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Electromagnetic Scattering Modeling Using Boundary Element Method

J. Chao, Y. J. Liu*, F. Rizzo*, L. Udpa and P. Martin**

Department of Electrical and Computer Engineering
Iowa State University
Ames, Iowa 50011

* Department of Aerospace Engineering and Engineering Mechanics
Iowa State University
Ames, Iowa 50011

** Department of Mathematics
University of Manchester
Manchester M13 9PL, U.K.

ABSTRACT

This paper presents a boundary integral equation (BIE) approach for the problem of electromagnetic scattering from a three dimensional homogeneous dielectric body. The boundary integral equations are formulated by applying the equivalence principle. The integral equation is then solved numerically by using the boundary element method. In particular, an isoparametric formulation which involves second order shape functions is used to represent both the surface geometry and the equivalent surface currents.

1. Introduction

The problem of electromagnetic scattering by an arbitrarily shaped dielectric object suspended in a medium is of interest in several applications. One such application is the detection of contaminants such as dust particles on semiconductor devices and optical surfaces during manufacture. The presence of such contaminants can affect yield and cause reliability problems. In optical devices, contaminants can degrade the image contrast and reduce the signal to noise ratio. Optical techniques are beginning to find widespread application as a tool for accurate detection of particles on surfaces. The approach primarily consists of propagating an electromagnetic wave and allowing the wave to interact with the scatterer. The scattered wave can be analyzed to determine the presence and characterize the properties of the contaminants.

This paper presents a BIE approach for modeling the propagation of an incident TEM wave onto the scatterer. The scattered fields around the body is calculated using the integral equation approach where a set of equivalent surface currents on the scatterer surface is viewed as sources producing the corresponding scattered fields [Mar84]. The formulated integral equations, however, contain the Green's function and spatial derivatives of the Green's function which are sources of high order singularities. The order of singularities can be reduced through regularization processes [Kri92].

The numerical reduction of integral equations to algebraic equations is done using the quadratic isoparametric elements where the geometry of the scatterer and the functions defined on the surface of the scatterer are approximated by a set of quadratic shape functions. This numerical technique commonly known as the boundary element method (BEM) is the focus of this paper. BEM has been shown to be extremely effective in solving scattering problems due to acoustic waves [Liu92] and elastic waves as shown in [Riz85] [Mar89]. The formulation of the electromagnetic scattering problem using BIE approach and the numerical implementation of the method are described next.

2. Boundary Integral Equations

Consider the problem where a scatterer having material properties of ε_i and μ_i is suspended in a medium with material properties of ε_e and μ_e as illustrated in Figure 1.

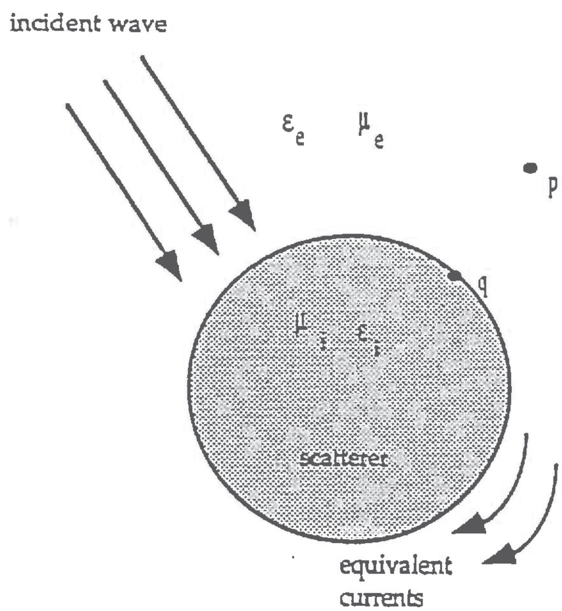

Figure 1. Scattering configuration

By using the vector potential equation

$$A = -\mu \int_s J(q)\, G(p,q)\, ds_q \qquad (1)$$

$$B = -\varepsilon \int_s M(q)\, G(p,q)\, ds_q \qquad (2)$$

wher

A	= magnetic vector potential
B	= electric vector potential
J	= surface electric current
M	= surface magnetic current
G	= Green's function

