Homework #11

From the textbook, problems 11.4, 11.8, 11.14.

Note: for 11.14, in the 3rd edition the problem references Fig. 11.29; in the 2nd edition the same figure is 11.31.
Problems

11.1. Yeast cells are recovered from a fermentation broth by using a tubular centrifuge. Sixty percent of the cells are recovered at a flow rate of 12 l/min with a rotational speed of 4000 rpm. Recovery is inversely proportional to flow rate.
   a. To increase the recovery of cells to 95% at the same flow rate, what should be the rpm of the centrifuge?
   b. At a constant rpm of 4000 rpm, what should be the flow rate to result in 95% cell recovery?

11.2. Gentamycin crystals are filtered through a small test filter medium with a negligible resistance. The following data were obtained:

<table>
<thead>
<tr>
<th>t (sec)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (l)</td>
<td>0.6</td>
<td>0.78</td>
<td>0.95</td>
<td>1.1</td>
</tr>
</tbody>
</table>

The pressure drop in this test run was 1.8 times that when water was used with a filter area of 100 cm². The concentration of gentamycin in solution is 5 g/l. How long would it take to filter 5000 l of gentamycin solution through a filter of 1.5 m², assuming the pressure drop is constant and μ = 1.2 centipoise?

11.3. Streptomycin is extracted from the fermentation broth using an organic solvent in a countercurrent staged extraction unit. The distribution coefficient of streptomycin at pH = 4 is \( K_D = Y/I_X = 40 \), and the flow rate of the aqueous (H) phase is \( H = 150 \) l/min. If only five extraction units are available to reduce the streptomycin concentration from 10 g/l in the aqueous phase to 0.2 g/l, determine the required flow rate of the organic phase (L) in the extraction unit.

11.4. A new antibiotic is separated from a fermentation broth by adsorption on resin beads in a fixed bed. The bed is 5 cm in diameter and contains 0.75 cm³ resin/cm bed. The overall mass transfer coefficient is 12 h⁻¹. If the antibiotic concentration in the feed is 4 g/l and is desired to be 0.1 g/l in the effluent in a 50 cm column, determine the liquid flow rate through the column. The equilibrium relationship is \( C_S^* = 20(C_L^*)^{0.5} \), and the operating-line relationship is \( C_S = 5C_L \), where \( C_S \) is g solute/l resin and \( C_L \) is g solute/l solution.

11.5. An activated carbon adsorption column is used for removal of phenolic compounds from wastewater. The feed and desired effluent adsorbate concentrations are \( C_o = 100 \) mg/l
and \( C_e = 1 \text{ mg/l} \), and the flow rate is \( F = 10 \text{ m}^3/\text{h} \). Adsorption column contains 3 m³ activated carbon with 30% void fraction and the column diameter is \( D_o = 0.5 \text{ m} \). Adsorption capacity and rate for activated carbon are \( N_o = 60 \text{ kg adsorbate/m}^3 \text{ bed and } K = 0.8 \text{ m}^3/\text{kg-h} \).

a. Determine the adsorption column height and service time.

b. Using the same column characteristics, determine column height for 10 days of operation.

11.6. Refractory organic compounds are removed from aqueous phase by using an activated carbon adsorption column. For the feed flow rate of 5 m³/h and the feed and effluent organics concentrations of 40 and 0.5 mg/L, the column can be operated for 3 days when column height was 3 m. The same column is operated for 6 days when \( H = 5 \text{ m} \). \( D_o = 0.6 \text{ m} \).

a. Determine adsorption capacity \( (N_o) \) and the rate constant \( (K) \) for the activated carbon adsorption column.

b. For the same column characteristics, determine the required column height when the flow rate doubles for 6 days of operation.

c. Determine the volume and the weight of required AC if void fraction is 35% and density of AC is 2 g/cm³ for the column in part (b).

11.7. In a cross-flow ultrafiltration unit, a protein of \( \text{MW} = 3 \times 10^5 \) da is separated from the fermentation broth by using a UF membrane. The flow rate of liquid through a tube of diameter \( d = 2 \text{ cm} \) and length \( L = 50 \text{ cm} \) is \( Q = 2 \text{ L/min} \). The flow regime is turbulent, \( f = 0.0005 \), and \( C_4 = 2 \text{ (atm (s/cm)²)} \). The inlet pressure is \( P_i = 2 \text{ atm} \). Protein concentrations in the solution and on gel film are \( C_B = 30 \text{ mg/l} \) and \( C_G = 100 \text{ g/l} \), respectively.

a. Determine the exit pressure \( (P_0) \).

b. Determine the transmembrane pressure drop \( (\Delta P_M) \).

c. If the mass transfer coefficient \( (k) \) for protein flux is \( k = 5 \text{ cm/s} \), determine the flux of liquid through the UF membrane \( (J) \).

d. If the resistance of the filter is \( R_M = 0.002 \text{ atm/cm}^2 \cdot \text{s/cm}^2 \), determine the cake resistance, \( R_C \).

