Shock wave propagation along constant sloped ocean bottoms

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The nonlinear progressive wave equation (NPE) is a time-domain model used to calculate long-range shock propagation using a wave-following computational domain. Current models are capable of treating smoothly spatially varying medium properties, and fluid-fluid interfaces that align horizontally with a computational grid that can be handled by enforcing appropriate interface conditions. However, sloping interfaces that do not align with a horizontal grid present a computational challenge as application of interface conditions to vertical contacts is non-trivial. In this work, range-dependent environments, characterized by sloping bathymetry, are treated using a rotated coordinate system approach where the irregular interface is aligned with the coordinate axes. The coordinate rotation does not change the governing equation due to the narrow-angle assumption adopted in its derivation, but care is taken with applying initial, interface, and boundary conditions. Additionally, sound pressure level influences on nonlinear steepening for range-independent and range-dependent domains are used to quantify the pressures for which linear acoustic models suffice. A study is also performed to investigate the effects of thin sediment layers on the propagation of blast waves generated by explosives buried beneath mud line.

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I. INTRODUCTION

In order to simulate weakly nonlinear shock wave propagation, the nonlinear progressive wave equation (NPE) was first developed by McDonald and Kuperman as a time-domain counterpart of the frequency-domain parabolic wave equation (PE), but that was capable of describing broadband linear and nonlinear propagation. The NPE is well suited for long-range underwater acoustic pulse propagation because upon implementation it provides a good balance of accuracy and efficiency. A critical aspect missing from the original NPE development is the accurate treatment of range-dependent sloping seafloors.

In underwater acoustics, a range-dependent environment is defined as one whose geoaoustic properties vary with range, and perhaps the most physically interesting case of range-dependence is non-flat ocean bottoms where the interface between the fluid and the sediment represents a sloping discontinuity. The NPE, like most time-domain, finite-difference (FD) models, is capable of having local medium properties specified at every grid point in the domain—thus allowing for treatment of depth and range dependence. However, discontinuities or jumps in properties between adjacent grid points often lead to numerical inaccuracies and instability. A brief discussion is provided of solutions to this problem offered from two different viewpoints: the seismology community (time-domain, FD models for elastic wave propagation) and the underwater acoustics community (frequency-domain models for fluid and elastic wave propagation).

Wave propagation through heterogeneous media, media whose properties vary spatially, has been studied in much detail in the field of seismology. Mishra et al. explored FD approximations to the wave equation with rough coefficients (non-smooth variation in space) and found the rate of convergence of the numerical approximations relies on the relative smoothness of medium properties. Sharp contrasts in properties such as density and sound speed necessitate the use of extremely fine grids—an impractical task for most long-range propagation problems. The accuracy of FD schemes is also dependent on whether interfaces align with the computational grid. Numerical experiments performed by Zahradnik revealed greater inaccuracies developed at diagonal discontinuities. Various methods have been employed to handle these interfaces. For cases where the interface is continuous and coincides with the grid, simple interface conditions can be applied. If the interface does not coincide with the grid, the coordinate system can be mapped so the grid aligns with a smooth surface. Finally, if the interfaces are discontinuous and highly sporadic throughout the domain, a method such as property averaging can be used to smooth out discontinuities across many grid points.

Heterogeneous methods that use the same formulas for all grid points, i.e., do not use interface conditions, are rarely

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seen in underwater acoustics. Ocean environments typically comprise of a water column atop layers of sediment, so applying interface conditions is possible. However, seafloors are often non-flat, with varying topographies that can significantly affect wave propagation, especially for shallow water environments. It is important to note the typical slope angles found in ocean bathymetry are generally less than 10°. Although these angles are small, they can greatly affect the wave propagation at long ranges and therefore cannot be disregarded. Traditionally, range-dependent environments are treated using either range discretization methods, coordinate transformation methods, or a combination of both. One range discretization method known as the stair-stepping approach splits the domain into a series of range-independent regions and then matches interface conditions between range-independent regions. This method has been successfully implemented with the PE solution. Coordinate transformation approaches consist of at least two main methods, terrain-following coordinates (also known as mapping solutions) and coordinate rotation solutions. Terrain-following coordinates are implemented by shifting the z-coordinate to match the ground height, \( h(x) \), which effectively shifts the computational domain up and down with the terrain surface. This approach is well suited for environments with rigid bottoms but is ineffective for most ocean acoustics problems where penetrable bottoms are present, as errors develop at the fluid-bottom interface. The rotated coordinate approach rotates the computational domain so that its axes are aligned with the sloped bottom; as a result the ocean surface becomes sloped and is approximated as a series of stair steps. The rotated coordinate approach has been successfully applied to PE models for both constant and variably sloping ocean bottoms.

