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Water waves in a simple inlet and bay system

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Abstract: Water waves in an symmetric inlet/bay system connected to the sea are solved using coupled boundary value problems. Fourier transforms and the eigenfunction expansion method is used with the Helmholtz equation for the constant-depth system. The methodology gives a simple model for determining possible system resonances to short and long wave forcing and serves as a didactic tool.

1 INTRODUCTION

Melo and Guza (1991 a, b) measured a remarkable dissipation of wave energy within a jettied entrance channel and modelled it with a set of parabolic wave models. Dalrymple (1992), intrigued by this rapid decrease in wave energy in the channel, developed an simple analytic model for an infinitely long inlet, using an impedance boundary condition at the jetties. This boundary condition allowed wave energy to propagate into the jetties. A disadvantage of this model was the use of a "wavemaker" at the mouth of the inlet to represent the incoming ocean waves. This caused an artificial enhancement of the wave height near the mouth of the inlet.

Dalrymple and Martin (1996) overcame the artificiality of the wavemaker condition at the mouth of the inlet by including the ocean into the problem, allowing for the interaction between the waves in the ocean and those in the channel. The mathematical problem was divided into a two regions (ocean and inlet) with a boundary value problem for each. The regions were joined by matching conditions at the junction of the ocean and inlet. The addition of the ocean did remove the spurious wave focussing at the channel entrance.

More recently, Dalrymple, Martin, and Li (1999) have added an infinitely large bay to the far end of the inlet, so as to have a finite length inlet. Once again, wave focussing appears, this time at the end of the channel, as the waves enter the bay. This effect, due to diffraction, is not artificial, however.

In this paper, the infinite bay is replaced by a symmetric rectangular harbor, which is a more realistic case. However, there are plenty of artificialities in the model: vertical reflective sidewalls to the channels, harbor, and ocean shorelines; constant depth within all regions; and the assumption of linear water waves, and no energy losses at the abrupt channel junctions or the bottom.

Now, the problem is one of a rectangular harbor connected to the sea by an straight entrance channel, which has been a problem long important to coastal engineers. There have been a large number of studies of the harbor resonance problem (see, *e.g.*, Raichlen, 1966; Lee (1971), to name a few) and more sophisticated models have been applied to these types of problems.

The study of harbor response has evolved over a similar path. Early analytical studies by McNown (1952) and Kravchenko and McNown (1955) examined the response modes of a harbor assuming that the waves in the harbor could be approximated by the eigenmodes of a closed basin. Raichlen and Ippen (1961) examined a rectangular harbor that was attached to an ocean that was represented by a wide channel. The eigenfunctions in each region were matched by a cross-channel mean water surface slope at the harbor mouth. See Raichlen (1966) for a review of the earlier works in this area. Miles and Munk (1961) were the first to include the full influence of an ocean (by embedding the harbor into an infinitely long coastline), allowing the complete interaction between the harbor and the ocean. This provided the resolution of the so-called harbor paradox, which is that the resonant response of the harbor becomes larger as the channel into the harbor becomes narrower (which would seemingly let less energy into the harbor, but in fact drives the harbor towards more complete resonance). The radiation of energy from the harbor to sea, included by Miles and Munk, permitted a more accurate analysis than previously and prevented resonance to infinite wave height in the harbor.

Since these early modelling efforts, more sophisticated harbor models have evolved, first using Greens function approaches (Lee, 1971) and now including the use of the mild slope equation or the Boussinesq equations. However, the purpose of this paper is to develop a simple model for illustrative use and to highlight some interesting possibilities that occur in these situations.

2 THEORETICAL CONSIDERATIONS

In this simple system, the water depth will be taken as constant with depth h and all the system walls are vertical and rigid.

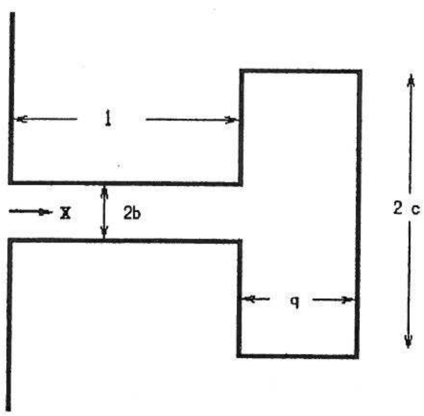

Figure 1: Schematic Diagram of the Ocean, Inlet, and Bay System.

