WAR with Auction Rounds
Assumptions

Since:

1. The starting state of each player is known to all other players
2. Every player knows what all other players played

We may assume:

Perfect Information Principle

Since every player is capable of “counting cards”, every player is theoretically capable of knowing the cards in every other player $i$’s hand ($H_i$) and wins pile ($W_i$).
WAR as a Repeated Static Game

WAR can be represented as a repeated static game; that is, the outcome of a player’s move depends on the other players’ moves, but state is maintained between moves to formulate the outcome of game.
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Given a game of WAR with $N$ players, devise a strategy to “do well” at WAR.
Dead Simple Strategy: DummiePlayer (DP)

**DummiePlayer**$_i$ (DP):

- To play a normal move, select randomly from $H_i$
- To play a tie, choose 4 cards randomly from $H_i$ and discard the first 3, play the last
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**How well does DP do?** When playing against \(N\) other DP’s, a DP will have a \(\frac{1}{N}\) chance of winning.
SimpleMindedPlayer (SMP):

To determine a normal play:

1. Compute $S = \{H_{ic} \mid \forall j, d[H_{ic} > H_{jd}]\}$.
2. If $S \neq \emptyset$, play $\min(S)$.
3. Otherwise if, $|H_i| \geq 5$, $\exists j, c[H_{ic} = \max(H_j) \wedge \max_2(H_i) \geq \max_2(H_j)]$, then play the corresponding $H_{ic}$.
4. Otherwise, play $\min(H_i)$.

To play a tie, follow strategy similar to above, but remove 3 smallest cards from hand first to be discarded.
**Example:** SMP Move Computation \((S \not= \emptyset)\)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.
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Compute \[ S = \{H_{ic} \mid \forall j, d[H_{ic} > H_{jd}]\} = \{10, 10, Q\}. \]
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1. Compute \( S = \{H_{ic} \mid \forall j, d[H_{ic}] > H_{jd}\} \) = \{10, 10, Q\}.
2. \( S \neq \emptyset \), so we play \( \min(S) = 10 \).
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   1. \( |H_i| \geq 5 \)? Yes.
   2. \( \exists j, c[H_{ic} = \max (H_j) \land \max_2 (H_i) \geq \max_2 (H_j)] \)? Yes, \( j = 2, H_{jc} = Q \).
   3. Play \( Q \).
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How well does a SMP do against $N$ DPs? Too hard to figure out mathematically, so we wrote a computer simulation to answer this.
Computer Simulation

- https://github.com/psattiza/war
- 100,000 simulated games
- A single SMP battles $N$ DPs, for all $N \in \{1, \ldots, 100\}$. 
Computer Simulation

SMP vs. $N$ DPs: Win Ratio

Number of DPs ($N$)

SMP Win Ratio
Computer Simulation

SMP vs. \( N \) DPs: Mean Rounds Played

- Mean Rounds Played
- Number of DPs (\( N \))

Graph showing the relationship between the number of DPs and the mean rounds played, indicating an increasing trend.
Computer Simulation

SMP vs. $N$ DPs: Variance of Rounds Played

$\sigma$ (Rounds)

Number of DPs ($N$)
SMP does not take into account $W_i$ for either themselves, or other players. Design a CMP (ComplexMindedPlayer) which adds in factors from $W_i$. When an SMP determines it cannot win, it chooses its lowest card, which may have a high probability of causing a tie against DPs. Design a player good for defeating large quantities of DPs that takes this into account. Throw the game at a ML algorithm and see what happens? More simulation runs!
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Questions?