PHGN 422: Nuclear Physics
Lecture 17: $\gamma$ Decay

Prof. Kyle Leach
Last Class...

- $\beta$ decay converts a proton to a neutron (or vice versa) within a bound nucleus.
- That process is mediated by the weak interaction.
- $\beta^\pm$ are three body decays, and EC is a two-body decay.
- Fermi theory of beta decay allowed us to estimate the decay rates from Fermi’s golden rule.

- Assignment 3 is now online!
- Topic selection on my door
**γ Decay**

- γ decay is an *electromagnetic* process where the nucleus decreases in excitation energy, but does not change proton or neutron numbers.

- This decay process only involves the emission of photons.
The Spectrum of Electromagnetic Radiation

- **Increasing Frequency (ν)**
  - γ rays
  - X rays
  - UV
  - IR
  - Microwave
  - FM Radio waves
  - AM Radio waves
  - Long radio waves

- **Increasing Wavelength (λ)**
  - Visible spectrum
  - Increasing Wavelength (λ) in nm
\( \gamma \) Decay

There are only two-bodies in the final state for \( \gamma \) decay:

\[
\frac{A}{Z} X_N^* \rightarrow \frac{A}{Z} X_N^{(*)} + \gamma
\]
\( \gamma \) Decay in a Nutshell

- The photon emission of the nucleus essentially results from a re-ordering of nucleons within the shells:

Source: Krane, Fig. 5.11
 Decay in a Nutshell

- This re-ordering often follows $\alpha$ or $\beta$ decay, and moves the system into a more energetically favourable state:
Classical Electrodynamics

First let’s think of this in broader terms:

- The nucleus is a collection of moving charges, which can induce magnetic/electric fields
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\[ P = \frac{1}{12\pi\varepsilon_0} \frac{\omega^4}{c^3} d^2 \]

- For a magnetic dipole is:

\[ P = \frac{1}{12\pi\varepsilon_0} \frac{\omega^4}{c^5} \mu^2 \]
Electric/Magnetic Dipoles

Electric and magnetic dipole fields have opposite parity: Magnetic dipoles have even parity, and electric dipole fields have odd parity.

\[ \pi(ML) = (-1)^{L+1} \text{ and } \pi(EL) = (-1)^L \]
Higher Order Multipoles

It is also possible to describe the angular distribution of the radiation field as a function of the *multipole order* using Legendre polynomials. So, as a refresher:

- $L$: The index of radiation
  - $2^L$: The multipole order of the radiation
- $L = 1 \rightarrow$ Dipole
  - $L = 2 \rightarrow$ Quadrupole
  - $L = 3 \rightarrow$ Octopole
  - ....
- The associated Legendre polynomials $P_{2L}(\cos(\theta))$ are:
  - For $L = 1$: $P_2 = \frac{1}{2}(3\cos^2(\theta) - 1)$
  - For $L = 2$: $P_4 = \frac{1}{8}(35\cos^4(\theta) - 30\cos^2(\theta) + 3)$
  - etc....
Classical to Quantum Electrodynamics

Nuclei are (of course) quantum mechanical object. So, we need to adjust our theory to properly describe $\gamma$ emission from nuclear decay. How do we do this?

- First, we need to quantize the electromagnetic field. We first start by replacing our electromagnetic moments by the relevant electromagnetic operators:

$$m(\sigma L) \rightarrow m_{fi}(\sigma L)$$

Where $m_{fi}(\sigma L) = \int \psi_f^* \, m(\sigma L) \, \psi_i \, dV$

- But we’ve done this before! Let’s head to the chalkboard again...
Electric Transitions

Electric Transition Probability

\[ \lambda(EL) = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{e^2}{4\pi\epsilon_0\hbar c} \left( \frac{E}{\hbar c} \right)^{2L+1} \left( \frac{3}{L+3} \right)^2 cR^{2L} \]