Knowledge gained from the seismology and underwater acoustics communities offers great insight as to the steps needed to enhance the current NPE model to accurately treat range-dependence. In the original NPE model, it is possible to introduce range-dependence by specifying local geoacoustic properties at each grid point. However, even gradual changes in medium properties in the range direction leads to unacceptable error due to the wave-following feature of the NPE. McDonald and Piacsek introduced a correction term, referred to as a “stratification” term, to accommodate smoothly varying density and sound speed profiles in the range direction, but this technique can only treat smoothly varying medium properties and not the discontinuous interfaces seen in sloping ocean bottoms. If the changes in properties are considered discontinuous jumps, then a range discretization method can be employed, but such methods are not easily applied to the NPE because vertical interface conditions are difficult to enforce in the wave-following domain. This leaves two possible options from the previous discussion on range-dependence: average the local medium properties to smooth out any discontinuity, or apply a coordinate transformation to the computational grid and enforce interface conditions. The latter option provides greater accuracy, and it has previously been implemented in PE models with great success. Also, the narrow-angle assumption used in the derivation of the NPE presumes that all energy propagates at a narrow angle to the range direction; this implies that, for sufficiently small slope angles, the original nonlinear progressive wave equation will suffice.

The main goal of this work is to enhance current NPE code implementations to be able to treat more realistic ocean environments. The NPE model is adapted to treat constant sloped ocean bottoms using a rotated coordinates approach for which linear solutions will be compared to a Fourier-transformed PE model. The model is then used to study two problems: The effect of range-dependence on nonlinear steepening, and the effect of thin sediment layers on shock propagation in both range-dependent and range-independent environments.

II. THE NONLINEAR PROGRESSIVE WAVE EQUATION

The NPE is derived by taking Euler’s fluid dynamical equations and combining them, using the adiabatic equation of state, into an equation of a single unknown dependent variable. This equation is useful in underwater acoustics because it allows for efficient computation, in particular when considering long-range propagation (greater than 1 km). The NPE is highly applicable because it is capable of describing several different physical effects of propagation such as diffraction, refraction, and nonlinear steepening. The approach presented here is taken from previous work in which the NPE is derived in terms of a small scaling variable \( \varepsilon \). It is assumed that the background medium velocity is zero and that the pulse is at a sufficiently far distance from the source (a distance where pressures may be assumed to have “acoustic” values). The NPE in Cartesian coordinates is given by

\[
\frac{\partial R}{\partial t} = -\frac{\partial}{\partial x} \left( c_1 R + \frac{\beta c_0}{2} \frac{\partial R}{\partial x} \right) - \frac{c_0}{2} \int_{y}^{x} \nabla^2 R \, dx, \tag{1}
\]

where \( R = \rho' / \rho_0 \) for an acoustic density fluctuation \( \rho' \). \( \rho_0 \) is the constant ambient density in the medium, and the sound speed \( c = c_0 + c_1(x, y, z) \). The lower limit of the integral, \( x_f \), is taken to be a point ahead of the pulse where \( R = \partial_x R = 0 \). The constant \( \beta \) is dimensionless and is known as the coefficient of nonlinearity with a value of 3.5 for the ocean. Here \( c_1 \) is a small local difference from the background constant sound speed \( c_0 \), \( \nabla^2_x = \partial_x^2 + \partial_y^2 \) is the transverse Laplacian, and the time derivative in a wave-tracking frame is defined using the material derivative:

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + c_0 \frac{\partial}{\partial x}. \tag{2}
\]

The standard NPE is written in terms of a dimensionless density perturbation, \( R \), which is not useful when applying boundary conditions. It is possible to reformulate the NPE in terms of a dimensionless pressure variable \( Q = \rho' / \rho_0 \varepsilon^2 \) (Ref. 14):

\[
D_t Q = -\partial_x \left[ c_1 Q + \frac{\beta c_0}{2} Q^2 \right] - \frac{c_0}{2} \int_{y}^{x} \left( \partial_y^2 + \partial_z^2 \right) Q \, dx. \tag{3}
\]

This form of the equation is no different than the original formulation. The reason for this is because \( Q = R + O(\varepsilon^2) \), and substitution of this term into (1) leads to an error of size...
wavespeed approach is nontrivial and therefore not consid-
ered to attenuate all energy. The NPE is not complex-
ical wave speed can be increased near the bottom of the
dispersive sediments. The imaginary component of the
frequency-domain solutions use a complex wavespeed to
energy propagating to the base of the computational domain.