The ocean/inlet/bay system is separated into three distinct regions, each described by a velocity potential: Φ_1, Φ_2, and Φ_3, respectively. The ocean and inlet systems have been described by Dalrymple and Martin (1996), who separated the problem into a symmetric problem and an antisymmetric problem, just as we will do here. In the ocean, Fourier transforms are used to represent the wave field. In the inlet, an eigenfunction expansion is used, with a similar expansion in the rectangular bay. Both problems are solved by matching the water surface elevations and normal velocity across the mouth of the inlet and where the inlet meets the bay.

All of the potential functions in this constant-depth problem are assumed to be separable, and the time and depth dependencies are given explicitly. Thus, we have

$$\Phi_i(x,y,z,t) = \mathrm{Re}\left\{-\frac{\mathrm{i}gH}{2\omega}\,\phi_i(x,y)\frac{\cosh k(h+z)}{\cosh kh}\,e^{-\mathrm{i}\omega t}\right\}$$

$$\text{where } i = 1,2,3,\ldots, \qquad (1)$$

H is the wave height, g is the acceleration of gravity, and the wavenumber k and the frequency ω are related by the dispersion relationship

$$\omega^2 = gk\tanh kh.$$

Here, z is the vertical coordinate, with the mean free surface at $z = 0$ and the rigid bottom at $z = -h$. The potential terms, ϕ_i, are dimensionless.

In the ocean, the incident waves reflect from the shoreline, creating a standing wave system; therefore the total wave field in the ocean is

$$\begin{aligned}\phi_1(x,y) &= 2\cos(ky\sin\theta)\cos(kx\cos\theta) \\ &+ \frac{1}{\pi}\int_0^\infty A^s(\lambda)\,e^{-\mathrm{i}\sqrt{k^2-\lambda^2}x}\,\cos\lambda y\,d\lambda \\ &+ 2\mathrm{i}\sin(ky\sin\theta)\cos(kx\cos\theta) \\ &+ \frac{1}{\pi}\int_0^\infty A^a(\lambda)\,e^{-\mathrm{i}\sqrt{k^2-\lambda^2}x}\,\sin\lambda y\,d\lambda, \qquad (2)\end{aligned}$$

where θ is the angle of incidence. The first line on the right-hand side is the symmetric part of the solution (an even function of y) and the second line is the antisymmetric part (an odd function of y).

Within the channel, the potential is

$$\begin{aligned}\phi_2 &= \sum_{n=0}^{\infty}\left(\zeta_n^s\,e^{\mathrm{i}\beta(\lambda_n^s)x} + \overline{\zeta_n^s}\,e^{-\mathrm{i}\beta(\lambda_n^s)(x-\ell)}\right)\cos(\lambda_n^s y), \\ &+ \sum_{n=0}^{\infty}\left(\zeta_n^a\,e^{\mathrm{i}\beta(\lambda_n^a)x} + \overline{\zeta_n^a}\,e^{-\mathrm{i}\beta(\lambda_n^a)(x-\ell)}\right)\sin(\lambda_n^a y), \\ &\text{for } 0 \le x \le \ell \text{ and } -b < y < b. \qquad (3)\end{aligned}$$

The first term again represents the symmetric part of the solution and the second term is the antisymmetric part. The terms with the unknown coefficients ζ_n^s and ζ_n^a are waves propagating down the channel in the positive x direction, while the terms involving $\overline{\zeta_n^s}$ and $\overline{\zeta_n^a}$ represent reflected waves propagating from the end of the channel towards the ocean. λ_n^s and λ_n^a are defined by

$$\lambda_n^s = n\pi/b \quad\text{and}\quad \lambda_n^a = (n+\tfrac{1}{2})\pi/b,$$

respectively, which guarantees that the assumed solutions satisfy a no-flow boundary condition at the channel sidewalls, $\partial\phi_2/\partial y = 0$ at $|y| = b$. The function $\beta(\lambda)$ is defined by

$$\beta(\lambda) = \sqrt{k^2-\lambda^2} \quad \text{for } \lambda \le k$$

$$= \mathrm{i}\sqrt{\lambda^2-k^2} \quad \text{otherwise.}$$

The bay solution (symmetric + antisymmetric) is given by

$$\phi_3 = \sum_{n=0}^{\infty} c_n^s \cos\{\beta(\alpha_n^s)\,[x-(\ell+q)]\}\cos(\alpha_n^s y)$$

$$+ \sum_{n=0}^{\infty} c_n^a \cos\{\beta(\alpha_n^a)\,[x-(\ell+q)]\}\sin(\alpha_n^a y),$$

$$\text{where} \quad \alpha_n^s = n\pi/c \quad \text{and} \quad \alpha_n^a = (n+\tfrac{1}{2})\pi/c. \qquad (4)$$

By construction, ϕ_3 satisfies the no-flow boundary condition at the harbor sidewalls, $|y| = c$, and at the end-wall, $x = \ell + q$.