Where \( R = r_0a^{1/3} \), as usual. From here, we can now make some estimates for the various multipoles:

\[ \lambda(E1) = 1.0 \times 10^{14} A^{2/3} E^3 \]  
(1)

\[ \lambda(E2) = 7.3 \times 10^7 A^{4/3} E^5 \]  
(2)

\[ \lambda(E3) = 3.4 \times 10^1 A^2 E^7 \]  
(3)

\[ \lambda(E4) = 1.1 \times 10^{-5} A^{8/3} E^9 \]  
(4)

where \( \lambda \) is in \( s^{-1} \) and \( E \) is in MeV.
Magnetic Transitions

Magnetic Transition Probability

\[ \lambda(ML) = \frac{80\pi(L+1)}{L[(2L+1)!!]} \left( \frac{\hbar}{m_pc} \right)^2 \frac{e^2}{4\pi\epsilon_0\hbar c} \left( \frac{E}{\hbar c} \right)^{2L+1} \left( \frac{3}{L+2} \right)^2 cR^{2L-2} \]

Therefore, the magnetic multipole estimates are:

\[ \lambda(M1) = 5.6 \times 10^{13} E^3 \]  
\[ \lambda(M2) = 3.5 \times 10^7 A^{2/3} E^5 \]  
\[ \lambda(M3) = 1.6 \times 10^1 A^{4/3} E^7 \]  
\[ \lambda(M4) = 4.5 \times 10^{-6} A^2 E^9 \]

where \( \lambda \) is in \( s^{-1} \) and \( E \) is in MeV.
The Weisskopf Estimates

We have made the assumption here that only a single particle (proton) changing states in the shell-model is responsible for the de-excitation via $\gamma$ decay.

**Electric Transitions**

\[
\begin{align*}
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\end{align*}
\]

These estimates (for both ML and EL) are known as the *Weisskopf Estimates*. What can we learn from these?
Interpreting the Weisskopf Estimates

The Weisskopf estimates are only intended to be a comparative guideline, and are not meant to be exact representations of all cases (remember, we have removed our dependence on the nuclear structure). So, how do we actually interpret a comparison between these estimates, and what we would observe experimentally (the transition rate):

\[ \lambda(\sigma L)_{\text{Exp.}} \ll \lambda(\sigma L)_{\text{Weiss.}}. \]

A poor overlap between \( \psi_i \) and \( \psi_f \) which decreases the probability for the transition to occur.

\[ \lambda(\sigma L)_{\text{Exp.}} \gg \lambda(\sigma L)_{\text{Weiss.}}. \]

More than one single particle contributes to the decay, thus increasing the probability for the transition.
Angular Momentum in $\gamma$ Decay

- The photon is a spin-1 Boson.
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- Like $\alpha$ decay and $\beta$ decay the emitted $\gamma$ can carry away units of angular momentum $L$, which has given us the different multipolarities for the transitions described above.
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  For orbital angular momentum, we can have values $L = 0, 1, 2, 3...$ that correspond to our multipolarity.

- Therefore, our selection rule is:

  $$|J_i - J_f| \leq L \leq |J_i + J_f|$$

Let’s head to the chalkboard for an example...
Internal Conversion

Each of the above cases were for $J_i \neq J_f$, but what about $J_i = J_f$?
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There are no \( L = 0 \) photons! So what about \( 0 \to 0 \) transitions within the nucleus? Do they occur?
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Yes!, but they cannot decay by photon emission....
Internal Conversion
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There are no $L = 0$ photons! So what about $0 \rightarrow 0$ transitions within the nucleus? Do they occur?
Yes!, but they cannot decay by photon emission.

- The decrease in energy from state $i \rightarrow f$ can instead "kick" an electron out of the atom
- This process is called *internal conversion*
- In general, this process competes with $\gamma$ decay..
- However, for $0 \rightarrow 0$ transitions, it is the only possible decay mode

We won’t cover IC any further, but see Krane 10.6 for a detailed discussion.
Next Week...

Reading Before Next Class

- Chapter 7.1 in Krane

Next Class Topics

- Rutherford scattering and introduction to nuclear reactions
- Radiation interaction with matter