In this effort, sediment layers in the ocean bottom are
treated as fluids with densities and sound speeds that differ
from those of the water column. Two interface conditions that
need to be satisfied at the fluid-fluid interface at the ocean bot-
tom are continuity of pressure and continuity of normal particle
speed. Mathematically, these conditions are given by

\[ Q_w = Q_b \quad \text{and} \quad \frac{1}{\rho_w} \frac{\partial Q_w}{\partial z} = \frac{1}{\rho_b} \frac{\partial Q_b}{\partial z}, \]

where the subscripts \( w \) and \( b \) denote the water column and bottom sediment.

The following paragraphs provide details on numerical
procedures used to evaluate the NPE. Results of this model
are compared to those of a fluid PE solution for a range-
dependent case; the nonlinear effects of the NPE are sup-
pressed. An implementation such as this has previously been
benchmarked against the PE in the frequency-domain, but
comparisons in the time-domain have been chosen for this
paper to better illustrate pulse and shock propagation.

The NPE is implemented using two types of numerical
schemes: FD methods are used for linear terms, and a flux-
corrected transport (FCT) algorithm for nonlinear terms.
Such an algorithm requires that for each time step, at each
grid point, a flux limiter be calculated and used to refine the
nonlinear behavior of the model. The numerical scheme
implemented in this work is a combination of the two numeri-
cal methods (FD and FCT) into a Crank-Nicolson approach.

A penetrable ocean bottom allows for the possibility of
energy propagating to the base of the computational domain
and causing artificial reflections. An artificial absorbing layer
is used to damp out any energy near the bottom of the com-
putational grid to prevent these unrealistic reflections. Most
frequency-domain solutions use a complex wavespeed to
attenuate energy in the seafloor, which is an effective model
for dispersive sediments. The imaginary component of the
complex wave speed can be increased near the bottom of the
domain to attenuate all energy. The NPE is not complex-
valued and is solved in the time-domain, and so the complex
wavespeed approach is nontrivial and therefore not consid-
ered here. The absorbing layer used here is implemented
numerically by adding a \( -R/\tau \) term to Eq. (1) [or, equiva-
ently, adding a \( -Q/\tau \) term to Eq. (3)]. The numerical damping
coefficient \( \tau \) varies as a function of depth and is defined
solely in the artificial absorbing region. Future work will
consider a more physically realistic bottom model.

The NPE solution is benchmarked against a parabolic equa-
tion solution for a linear, range-independent case. The parabolic
equation model used is the Range-dependent Acoustic Model
(RAM) developed at the Naval Research Laboratory by
Michael Collins. It is a linear, frequency-domain solution
known for producing accurate results for both range-
dependent and range-dependent environments. As RAM is a
frequency-domain model, a Fourier transform is required for
solutions in the time-domain, using a discrete Fourier synthesis.

The environment for comparisons is the Pekeris waves-
guide with a water column depth of 100 m and an additional
100 m of sediment depth. The water column has a density of
1000 kg/m\(^3\) and a sound speed of 1500 m/s, whereas the sedi-
ment has a density of 1500 kg/m\(^3\) and a sound speed of
1650 m/s. The source is located at \( z_s = 50 \) m and emits a
pulse described by

\[ Q(t) = \begin{cases} 
\sin(\omega_c t) - \frac{1}{2} \sin(2\omega_c t) & \text{for } 0 < t < 1/f_c \\
0 & \text{otherwise,} \end{cases} \]

where the center frequency \( f_c \) is chosen to be 100 Hz. This
problem is analyzed in a moving window of depth 200 m and
width 90 m. The x-direction grid spacing is 0.225 m, the
z-direction grid spacing is 1 m, and the time step is 0.0005 s.
Only the linear case is considered and as such the coefficient of
nonlinearity \( \beta \) is set to zero.