The matching between ϕ_2 and ϕ_3 proceeds as follows. First, we require that the velocities in the x direction at the junction of the channel and the bay ($x = \ell$) match:

$$\frac{\partial\phi_3}{\partial x} = \begin{cases} \dfrac{\partial\phi_2}{\partial x} & \text{for } |y| \le b, \\ 0 & \text{for } b < |y| < c. \end{cases}$$

Examining the symmetric part first, we have

$$\sum_{n=0}^{\infty} c_n^s\beta(\alpha_n^s)\sin\{\beta(\alpha_n^s)q\}\cos(\alpha_n^s y) =$$

$$\begin{cases} \displaystyle\sum_{n=0}^{\infty}\left(\zeta_n^s\, e^{\mathrm{i}\beta(\lambda_n^s)\ell} - \overline{\zeta_n^s}\right)\mathrm{i}\beta(\lambda_n^s)\cos(\lambda_n^s y), \\ \qquad\qquad \text{for } 0 \le y \le b, \text{ or} \\ 0, \qquad\quad \text{for } b < y < c. \end{cases}$$

This equation with its infinite number of unknowns, ζ_n^s, $\overline{\zeta_n^s}$ and c_n^s, can be reduced to an infinite set of equations by using the orthogonality of $\cos(\alpha_n^s y)$ over $0 < y < c$. Multiplying both sides by $\cos(\alpha_m^s y)$ and integrating, we obtain

$$c_m^s = \frac{\epsilon_m}{c\beta(\alpha_m^s)\sin\{\beta(\alpha_m^s)q\}}\cdot$$

$$\textstyle\sum_{n=0}^{\infty}\left(\zeta_n^s\, e^{\mathrm{i}\beta(\lambda_n^s)\ell} - \overline{\zeta_n^s}\right)\mathrm{i}\beta(\lambda_n^s)L_{mn}^s,$$

where $\epsilon_m = 1$ for $m = 0$ and 2 otherwise, and

$$L_{mn}^s = \int_0^b \cos(\lambda_n^s y)\cos(\alpha_m^s y)\,dy$$

$$= \frac{-\alpha_m^s(-1)^n\sin(\alpha_m^s b)}{(\lambda_n^s)^2-(\alpha_m^s)^2}$$

The second matching condition is that the water surfaces must be continuous between the channel and bay, or $\phi_2 = \phi_3$ at $x = \ell$ for $-b < y < b$. This gives

$$\sum_{n=0}^{\infty} c_n^s\cos\{\beta(\alpha_n^s)q\}\cos(\alpha_n^s y)$$

$$= \sum_{n=0}^{\infty}\left(\zeta_n^s\, e^{\mathrm{i}\beta(\lambda_n^s)\ell} + \overline{\zeta_n^s}\right)\cos(\lambda_n^s y),$$

$$0 \le y \le b. \qquad (5)$$

In this case, we use the orthogonality properties of the $\cos(\lambda_n^s y)$ over the range $0 \le y \le b$. Multiplying both sides by $\cos(\lambda_m^s y)$ and integrating, we obtain

$$\frac{b}{\epsilon_m}\left(\zeta_m^s\, e^{\mathrm{i}\beta(\lambda_m^s)\ell} + \overline{\zeta_m^s}\right) = \sum_{n=0}^{\infty} c_n^s\cos\{\beta(\alpha_n^s)q\}L_{nm}^s. \qquad (6)$$

Substituting for c_n^s from equation (5) and rearranging, we have

$$\sum_{n=0}^{\infty}\left\{\zeta_n^s\, e^{\mathrm{i}\beta(\lambda_n^s)\ell}\left(-\mathrm{i}\beta(\lambda_n^s)F_{nm}^s + \frac{bc}{\epsilon_m}\delta_{nm}\right)\right.$$

$$\left. + \overline{\zeta_n^s}\left(\mathrm{i}\beta(\lambda_n^s)F_{nm}^s + \frac{bc}{\epsilon_m}\delta_{nm}\right)\right\} = 0 \qquad (7)$$

where

$$F_{nm}^s = \sum_{r=1}^{\infty}\frac{\epsilon_r\cos\{\beta(\alpha_r^s)q\}}{\beta(\alpha_r^s)\sin\{\beta(\alpha_r^s)q\}}L_{rn}^s L_{rm}^s$$

and $\delta_{nm} = 0$ unless $n = m$ in which case it is equal to unity.

