II. CONSERVATION OF ENERGY

2-1 IT ALL STARTS WITH ENERGY

As we begin our exploration of the science of engineering let's summarize the essence of the design/engineering paradigm presented in the last chapter. This can be done quite compactly in pictorial form as in Figure 2-1. Here we see that design flows from the top-down through performance, properties, structure, and process, while science proceeds from bottom up through the discovery and understanding of process-structure-property-performance relationships. Simply, the science of engineering starts by understanding how the energy of a process is transmitted to and stored in structures, which requires that we first understand the basic facts of energy, and the most fundamental of these is that energy is conserved.

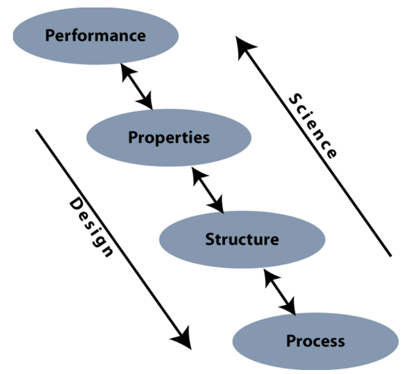


Fig 2-1

The fact that energy is conserved is often referred to as the First Law of Thermodynamics, or more commonly as the first law. Now a law is not something that one can derive (at least no one has succeeded so far). It is merely a statement about the way our universe behaves. The First Law of Thermodynamics is a mathematical principle stating that a numerical quantity called energy does not change as a result of a process. Energy can be taken from one place and moved to another, but its total numerical value does not change. It appears that this law is strictly enforced—it is what physicist call an exact law—because we know of no exceptions.

The discovery that energy is conserved took many years and divided the scientific community—including such notables as Newton and Leibniz—during the Age of Enlightenment (1685-1815). (You are invited to read a little about those who were instrumental in discovering the first law in the supplementary reading "Energy Conservation.") The reason for the division is that energy takes several forms, some of them difficult to recognize.

2-2 KINETIC AND POTENTIAL ENERGY

The most obvious form of energy is the energy of motion, what we call kinetic energy. Kinetic energy is one of several numerical quantities that can be associated with a moving object and you will investigate these in much greater detail in your physics classes. For our purposes, we note that the numerical quantity we call kinetic energy is given by the formula $Ke = \frac{1}{2}mv^2$ where m is mass and v is velocity. For a system of rigid particles bouncing around in space, the total kinetic energy of all the particles in the system is conserved. While the kinetic energy of a single particle may change as the result of a collision, the total energy will not. However, here on Earth kinetic energy, by itself, is not always conserved, as the example of a pendulum illustrates.

If we lift a pendulum bob to some height, h, and release, it will accelerate from zero to a maximum velocity at the bottom of its swing and then decelerate to zero as it ascends to h on the other side. A well-made pendulum will continue to swing back and forth for quite some period of time. And so, while the bob's kinetic energy is changing some other aspect of its motion is conserved. The missing part is of course the potential energy due to gravity. The change in potential energy from place to place is experienced as a force, e.g., the force of gravity.

Though the force of gravity is most familiar to us, we know of four fundamental interactions in nature: gravitation, electromagnetism, the strong interaction and the weak interaction. Gravity arises from the attractive potential between masses. The electromagnetism arises, in part, from the attractive and repulsive potential between bodies of opposite and similar charge respectively. The strong interaction acts over very short distances and holds together atomic nuclei. The weak interaction is responsible for various kinds of radioactive decay. While the last two interactions will not concern us, all these interactions share a common feature in that there is a potential energy arising from an interaction that depends on the distance between interacting particles.

As an example, close to the earth's surface the potential energy of a mass m is given as $Pe = mgh$ where g is the acceleration of gravity and h is the height above an arbitrary reference point (see problem 2-1). Returning to our pendulum, when the bob was lifted and held steady at height h above the lowest possible point in its swing, its potential energy became mgh and its kinetic energy was 0, making its total energy $Te = Pe + Ke = mgh$. At the bottom of the swing the potential energy is 0 but the kinetic energy is $\frac{1}{2}mv^2$, where v is the bob's measured velocity, and hence $Te = \frac{1}{2}mv^2$. If total energy is to be conserved, then $\frac{1}{2}mv^2 = mgh$ and $v = \sqrt{2gh}$ where the negative root would indicate the bob was swinging to the left and the positive to the right. If you performed this experiment, you would find that the measured velocity is the one that conserves energy. A very neat fact!

