Exam I Review

1 Classification

Classify the following initial value problems: state the order of each equation, whether they are linear or non-linear, whether they separable, and classify the coefficients. Are any of them Bernoulli?

1. \((yy')' + x^2 y = 5;\ y(0) = 0\)
2. \(y'' + x^{-1/2} y = e^{x^2};\ y(0) = 1\)
3. \(x \frac{dy}{dx} - (1 + x)y = xy^2 + 5;\ y(0) = 1\)
4. \(y' + \tan(x)y = \cos^2(x);\ y(0) = -1\)
5. \(-y'' + 2y' - 5y = 0;\ y(0) = 0, y'(0) = 1\)
6. \(\frac{dy}{dx} + 3x^2 y = x^2;\ y(0) = 5\)
7. \(y^{1/2} \frac{dy}{dx} + y^{3/2} = 1;\ y(0) = 4\)
8. \(\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0;\ y(0) = 1, y'(0) = 0\)

2 Solution Techniques

For equations (4)-(8) above, (a) if the equation is first order, state all possible solution techniques (give reasons), (b) solve each of the equations, regardless of order, by an appropriate technique.

3 Existence and Uniqueness

Determine a region of the \(xy\)-plane for which the given differential equations would have a unique solution passing through a point \((x_0, y_0)\) in that region.

1. \(\frac{dy}{dx} = y^{2/3}\)
2. \(x \frac{dy}{dx} = y\)
3. \((1 + y^3)y' = x^2\)
4. \((x - y)y' = y + x\)
4 Phase Lines, Fixed Points, etc.

Consider the following first order differential equation:

\[ y' = y^3 - 15y^2 + 50y. \]  \hspace{1cm} (1)

Perform the following tasks: (a) find all fixed points and classify their stability, (b) draw the phase line, (c) next to the phase line, draw sample solution curves in each region.

5 Heating/Cooling

A small metal bar, whose initial temperature was 20° C, is dropped into a large container of boiling water. How long will it take the bar to reach 90° C if it is known that its temperature increases 2° in one second?

6 “Challenge” Questions

6.1 Write your own tank problem.

Given the following differential equation,

\[ \frac{dy}{dt} = 2 - \frac{y}{100 + t}, \quad y(0) = 10, \]

write an appropriate “tank problem”. Solve it.

6.2 Bifurcation

Consider the harvesting model,

\[ P' = P(4 - P) - h. \]

Find the value of \( h \) for which there is exactly one fixed point.

7 One More Thing

There are a few topics that are not included on this review, but that doesn’t mean they won’t be on the exam. You are still responsible for all material covered in lecture!