To benchmark the NPE solution, the Fourier-transformed
PE solution is used to generate the initial waveform for the
NPE code, and the simulation marches the domain out to a
final range of 1000 m. The pressure data is saved at four differ-
ent receiver locations: 500 m and 1000 m in range (both at
20 m and 50 m in depth). The time series data at these receiver
locations is then compared to those of the Fourier-transformed
RAM solution. These comparisons are detailed in the plots of
Fig. 1 which show that the PE (solid curve) and NPE (dashed
curve) solutions agree very well. This indicates that the pene-
trable bottom NPE model is correct and can be compared to
the Fourier-transformed PE solution for linear cases.

III. SLOPED OCEAN BATHYMETRY

Most ocean environments do not have perfectly flat sea-
floors. In fact, while many ocean bottoms appear to be flat,
they often are sloped to a small degree, and in shallow
waters, even a small slope can have a significant effect on
acoustic pulse propagation over long ranges. A critical as-
pect missing from the NPE model has been efficient treat-
ment of range-dependent ocean bathymetries. This issue is
addressed in this work by rotating the coordinate system in
the standard NPE model to align with the slope of the sea-
floor characterized by constant-slope ocean bottoms. Results
of this rotated solution are then compared to those of Fourier
synthesized parabolic equation solutions for select range-
dependent environments. The topography of the seafloor is
assumed to be a constant sloped surface with a slope angle \( \theta \)
positive for upslope propagation and negative for downslope
propagation. The standard domain defined in the \( xz \)-plane
is rotated by the slope angle \( \theta \) so that the \( x \) axis runs parallel to
the sloped ocean bottom. This is achieved with the coordi-
nate transformation given by

\[ x' = x \cos \theta - z \sin \theta, \quad z' = x \sin \theta + z \cos \theta. \]
\[
\begin{aligned}
\begin{bmatrix}
\tilde{x} \\
\tilde{z}
\end{bmatrix} &=
\begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
z
\end{bmatrix},
\end{aligned}
\]  

(6)

where \(\tilde{x}\) and \(\tilde{z}\) are the tangential and normal coordinates relative to the ocean bottom, respectively. This coordinate transformation is applied to the standard NPE derivation used by McDonald and Kuperman, and doing so yields an equation which is exactly the same form as the non-rotated NPE. This is because the Laplacian operator does not change with coordinate rotation, and the narrow-angle assumption was made in the plane wave velocity equation [Eq. (3) of McDonald and Kuperman]. The narrow-angle assumption, which assumes that energy propagates at a small angle to the \(\tilde{x}\) direction, allows the propagation direction to be specified along the sloping interface. If the slope angle is greater than approximately 10°, the narrow-angle assumption is violated and trigonometric correction terms need to be applied.

The governing equation may remain the same for the medium, but the initial condition and boundary and interface conditions must be transformed to the rotated coordinate system. If the initial condition is a plane wave, then it is rotated, but no rotation is needed for the spherical wave case as it is axisymmetric. The coordinate rotation results in a flat ocean-bottom interface which allows for explicit application of the interface conditions. The ocean surface becomes a sloped line, but the Dirichlet pressure release boundary condition is easily imposed imposed there discretely.

Numerically, the NPE is implemented in the manner described in the two previous sections, but now special consideration is taken when applying the upper boundary condition. The sloping ocean surface is approximated as a series of range-independent stair steps, and the pressure release boundary condition \(Q = 0\) is applied to the horizontal areas of the steps. As the lengths of the vertical areas of the steps become small, \(|Q|\) becomes small in these areas by continuity. This means, for small slope angles, conditions at the vertical interfaces of the steps need not be explicitly imposed. However, the NPE model uses a moving computational domain that follows the ocean bottom, so the sloped upper boundary will shift vertically as the domain traverses along the waveguide. The amount the boundary moves vertically, the domain’s change in height, is \(\Delta H = c_0 n \tan \theta\), where \(n\) is the time step. Figure 2 shows how the upper boundary is handled numerically. For every time step, the equation of the sloping line is calculated and then the routine loops through every grid point and determines if the centroid of the grid point lies above or below the line. If the centroid lies above the line, then the pressure value of that grid point is set equal to zero. There are slight differences in the numerical scheme for upslope and downslope propagation which are outlined in the paragraphs below.