2-3 A FLY IN THE OINTMENT

It makes no difference the form of the potential. Just as the total energy of a falling mass is conserved through its fall, so too is the total energy of charged particles accelerating toward each other under the force of electrostatic attraction. That is, until they collide. The apple falling from Newton's tree has the same total energy at every point along its path to the ground, where it comes to a stop, making its kinetic, potential and total energy appear to vanish. It seemed that energy is not always conserved.

In the middle of the nineteenth century the mystery of the vanishing energy was solved. The energy of the falling apple didn't vanish, it merely took on yet another form—heat.

How we came to understand that heat is energy is a fascinating story—you may read about it in the supplementary reading—but setting that story aside, the discovery itself presented an interesting complication. Heat is perceived through temperature. The conversion of kinetic or potential energy to heat is accompanied by a temperature change, and it is through temperature change that heat energy is measured. By way of illustration, a unit of heat energy is the kilocalorie, it is defined as the amount of energy needed to change the temperature of one kilogram of water by 1^o Celsius. On the other hand, mechanical energy is defined in terms of the movement of mass, e.g., the mechanical energy needed to lift a one kilogram mass one meter is 9.8 joules. Heat energy and mechanical energy are both energy, but how many kilocalories are there in a joule?

James Prescott Joule answered this question with an ingenious experiment and reported his results in 1843. Joule reasoned that if he could transfer the energy of a falling weight to a known amount of water and measure the temperature change of the water, he could ascertain the mechanical equivalent of heat. Accordingly, he constructed the devise shown schematically in Figure 2-2, where a falling weight turns a paddle wheel in a sealed water reservoir. Joule found that, in modern units, 4184 joules of mechanical energy are needed to raise the temperature of one kilogram of water 1^o C.

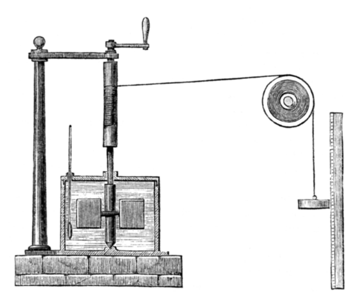


Fig 2-2

With his experiment, Joule determined what today is known as the heat capacity of water (it is really the thermal energy capacity) and is denoted as *Cw*. The heat energy, Q, accompanying any process causing the temperature to change is now easily determined through the equation $Q = m\ Cs\ \Delta T$, where Cs is the heat capacity of the substance changing temperature and m is the mass of that substance.

Joule's discovery provided the evidence to confirm the conservation of energy. In any collision between particles, some energy is retained in motion, balls bounce off each other, and some is converted to heat, the balls warm up. However, the sum of the two does not change.

2-4 Some fun implications

One of the stories about Joule, probably apocryphal, is that on his honeymoon he took time to measure the temperature of the water at the top and bottom of a waterfall. If energy is heat, then the water temperature at the bottom of the falls should be higher than at the top. Let's calculate the theoretical maximum temperature change that could be measured at a water fall. For this problem, we will go to the highest fall in the world, Angel Falls in Venezuela, with a height of 979 m.

We will assume that the flow in the river at the top and bottom of the falls is about the same, so we only need to consider the change in potential energy of the water at the top and bottom of the fall, which must be transformed into heat energy in the water, hence we know that,

$$\Delta Pe = Qw\ .$$

The change in potential energy of some arbitrary mass of water is

$$\Delta Pe = m\ g\ \Delta h = m\ 9.8 \frac{\mathrm{m}}{\mathrm{sec}^2} 979\ \mathrm{m} = m\ 9594.2\ \frac{\mathrm{m}^2}{\mathrm{sec}^2}$$

and the added heat energy in the same arbitrary mass is given by,

$$Qw = m\ Cw\ \Delta T = m\ 4184\ \frac{\mathrm{J}}{\mathrm{kg\ K}}\ \Delta T.$$

(Because we are working in MKS units—where distance, mass, and time are expressed in meters, kilograms and seconds respectively—we must use the MKS value for the heat capacity of water.) Equating Pe and Qw and solving for the change in temperature gives,

