

Review of uncertainty analysis

According to ISO (International Organisation for Standardisation) (sorry about European spelling!)

There are 2 types of uncertainty: *measured* and “*all other,*”
or *estimated*

1. Measured uncertainty

When you have a *series of measurements* of the same quantity (taken under the same conditions, but never mind)

(a) Calculate the *mean* μ of the measurements

(i) Mean is *estimator of true value*

(b) Standard deviation *of the sample*

$$s = \sqrt{\frac{\sum (x_i - \mu)^2}{(N - 1)}}$$

- (i) The SD you are familiar with (N is no. of measurements)
- (ii) Is approximately constant with increasing N
- (iii) Measures range of values of data
- (iv) Is *not* the uncertainty (if you have a lot of data, you can locate the peak well within $\pm s$)

(c) Standard deviation *of the mean* (Baird's S_m)

$$\sigma \equiv \frac{s}{\sqrt{N}} = \sqrt{\frac{\sum (x_i - \mu)^2}{N(N-1)}}$$

(i) *Is* measure of the uncertainty

(ii) Decreases slowly with increasing N

(d) σ is renamed *standard uncertainty*, symbol u (*not* μ)

(i) Thus, $u_r = \sigma$; r for *random*

(ii) One *component* of uncertainty

2. Estimated (all other) uncertainties

(a) Uncertainties you cannot measure

(b) Estimate or guesstimate *greatest probable error* Δx

(i) Often 1/2 smallest scale division (least count)

0.5 mm, 0.5 mV, 0.05° (think DVM with 1-mV least count)

(ii) Inexpensive electronics: Look up on Web or use estimate such as 2 % or 2% of full scale reading

(iii) Example: Voltage on Tek TDS 1000-, 2000-series oscilloscopes specked at 3 % of reading

(c) Δx is not the uncertainty!

(i) To calculate uncertainty, divide Δx by $\sqrt{3}$:

(ii) $u_x = \frac{\Delta x}{\sqrt{3}}$ (rabbit out of hat; sorry)

(iii) u is a *component* of uncertainty

3. *When x is function of some other variable z*

And you can measure or estimate Δz (but not Δx)

(a) Use $\Delta x = \left| \frac{\partial x}{\partial z} \right| \Delta z$ (*propagation of uncertainty*)

(b) *Then divide by $\sqrt{3}$*

(c) Component of uncertainty is again $u_x = \frac{\Delta x}{\sqrt{3}}$

(i) You could also call it u_{xz} or $u_{x,z}$ or ... to remind yourself that it is uncertainty of x due to uncertainty in z (be inventive!)

4. *Calculate* each component of uncertainty separately

(a) You get *useful information* by comparing them

5. *Only now add components of uncertainty in quadrature:*

$$u_c = +\sqrt{u_r^2 + u_x^2 + u_y^2 + \dots} \quad (\text{combined standard uncertainty})$$

(a) One or more of u 's may be an SDOM

(b) One or more of u 's may be an estimated uncertainty

(c) u_c is always positive, by convention

6. Expressing result of measurement

(a) Define *expanded* uncertainty

$$U = 2 u_c$$

(U like u_c is *positive* by convention)

(b) Express result of measurement as

$$\mu \pm U$$

- (i) μ = measured value or mean of measured values
(depending whether you have 1 measured or an average)
- (ii) $\mu - U \rightarrow \mu + U$ defines 95 % *confidence interval*
- (iii) U typically has only 1 or 2 significant digits

7. 95 % confidence interval

- (a) We think (hope? pray?) that there is 95 % chance that true value lies within 95 % confidence interval

- (b) Uncertainty analysis is imprecise
 - (i) May tell more what you think about measurement than about measurement itself

- (c) But we do the best we can

8. *Simplified calculation.* When

$$y = x^a \quad (\text{but not when } y = A + Bx, y = e^x, y = \sin x, \text{ etc.})$$

I.C.B.S.T.:

$$\frac{\Delta y}{y} = |a| \frac{\Delta x}{x}$$

OR

$$\Delta y = |a| y \frac{\Delta x}{x}$$

Useful in, say, e/m expt.

9. *New or newish terms*

Standard deviation of the mean (s/\sqrt{N} , where s is standard deviation of sample or population)

Error (deviation of single measurement from true value)

Greatest probable error (Δx)

Standard uncertainty (u , estimator of mean error)

Combined standard uncertainty (u_c ; not *total* uncertainty)

Expanded uncertainty (U ; for our purposes, $2u_c$)

95 % confidence interval (region around mean or measured value wherein we will find true value with 95 % probability (we *think!*); 5 % chance that true value lies outside 95 % C.I.)