046

Endnotes

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[1] The ZA approach is only one of several in the literature on selecting break points. For example, Perron (1997) used the maximum t statistic (over all possible break dates) on the level break dummy or the growth break dummy to identify the break point. Nunes, Newbold and Kuan (1997) and Lee and Strazicich (2001) use the Schwarz Bayesian Criterion (SBC) in an ADF-style testing equation to select the break points. However, this method depends on nuisance parameters indicating the magnitude and date of break under the null hypothesis. Maddala and Kim (1996) explore the use of SBC for selecting the number of breaks, regardless of the stationarity of regressors.

[2] Lumsdaine-Papell (1997) extend ZA to allow for two breaks at unknown dates and use the min-τ criterion to choose the break dates. LS (2003) use the min-τ criterion in their LM unit root tests with two unknown break dates. Cuddington-Nishioka (2005b) extend the LM-supF tests to two breaks at unknown dates.

[3] The literature also considers the possibility of multiple breaks. The approach proposed in this paper for LM unit root tests can easily be generalized to two or more breaks.

[4] Note that in our context where all regression specifications for break A or C models have the same number of parameters, the use of the supF criterion yields the same results as the SBC or Akaike criterion. What is critical is that these criteria are applied to the first-difference specification.

[5] Throughout this paper, we will use the term 'detrending' to refer to removing all deterministic components of the time series, including the mean, trend, and segments with differing means and/or trends reflecting 'breaks'.

[6] Moreover, it is easy to generalize the analysis to consider possible breaks of unknown type. See Cuddington-Nishioka (2005a). The existing literature, in contrast, assumes that the timing of the break may be unknown but the investigator has *a priori* information about whether the break is a level shift (Perron's Model A) or a level and growth rate shift (Perron's Model C).

[7] Benergee et al (1992) use 15% trimming and Lee-Strazicich (2003) use 10% trimming for obtaining critical values. Perron (1997) show that ZA is valid without any trimming at the end points (although critical values for the unit root test depend on sample size and trimming selected).

[8] It is easy to show that x_T=0 as well; i.e., the detrended series equal the actual series for both the first and the last observation.

[9] Amsler-Lee (1995) test model A by using critical values from Schmidt and Phillips (1992) after demonstrating that the critical values in their specification are invariant to existence of a level-shift break. They show that Model C is not invariant.

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[10] Alternatively, one could measure mean squared deviations from the λ=0 case in the first column.

[11] Cuddington-Nishioka (2005a) generalizes the analysis to select the type of break (A, C or none).

[12] To see that the λ invariance property does not hold for the ZA test, look across row 1 or row 2 in Table 5. The critical values increase dramatically as λ increases, especially when the break is big. The huge range of critical values in the table cell range C1-H2 clearly indicates that the ZA test is neither λ nor *d* invariant. The size of the break (*d*) is only irrelevant in column C where there is, in fact, no break! (This *d*-invariance is indicated by shading the column average in yellow. The difference between C1 and C2 is merely sampling variation. It gives some idea of how precise our estimated critical values are when our simulations are based on 50,000 iterations.)

[13] Amsler and Lee (1995) and Lee and Strazicich (2003) showed that the invariance property holds *asymptotically* in the case of Model A, but not Model C, for the LM-AL test. Our simulations use sample size of 100 and the critical values are based on 50,000 replications.

[14] Amsler-Lee emphasized that the invariance property does not strictly hold for their LM test in the case of Model C.

[15] The Nelson and Plosser data were obtained from *the Journal of Business and Economic Statistics* Web Site: January 1994 p106 Koop and Steel: all series are for U.S., annual frequency, and in natural logs (except for the interest rate).