V. MECHANICAL ENERGY AND PROPERTIES

5-1 Mechanical energy

Mechanical energy acts to change the position of atoms more so than their motion. Mechanical energy storage comes in two forms. The first is called elastic energy. This is a form of reversible work, wherein structure is changed by applying a force (remember force acting through distance is work) but the original structure can be recovered by simply removing the force—meaning that no net work has been done and hence no net energy storage. The second form of mechanical energy storage is called plastic deformation. In this form, work has been done that permanently changes the arrangement of atoms within the structure. Generally, plastic deformation involves the rearrangement of many thousands of atoms.

To help differentiate these two kinds of mechanical energy, imagine performing a tensile test on a cylinder of some material with a cross section of 1.0 cm$^2$ and a length of 10.0 cm by applying a force along the cylinder axis while measuring its change in length. Regardless of the material from which the cylinder is constructed, for small loads the resultant data will appear graphically as in Figure 5.1. As the axial load is increased, the specimen stretches in proportion to the load, that is $F = k \Delta L$, where $F$ is the applied force, $k$ is a proportionality constant and $\Delta L$ is length change. This linear relation was first noticed by Sir Robert Hooke in 1678 and is called Hooke's Law.

The energy stored in the stretched cylinder is given by the area under the force displacement curve, i.e., $E = \frac{k}{2} \Delta L^2$. If the load is removed, the material will return to its original length along the same linear curve. Thus, the energy stored as the load is applied can be fully recovered as it is removed. Such behavior is called elastic deformation. While all materials behave elastically for sufficiently small loads, we may observe
two different responses when the load exceeds some critical value as depicted in Figure 5-2.

In one instance (the upper curve) at some critical load the specimen breaks, at what is called its fracture point. In the other instance, the specimen begins to depart from its linear response at its yield point. For displacements up to the yield point, only the distance between a material’s atoms is changing; their relative orientation is virtually unchanged. Beyond the yield point, the arrangement of atoms can change dramatically. Hence some of the energy going into the metal is being converted to work. When the load is removed after yielding, the specimen will return along a curve parallel to the original elastic displacement curve (dashed line in Figure 5-2), and the specimen will permanently change length and is said to have deformed. The work of deformation cannot be recovered and is called plastic deformation. The yield point marks the boundary between the elastic and plastic response—between recoverable and non-recoverable energy of deformation.

The shape of the curves pictured in Figure 5-1 and 5-2 will depend on specimen shape. However, we can define a quantity called strain, $\varepsilon$, given by the elongation divided by original length, and another quantity called stress, $\sigma$, given by the force to achieve this elongation divided by the original cross sectional. If we then plot stress along the $y$-axis and the strain along the $x$-axis in the form of a stress versus strain, the resulting stress-strain diagram will be independent of the shape of the specimen and instead depend only on the material from which it is made. The various points on this diagram are measures for materials properties.

5-2 Mechanical properties

As with the previous figures the stress-strain diagram is separated into elastic and plastic regions. Where, as before, in the elastic region stress and strain are linearly related, i.e., $\sigma = k\varepsilon$. Broadly speaking, materials that do not undergo significant
plastic deformation before fracturing are said to be brittle, those that do are said to be ductile.

The slope of the stress-strain curve in the linear elastic region is the proportionality constant of Hooke’s law and is equal to a material’s modulus. For two materials, the one of greater modulus is the stiffer.

The boundary between ductile and brittle is not sharp and these terms simply provide a qualitative way to describe materials. A quantitative measure of “ductility” or “brittleness” is provided by another material property called **toughness**, which is the amount work done per unit volume in changing the structure of a material—the arrangement of its atoms—before it fractures.

In the stress strain diagram of Figure 5-3, toughness is given as the area of the region shaded blue. The curve above has been drawn a little exaggerated to emphasize the elastic region. In a real material, the toughness can be found by integrating the region under its stress strain curve.

Some other important materials properties that can be extracted from a stress strain diagram are: **ultimate strength**, **fracture strength**, and **resilience**. Ultimate strength is the maximum stress in the stress-strain diagram it is sometimes called tensile strength. Fracture strength is the stress acting on the material at the point of fracture (the endpoint of the stress-strain diagram). This is also known as the breaking strength. Resilience is the given by the area under the stress-strain curve in the elastic region. Resilience measures a materials ability to absorb energy without creating a permanent distortion.

**5-3 Problems for the curious**

1) Identify the performance criteria and an example application where tough materials are needed. Note that strength and toughness refer to quite different properties, it is
possible to have a strong material that is not tough and a tough material that is not strong.

2) Find stress strain diagrams for various materials and compare these. For example, Google “stress strain curve rubber” and “stress strain curve aluminum,” compare curves. What are their differences? How do these differences manifest? Give concrete examples, i.e. which has the higher modulus steel or aluminum? What does this mean in a practical sense?

3) Do some research to find: the strongest material, the toughest material, the most resilient material. What you find may surprise you.

4) The subject of this chapter is referred to as the science of the strength of materials. As with all science, the strength of materials involves discovering relationships between structure and properties. Of course, it is impossible to discover relationships if the property is unknown or if there is no established way to quantify the property. For example, when do you think that engineers came to know that a material has properties that we now call modulus, strength and toughness. Did the Egyptian, Greek, or Roman builders know about these properties? See if you can find out. (This is an excellent journaling opportunity. To really dig into the subject will require at least an hour or two. By dig in I do not mean simply answering the question posed here, but also consider the implications as they pertain to engineering science. If you want to pursue this problem and after spending 30 min or so you have come up with nothing, come see me for a hint.)