In the upslope propagation case, the computational domain is initially fully “submerged” in the ocean. As the domain moves in range, more and more of the domain is

FIG. 1. NPE-PE comparison for range-independent environment. Dashed line: linear PE solution; solid line: linear NPE solution. Receiver located at (a) \(r = 500\, \text{m}, z = 20\, \text{m}\), (b) \(r = 1000\, \text{m}, z = 20\), (c) \(r = 500\, \text{m}, z = 50\, \text{m}\), and (d) \(r = 1000\, \text{m}, z = 50\, \text{m}\).
displaced above the ocean surface. This corresponds to the sloped upper boundary line moving down the grid. The equation of the line is given by

$$z(x) = \Delta H + x\tan \theta,$$

(7)

where $x$ and $z$ are represented in the coordinate system of the computational domain and not global coordinates. Note that as the upper boundary line is shifted down, energy in the domain is truncated and lost. This truncation error must be taken into account when prescribing the vertical grid spacing.

In the case of downslope propagation, the computational domain initially starts with the top of the grid set a certain height above the sea surface, and as the domain moves in range, it becomes fully submerged. The initial height offset is given by $H_0 = r\tan \theta + W\sin \theta$, where $r$ is the total simulation range. The upper boundary line moves up the grid as the simulation ensues, and it is represented by

$$z(x) = \frac{H_0}{\cos \theta} - \Delta H - x\tan \theta.$$  

(8)

This downslope propagation case is free of the energy truncation error seen in upslope propagation, but the upward shift of the sloped boundary line creates an area on the grid devoid of energy. Time is spent for the wave to propagate into this void, thus producing error. A smaller vertical grid spacing will resolve this issue.
The rotated NPE model is benchmarked against a parabolic equation solution for two, linear range-dependent cases. The first environment for comparison is an upslope waveguide with a slope angle of $2^\circ$. Initially, the environment has a water column depth of 100 m and an additional 100 m of sediment depth which remains constant in range. The water column has a density of 1000 kg/m$^3$ and a sound speed of 1500 m/s, and the sediment has a density of 1500 kg/m$^3$ and a sound speed of 1650 m/s. The source is located at $z_s = 50$ m and emits a pulse described by Eq. (5). The $x$-direction grid spacing is 0.25 m, the $z$-direction grid spacing is 1 m, and the time step is 0.0005 s. Only the linear case is considered and as such the coefficient of nonlinearity $\beta$ is set to zero.

As was done for the range-independent problem, the starting field used by the NPE is generated by the PE solution. The initial waveform generated by the PE solution is essentially a plane wave source, so it is first rotated to match the rotated coordinates. The wave is then marched out in range to a final distance of 1000 m while saving data at ranges of 500 m, and 1000 m, each for depths of 20 m and 50 m. These pressure histories are then compared to the PE solution and the results are shown in Fig. 3. The results show that the PE (solid curve) and NPE (dashed curve) solutions agree, with only small error. The error is mostly in amplitude which may be due to the truncation that occurs with the upslope propagation scheme.

The second environment used for comparison is a downslope waveguide with slope angle of $-2^\circ$. Initially, the environment has a water column depth of 100 m and an additional 100 m of sediment depth which increases linearly in range. Geoaoustic parameters (density and sound speed), the initial condition, and computational parameters remain the same as in the upslope problem. The NPE and PE are compared in Fig. 4. The results show that the PE and NPE solutions agree, but there seems to exist an error associated with the phase of the NPE solution. This error may be due to the method in which the sloped upper boundary is shifted upward during propagation. One would expect an error of this type to build up over distance, but results at the 1000 m range appear to be more accurate than those of the 500 m range indicating a different source of error.

IV. NONLINEAR STEEPENING IN RANGE-DEPENDENT ENVIRONMENTS

The accurate treatment of range-dependent environments allows for the investigation of many interesting, physically relevant problems. This study looks at the effect of sloped ocean bottoms on nonlinear steepening to determine if shock effects are stronger in flat, upslope, or downslope environments. An initial sinusoidal pulse is scaled to different sound pressure levels to characterize the effect of pressure on nonlinear steepening for each environment. This
information is useful when studying long-range propagation of blast waves because it is often necessary to find the point at which a linear acoustic model can be implemented.