This set of equations, (7), for the symmetric coefficients, ζ_n^s and $\overline{\zeta_n^s}$, is combined with the symmetric equations from the channel mouth at $x = 0$ and solved simultaneously. From ζ_n^s and $\overline{\zeta_n^s}$, the solution in the inlet, bay and ocean are then known for the symmetric part of the solution; this is satisfactory only for normal wave incidence ($\theta = 0$). In practice, of course, the number of terms in the solutions have to be truncated to a reasonable value, defined as NN.

To treat the antisymmetric case, the equations that result from the two matching conditions at the inlet/bay junction are found to be

$$\sum_{n=0}^{\infty}\left\{\zeta_n^a\, e^{\mathrm{i}\beta(\lambda_n^a)\ell}\left(-\mathrm{i}\beta(\lambda_n^a)F_{nm}^a + \frac{bc}{2}\delta_{nm}\right)\right.$$

$$\left. + \overline{\zeta_n^a}\left(\mathrm{i}\beta(\lambda_n^a)F_{nm}^a + \frac{bc}{2}\delta_{nm}\right)\right\} = 0 \qquad (8)$$

where

$$F^a_{nm} = \sum_{r=1}^{\infty} \frac{\cos\{\beta(\alpha^a_r)q\}}{\beta(\alpha^a_r)\sin\{\beta(\alpha^a_r)q\}} L^a_{rn} L^a_{rm}$$

and

$$L^a_{mn} = \int_0^b \sin(\lambda^a_n y)\sin(\alpha^a_m y)\,dy$$

$$= \frac{\alpha^a_m(-1)^n \cos(\alpha^a_m b)}{(\lambda^a_n)^2 - (\alpha^a_m)^2}.$$

Combined with the ocean/inlet equations, equation (8) provides ζ^a_n and $\overline{\zeta^a_n}$, thus giving the antisymmetric part of the solution in the ocean, inlet and bay.

Combining the antisymmetric and symmetric solutions gives the complete solution for the general case of oblique wave incidence.

3 RESULTS

Figure 2 shows a normally incident wave train into a rectangular harbor. The harbor is 4.3 times wider than the entrance channel and 3.3 times longer. (The two horizontal scales are distorted in this figure.) The interaction between the harbor and the ocean is clear and, in this case, extends far offshore. (Note: the offshore area is plotted using a Hankel function representation of the offshore seastate–see Dalrymple and Martin, 1996). A measure of the matching between regions is the comparison of the velocities between regions and/or the comparison of water surface elevations (or velocity potential). Of the two, the cleanest match is with the water surfaces, as the velocities are contaminated by the Gibbs phenomena. This is due to the discontinuity in the bay $x-$velocity at the channel entrance, $x = \ell$, as a result of the no-flow conditions on the bay sidewalls and the flow in the channel. The water surface matching does not have that discontinuity as the eigenfunction expansion for pressure (and water surface) are done over the width of the channel alone. However, the matching is not as perfect as one would hope. Table 1 shows the mean squared error, ϵ_η in the matching as a function of the number of terms taken in the summations for the potentials, NN. The mean squared error is defined as:

$$\epsilon_\eta = \frac{1}{50}\sqrt{\sum_{i=1}^{50}(\phi_2(i) - \phi_3(i))^2}, \qquad (9)$$

where fifty equally spaced points across the channel junction with the harbor are used. For all the figures in this paper, $NN=$ 15, or 15 terms were taken in each summation.