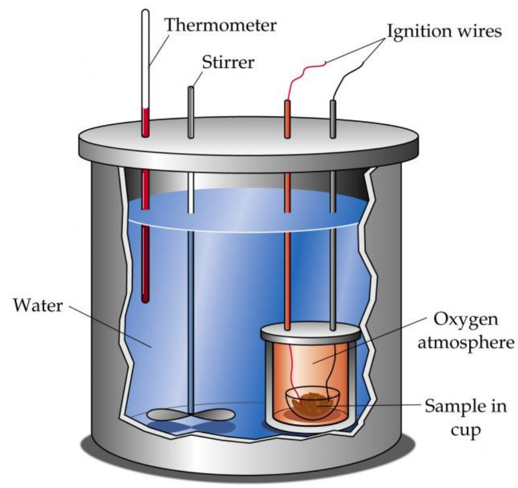
$$\Delta T = 2.3\ \mathrm{K} = 2.3^{\circ}\ \mathrm{C}$$

This is remarkable, assuming that no other factors come into play, such as evaporation during the fall, the water at the bottom of Angel Falls will be 2.3^{o} C warmer than at the top. All because energy is conserved.

2-5 Problems for the curious

1) Instead of measuring the height of the pendulum's bob from the bottom of its swing, let's measure it from the highest point of its swing. We will call this height zero. In that case, the bob is at a height $-h$ at the bottom of its swing and has a potential energy there of $-mgh$. Show that this choice for measuring height makes no difference to the total energy of the bob through it swing. Now call the zero of height a point n meters below the lowest point of the bob's swing. Again, show that this makes no difference to the total energy of the bob through its swing.

2) A bomb calorimeter (pictured right) is a common method used to determine the energy content of food, i.e., the number of calories it contains. Bomb calorimeters work by combusting a small food sample in a pure O_2 atmosphere inside what's known as the "bomb" – a resilient metal container. The bomb is placed in a large water bath and the sample is ignited. The combustion of the sample then raises the temperature of the water bath, which is measured using a thermometer. For this question, assume that the calorimeter contains 10L of water and that all of the energy released by the combustion goes into raising the temperature of the water.



   a. A 10g peanut raises the temperature by 9°C. How much energy was released by this peanut … in kJ? … in Cal (nutritional calories)?

   b. A LifeSaver© candy has a caloric content of 5 Cal/g. What would be the change in temperature (ΔT) from the combustion of a 2g sample

3) A typical candy bar provides ~1000 kJ of energy. Explain what this means in terms of our design paradigm. That is, identify the initial and final structure associated with extracting the energy from the candy bar. Explain how energy is conserved through this process.

4) The 1000 kJ of energy derived from that typical candy bar can be transferred as heat or work.

   a. Determine how much work can be done with the energy by determining the height of the mountain you could climb using this energy. Before you do the calculation, make a guess. After finishing the calculation, draw a "cartoon" showing the initial structure and the final structure resulting from the process of inputting the energy from the candy bar as work.

   b. As heat, determine how much the energy of the candy bar would raise your body temperature. Also make a guess. Again, draw a "cartoon" showing the initial structure and the final structure resulting from inputting the energy from the candy bar as heat.

c. Determine the mass of water that could be converted to vapor using the energy of the candy bar. (*You need an additional piece of information that we have not discussed to solve this problem. What is it?)*

d. Using these numbers, describe where the energy of a candy bar would go if you were walking up Lookout Mountain on a warm sunny day.

5) Two bicyclists go for a leisurely ride, climbing 510.2 m up Lookout Mountain. During the ride both cyclists remaining together and neither one is struggling to keep-up, arriving at the top together. At the bottom of the climb, both riders have an identical mass—they both weigh the same, have bikes of equal weight and performance, they even have identical amounts of water in identical water bottles—assume that the cyclists along with their bikes and gear weigh 100kg each. The only difference between the two is that one of them is a world-class professional cyclist while the other is significantly less athletic. Without asking them, how could you determine which one of the two was the world-class cyclist? Explain your answer.