The three environments—flat, upslope, and downslope—used in this study are taken directly from the linear calculations performed in the previous two sections. Each environment has a water column depth of 100 m and an additional 100 m of sediment depth. There is no slope angle for the flat case, and the slope angle is $+\frac{\pi}{4}$ and $-\frac{\pi}{4}$ for the upslope and downslope cases, respectively. The water column has a density of $1000 \text{ kg/m}^3$ and a sound speed of 1500 m/s whereas the sediment has a density of $1500 \text{ kg/m}^3$ and a sound speed of 1650 m/s. The source is located at $z_s = 50 \text{ m}$ and emits a pulse described by Eq. (5) with a center frequency of 100 Hz. However, the initial pulse is scaled to represent sound pressure levels ranging from 230 to 280 dB. This range was chosen because it crosses the acoustic, small-signal pressure limit of water—pressures below can be treated as acoustic disturbances and pressures above are considered nonlinear. The $x$-direction grid spacing is 0.3 m for the flat case and 0.225 m for the range-dependent cases. The $z$-direction grid spacing is 1 m and the time step is 0.0005 s for all cases. For each case, the PE-generated linear solution is given for comparison.

Results for the flat ocean bottom case are given in Fig. 5. The plots show that, at a range of 500 m, all source levels behave in a relatively linear fashion. At a range of 1000 m, the pulses with an initial amplitude of 270 dB and 280 dB show significant deviation from the linear solution because of a steep wavefront and an increase in velocity, both due to shock steepening. Also, the pulses at the 260 to 280 dB pressure levels see clipping in the dips of the waveforms at both ranges. Results for the upslope ocean bottom given in Fig. 6 show the high pressure cases, 270 and 280 dB, differ greatly from linear theory for all receiver locations. Again, valley clipping is observed in the pulses at the 260 to 280 dB pressure levels at both ranges. The results for the downslope propagation case are shown in Fig. 7. Results yield similar observations as in the previous cases: the high pressure pulses at 270 dB and 280 dB have steeper wavefronts, different amplitudes and phases, and clipped valleys when compared to the linear PE solution.

Analyzing the results from all three cases provides useful insight into the effect of range-dependence on nonlinear steepening. In all cases, it appears that high-pressure pulses had the greatest deviation from the linear solution at the 1000 m range. This is because the increased velocity of the steepened shock wave has time to outrun the linear solution. Also, more waveforms behaved linearly for the range-independent case than for the range-dependent cases. And, finally, in all cases it is safe to assume that pulses with an initial peak pressure of 250 dB or less can be treated using a linear acoustic model.

V. THE EFFECT OF THIN SEDIMENT LAYERS ON SEDIMENT-BORNE BLAST WAVE PROPAGATION

An additional study is presented here to investigate how thin sediment layers affect shock wave propagation. Thin layers of loosely consolidated soils and clays often line the ocean floor in shallow water environments. This study examines whether such layers play a significant role in the overall propagation of shock waves down shallow water waveguides. Numerical treatment of thin layers, defined as sediment layers where the ratio of the layer thickness to the

![Fig. 5](Image)

FIG. 5. (Color online) Nonlinear steepening study for a range-independent environment. The solid line represents a linear PE solution, and the markers represent NPE calculations where the initial pulses are scaled to the specified sound pressure level.
compressional wavelength is less than one, require finer depth grid spacing than is needed elsewhere in the domain to fully sample the effects of the layer. Studies such as this allow engineers to make better judgments as to the fidelity of environmental models they employ and whether or not such layers are necessary.

For this study, we consider the propagation of a blast wave generated from a buried explosive source. Such situations are common when looking at the explosive removals of offshore structures (EROS), and studies of this type may be useful in determining the impact of EROS on marine life. The blast wave is propagated in
range-independent and range-dependent environments, both with and without a thin sediment layer. Doing so resolves whether or not range-dependence may have an effect on the coupling of energy into the sediment layer.