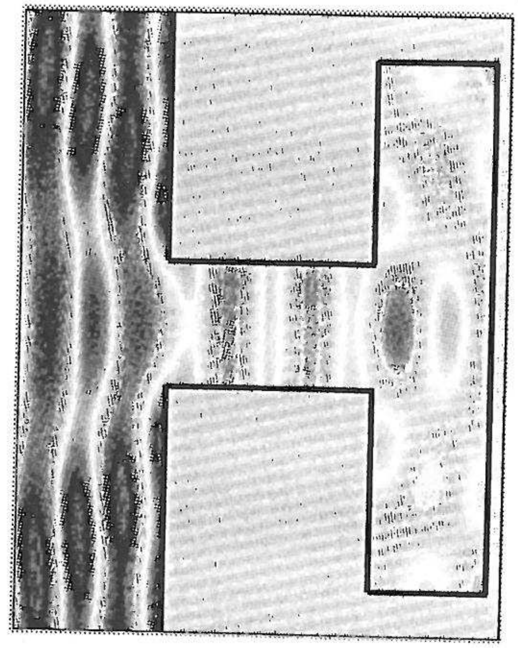

Figure 2: Normally Incident Waves into a Harbor: $kb = 2.5$, $\ell/b = 6.33$, $c/b = 4.3$, $q/b = 3.3$

3.1 Resonances and Wave Height Reduction

This symmetrical configuration of the ocean, inlet, and harbor is subject to a variety of resonances as is well-known. The first obvious ones are those that occur in the harbor, due to standing waves in the y direction or in the x direction. Mathematically this is stated: $k = (n+1)\pi/(2c)$ or $k = (m+1)pi/q$, where, again, c is the half-width of the harbor and q is the length of the harbor. These resonance conditions permit an integer number of half wave lengths to fit into a given dimension. For normal wave incidence, which dictates a symmetric solution, only whole wavelength can fit into the harbor width (c); or $k = n\pi/c$.

The first set of results are for normally incident waves. Figure 3 shows the incident wave field on a inlet/harbor system with $kb = 2.95$, $\ell/b = 6.33$, $c/b = 4.3$, $q/b = 3.3$. This situation is near reso-

Table 1: Root-mean-square Water Level Match (Potential) versus NN

NN	ϵ_η
3	0.02686
5	0.01250
7	0.00748
12	0.00394
15	0.00308
21	0.00206
30	0.00137

nance for both of the harbor dimensions.

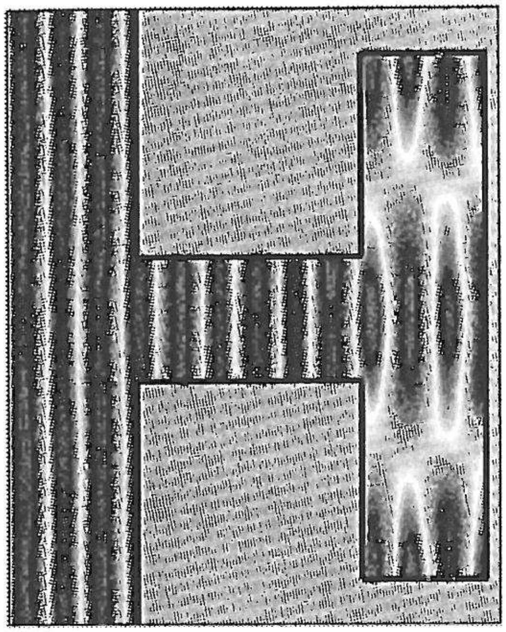

Figure 3: Normally Incident Waves: Near Resonance in Both Harbor Directions.

An interesting case occurs when the phase of the wave radiating from the inlet to the sea is 180°out of phase with the incident wave field. This occurs when the wave number is 4.38. This is shown in Figure 4, which shows the cancellation of the waves in the front of the harbor. (This cancellation is unlikely to be useful for wave design, due to the high degree of tuning necessary.) The distance seaward for which there is a noticable effect on the waves by the radiated waves from the harbor, for the cases here, is about twice the width of the harbor system (or $2*(\ell+q)$).

To show the effect of oblique wave incidence, Figure 5 shows the wave field for 8° degrees angle of incidence (from bottom left). Note the intensification of the wave height on the upwave side of the inlet channel and the downwave side of the harbor.

Figure 6 shows a wave angle of incidence of 45o at near resonance in the transverse harbor direction. The wave heights in the harbor are about 2.2 times those in the ocean.

