Consider a 1,000 L chemostat in which biomass is being produced with glucose as the substrate. The microbial system follows a Monod relationship with an unknown $\mu_m$, but we’re pretty sure \( K_s \) is 0.5 \( \text{g/L} \). When running with a feed of 100 L/h and 10 \( \text{g/L} \) substrate the reactor has 0.5 \( \text{g/L} \) residual substrate & 5 \( \text{g/L} \) biomass.

1. (3 points) What is the value of $\mu_m$?

2. (4 points) What is the value of the maximum yield of biomass from substrate, $Y_{X/S}$?

3. (2 points) You decide to recycle 10 L/h of the effluent after concentrating the cell mass by a factor of 3. Will the residual substrate be greater or less than what you have without the recycle?
Solution

1. For a non-recycled chemostat:

\[
\mu_g = \frac{D}{V} = \frac{100 \text{ L/h}}{1000 \text{ L}} = 0.1 \text{ h}^{-1}
\]

\[
\mu_g = \frac{\mu_m S}{K_s + S} \implies \mu_m = \frac{(0.1)(0.5 + 0.5)}{0.5} = 0.2 \text{ h}^{-1}
\]

2. Also for a non-recycled chemostat:

\[
\frac{D(S_0 - S)}{Y_{X/S}} = \frac{\mu_g}{Y_{X/S}} \implies Y_{X/S} = \frac{(0.1)(5)}{(0.1)(10 - 0.5)} = 0.53 \text{ g biomass/substrate}
\]

3. Because you've increased the biomass concentration the substrate will be more greatly converted, so its residual concentration will be less.

\[
\mu_g = D\left[1 + \alpha(1-C)\right] = (0.1 \text{ h}^{-1})\left[1 + \left(\frac{10 \text{ L/h}}{100 \text{ L/h}}\right)(1-3)\right]
\]

\[
= 0.08 \text{ h}^{-1}
\]

\[
S = \frac{\mu_g K_s}{\mu_m - \mu_g} = \frac{(0.08 \text{ h}^{-1})(0.5 \text{ g/L})}{(0.2 \text{ h}^{-1}) - (0.08 \text{ h}^{-1})} = 0.33 \text{ g/L}
\]