11.8. Components A and B of a binary mixture are to be separated in a chromatographic column. The adsorption isotherms of these compounds are given by

\[
m_A = f_A(c) = \frac{k_1 C_A}{k_1 + C_A} \quad \text{and} \quad m_B = f_B(c) = \frac{k_2' C_B}{k_2' + C_B}
\]

where \( k_1 = 0.2 \text{ mg solute A absorbed/mg adsorbent} \)
\( k_2' = 0.1 \text{ mg solute/ml liquid} \)
\( k_2' = 0.05 \text{ mg solute B adsorbed/mg adsorbent} \)
\( k_2 = 0.02 \text{ mg solute/ml liquid} \)

The bed contains 3 g of very fine support particles. The bed porosity is \( \varepsilon = 0.35 \), and the cross-sectional area of the bed is \( A = 6 \text{ cm}^2 \). The volume of the mixture added is \( \Delta V = 50 \text{ ml} \).

a. Determine the position of each band A and B in the column, \( L_A \) and \( L_B \) (or \( \Delta X_A \) and \( \Delta X_B \)).

b. Determine \( L_A/L_B ; R_{fa} = L_A/L_v ; R_{fb} = L_B/L_v \) when \( C_A = 10^{-1} \text{ mg/ml} \) and \( C_B = 0.05 \text{ mg/ml} \)
in liquid phase at equilibrium.

11.9. Consider the use of gel chromatography to separate two proteins, A and B. The partition coefficient \( (K_p) \) for A is 0.5 and for B is 0.15. The void volume in the column, \( V_v \), is
20 cm³. The void volume within the gel particles, \( V_v \), is 30 cm³. The total volume of the column is 60 cm³. The flow rate of eluent is 100 cm³/h. Ignoring dispersion and other effects, how long will it take for A to exit the column? How long for B?

11.10. Biomass present in a fermentation broth is to be separated by vacuum filtration. Following are the filter and broth characteristics:

\[ A = 50 \text{ m}^2, \quad \Delta P = 0.01 \text{ N/m}^2, \quad C = 15 \text{ kg/m}^3 \]

\[ \mu = 0.003 \text{ kg/m-s}, \quad \alpha = 2 \text{ m/kg} \]

a. If rate of filtration has a constant value of \( dV/dt = 50 \text{ l/min} \), determine the cake and filter resistances at \( t = 30 \text{ min} \).

b. Determine the filter surface area (\( A \)) required to filter 5000 l broth within 60 min with the same pressure drop across the filter.

11.11. A fermentation broth with a protein concentration of \( C_0 = 100 \text{ mg/l} \) and flow rate of \( Q = 4 \text{ m}^3/\text{h} \) is passed through two downflow adsorption columns connected in series. The adsorption isotherm is \( q = 4C^{0.4} \). Assume the system is in equilibrium.

a. Calculate the minimum amount of activated carbon required for two days of operation if removal efficiency for the column is \( E = 50\% \).

b. Protein concentration in the effluent of the second column is desired to be \( C = 0.5 \text{ mg/l} \). Determine the minimum amount of activated carbon required for two days of operation.

11.12. In a cross-flow ultrafiltration system used for filtration of proteins from a fermentation broth, gel resistance increases with protein concentration according to the following equation:

\[ R_a = 0.5 + 0.01(C), \text{ where } C \text{ is in mg/l}. \]

Pressure at the entrance of the system is \( P_1 = 6 \text{ atm} \) and at the exit is \( P_0 = 2 \text{ atm} \). The shell side of the filter is open to the atmosphere, resulting in \( P_f = 1 \text{ atm} \). The membrane resistance is \( R_M = 0.5 \text{ atm/(mg/m}^2\text{-h)} \), and protein concentration in the broth is \( C = 100 \text{ mg/l} \).

a. Determine the pressure drop across the membrane.

b. Determine the filtration flux.

c. Calculate the rejection coefficient of the membrane for effluent protein concentration of \( C_f = 5 \text{ mg/l} \).

11.13. A solute protein is to be separated from a liquid phase in a chromatographic column. The adsorption isotherm is given by

\[ C_s = kC_L^2 \]

where \( C_s \) is the solute concentration in solid phase (mg solute/mg adsorbent) and \( C_L \) is the liquid phase concentration of solute (mg solute/ml liquid). Use the following information:

\[ k = 0.4, \quad \varepsilon = 0.3, \quad A = 25 \text{ cm}^2, \]

\[ M = 10 \text{ g ads/100 ml column} = 100 \text{ mg/ml} \]

a. For \( V = 400 \text{ ml} \) and \( X = 25 \text{ cm} \), determine the equilibrium solute concentrations in liquid and solid phases.

b. Determine the ratio of travel distances of solute to solvent, \( R_f \).
11.14. Proteins A and B are to be separated using ion-exchange chromatography. When using a superficial fluid velocity of 30 cm/h, peak A exits with \( y_{\text{max}} \) at 50 min and peak B exits with \( y_{\text{max}} \) at 100 min. For peak A, the standard deviation is 12 min, and for peak B, 18 min.

a. Assuming that Taylor dispersion controls and that \( v \) is increased to 60 cm/h, what will be the values for the peak elution times and standard deviations for A and B?

b. For \( v = 60 \) cm/h, sketch the two peaks and calculate the resolution of the two peaks \( (R_s) \). (You may find Figure 11.29 to provide helpful guidance).

c. Would you recommend increasing \( v \) to 60 cm/h to increase throughput? Why or why not?

11.15. Consider the scale-up of a chromatography column for purification of a protein. A column of 40 cm length is used with a superficial velocity of 40 cm/h. The peak concentration of the target protein exits at a time of 100 min. The standard deviation of the peak is 14 min.

a. How long must you wait to collect 90% of the protein?

b. If the same column is used, but velocity is increased to 60 cm/h and external or Taylor dispersion controls, what will be the value of \( \sigma \)?

c. If the column is lengthened to 60 cm while the velocity is at 60 cm/h, how will \( \sigma \) change? Will the peak be sharper or broader than at 40 cm/h with a 40 cm column?