The details of the calculations are as follows: an explosive source is located at a depth of 10 m below mud line above which sits a 25 m water column with constant sound speed and density of $c_w = 1500 \text{ m/s}$ and $\rho_w = 1000 \text{ kg/m}^3$. For the case without the thin sediment layer, the sediment half space lies below the water column and contains the following parameters: $c_b = 2200 \text{ m/s}$ and $\rho_b = 1700 \text{ kg/m}^3$. For the case including the thin sediment layer, the 3 m layer starts at $z = 25 \text{ m}$ and its parameters are: $c_l = 1700 \text{ m/s}$ and $\rho_l = 1300 \text{ kg/m}^3$. The pressure waveform used for the initial condition of the study is given in Fig. 8(a). This simplified waveform represents the detonation of approximately 100 kg of TNT. The parameters of peak pressure, arrival distance, and shock duration are taken from an NSWC report. The frequency spectrum for the initial blast wave is given in Fig. 8(b). The plot shows that the signal is broadband with the majority of the energy existing below 5 kHz. Most of the explosive energy is in the low frequencies, so it is accurate to describe the sediment thickness of 3 m as thin because the ratio of layer thickness to the compressional wavelengths is less than one. The blast wave is propagated 200 m down a shallow water waveguide to simulate propagation into the open ocean. For the range-dependent case, the wave follows a $-3^\circ$ downsloping ocean bottom. The computational grid has a width of 100 m and depth of 70 m, including a 15 m thick artificial absorbing region. The x-direction grid spacing is 0.2 m, the z-direction grid spacing is 0.2 m, and the time step is 0.02 ms (the use of a short time step provides better resolution when generating the pressure time histories). The coefficient of nonlinearity, $\beta$, is set to 3.5.

Pressure histories at various depths in the water column, for the range-independent case, are given in Fig. 9. The pressure traces show that including the thin layer slightly delays the arrival of the pulse, but the amplitude of the initial arrival remains the same. However, incorporating the thin sediment layer does significantly alter the pulse’s reflections. Results for the downslope environment, presented in Fig. 10, show similar characteristics—a slight delay in the initial pulse and notable difference in the reflections. Range-dependence does not appear to either enhance nor diminish the effects the thin sediment layer.

This investigation suggests that it is possible to neglect thin sediment layers with densities and sound speeds lower than that of the basement when the primary pulse is of sole concern. Such is the case with EROS studies where peak pressure is the main factor being considered. However, if long-range acoustic propagation is of interest then it is necessary to include such layers as multiple reflections down the waveguide lead to changes in transmission loss.

VI. DISCUSSION

The NPE was used to investigate shock propagation along constant sloped ocean bottoms by applying a coordinate rotation to align the computational domain with the sloped interface. This led to a sloping top boundary that was approximated as a series of range-independent steps, easily satisfying the pressure release condition there. Care was needed to numerically implement the upper boundary condition, but the scheme used was shown to be efficient and to introduce only small errors. Comparisons with the Fourier-transformed PE solution showed that the rotated NPE model is accurate for shallow water environments with small slope angles (that comply with the narrow-angle assumption) in both upslope and downslope propagation scenarios. The rotated solution was then applied in two studies: investigating nonlinear steepening in range-dependent environments and investigating the effect of thin sediment layers on blast wave propagation. In the first study, results indicated that range dependence caused the high pressure pulses to deviate from the linear acoustic solution more so than the range-independent case. In the second study, results showed thin sediment layers did affect the blast waveform, but only in reflections.

There are a number of enhancements that can be made to the NPE model. A major enhancement would be to allow for elastic sediment layers in the seafloor. Currently, the model treats ocean sediment layers as fluids, but this is often unrealistic since many sediments can support shear waves. The current model is also not able to physically allow for a
loss mechanism in sediment layers. Waves propagating in sediments have a tendency to lose energy as they are dispersed to the surrounding media, so a physics-based loss mechanism is needed to be applied to the NPE. In order to increase accuracy near the source, a high-angle correction can be implemented into the model. This is a first-order correction to the narrow angle assumption made in the derivation of the NPE. This facilitates a more accurate treatment of wave energy traveling in the \( z \) direction, perpendicular to the direction of propagation.

The main enhancement to be made is to adapt the model to handle variably sloped ocean bottoms. One method for achieving this would be to use an interpolation-extrapolation scheme such as was done with the PE method. This method marches the solution past the end of one slope to generate the initial waveform used for the following slope. In the case of the PE, only a line of values are interpolated and extrapolated to form the starting field, but for the NPE, an entire plane of values would be needed.