4 CONCLUSIONS

An analytical solution has been provided for waves into a symmetric inlet/harbor system. The linear analysis, using eigenfunction expansions and Fourier transforms is a very efficient method to develop wave field (both long and short) within a

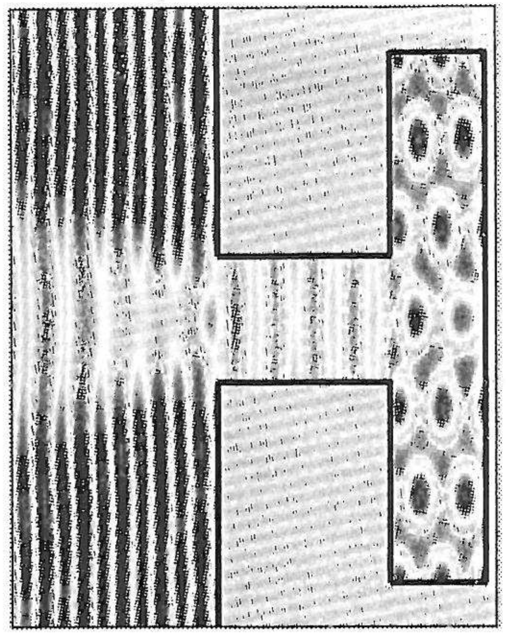

Figure 4: Normally Incident Waves: Near Resonance in Transverse Direction of Harbor. $kb = 4.38$, other geometry the same.

harbor system.

The interaction with the ocean is a critical aspect of treating the waves into the harbor. As previously shown by Miles and Munk, the wave energy radiating from the harbor is important. Further it can significantly alter the wave heights in front of the harbor for a distance offshore comparable to about two times the length of the inlet/harbor system.

As is well-known, there are a variety of resonances within the harbor. The resonances occur in both directions in the channel, and both directions in the harbor. Discontinuities in water depth at any transition (inlet to harbor, for example) will lead to stronger resonances. Since there are an infinite number of resonances in all these cases, there is high likelihood that waves are reinforced within the system in more than one direction.

There is the possibility of including channel damping in this case, following the eigenfunction expansion of Dalrymple (1992), or using Momoi's method as described in Dalrymple, Martin, and Li (1999). This has yet to be done, but would provide for far more realistic decay of wave height within the harbor system, due to sidewall energy absorption.

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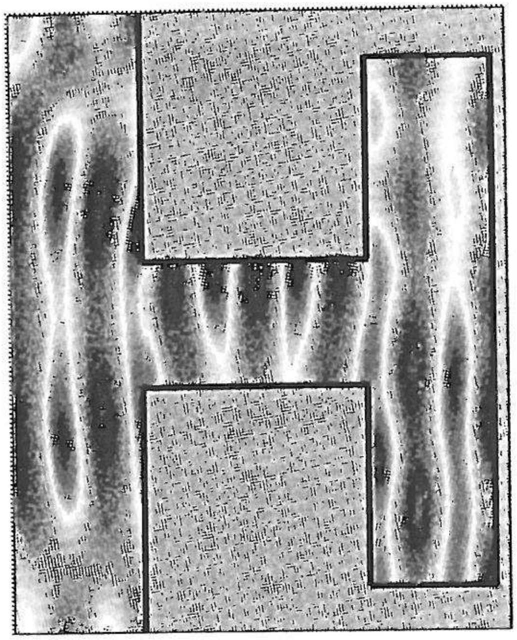

Figure 5: Obliquely Incident Waves at 8°: Near Resonance in Harbor. $kb = 2.85$, other geometry the same.

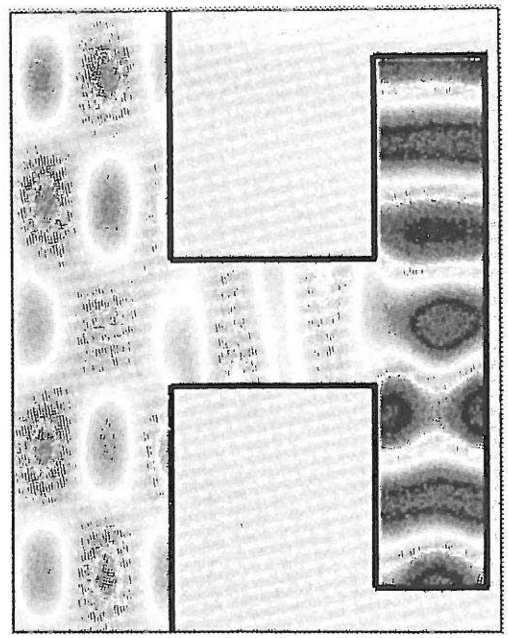

Figure 6: Obliquely Incident Waves at 45°: Near Resonance in Transverse Direction of Harbor. $kb = 2.198$, other geometry the same.

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