and the vector identity

$$\nabla \times \nabla \times \quad = \nabla(\nabla \cdot\ ) - \nabla^2 \qquad (3)$$

we can obtain the Stratton-Chu representation of the electromagnetic waves

$$E(p) = \nabla \times \int_s M(q)\, G(p,q)\, ds_q + \frac{j}{\omega\varepsilon} \nabla \times \nabla \times \int_s J(q)\, G(p,q)\, ds_q \qquad (4)$$

$$H(p) = \nabla \times \int_s J(q)\, G(p,q)\, ds_q - \frac{j}{\omega\mu} \nabla \times \nabla \times \int_s M(q)\, G(p,q)\, ds_q \qquad (5)$$

where

E	= electric field
H	= magnetic field
μ	= permeability
ε	= ermittivity
p	= an observation point either inside or outside the scatterer
q	= a source point on the surface of the scatterer.

The governing integral equations for computing the equivalent currents are obtained by approaching the surface of the scatterer from both inside and outside the scatterer. This will result in singularities within the integral equations. By taking the limit as p approaches the surface, we obtain the boundary integral equations for the problem as

$$2\, n \times H_{inc}(p) = J - \nabla \times \int_s J(q)\, G_e(p,q)\, ds_q - \frac{j}{\omega\mu_e} \nabla \times \nabla \times \int_s M(q)\, G_e(p,q)\, ds_q \qquad (6)$$

$$-2\,\mathbf{n} \times \mathbf{E}_{inc}(p) = \mathbf{M} - \nabla \times \int_s \mathbf{M}(q)\, G_e(p,q)\, ds_q + \frac{j}{\omega \varepsilon_e} \nabla \times \nabla \times \int_s \mathbf{J}(q)\, G_e(p,q)\, ds_q \qquad (7)$$

$$0 = \mathbf{J} + \nabla \times \int_s \mathbf{J}(q)\, G_i(p,q)\, ds_q + \frac{j}{\omega \varepsilon_i} \nabla \times \nabla \times \int_s \mathbf{M}(q)\, G_e(p,q)\, ds_q \qquad (8)$$

$$0 = \mathbf{M} + \nabla \times \int_s \mathbf{M}(q)\, G_i(p,q)\, ds_q - \frac{j}{\omega \mu_i} \nabla \times \nabla \times \int_s \mathbf{J}(q)\, G_e(p,q)\, ds_q \qquad (9)$$

where

E_{inc} = incident E - field
H_{inc} = incident H - field

The equivalent surface currents are computed from the tangential components of the electromagnetic waves on the surface. In general, either Equations (6) and (8) or Equations (7) and (9) are sufficient to solve for the surface currents. However, at certain frequencies where the exterior wavenumber is an eigenvalue of the interior Maxwell problem, the solution becomes nonunique. This problem, has been very efficiently alleviated through the use of linear combinations of the four equations [Mar92].

A second difficulty in solving the above integral equations is the problem of high order singularities inherent in the Green's function and its spatial derivatives. For example, all four equations contain a term which has singularity of order $O(1/R^3)$. This type of boundary integral equations is commonly referred as hypersingular boundary integral equations (HBIEs). However, the order of singularity can be reduced to a weakly singular case via regularization process developed by Rizzo et. al. In the numerical calculation, a weakly singular integral equation can be solved by performing a coordinate transformation.

3. Boundary Element Method

The boundary element method (BEM) is a numerical technique which solves BIEs numerically by dividing the domain of integration into a set of surface elements. The surface elements can be chosen to be either conforming or nonconforming. In nonconforming elements, the nodes are shared by adjacent elements, thereby, requiring the unknowns to be continuous across the inter-element boundary. In nonconfoming elements, nodes lie in the interior of each element as shown in Figure 2. The elements used in this paper are of nonconforming type. Two commonly used element types are the curvilinear quadrilaterals and the curvilinear triangles as shown in Figure 2. A typical discretization of a spherical scatterer is illustrated in Figure 3. This, in essence, transforms the continuous integral equations into discrete linear matrix equations. The linear equations with nodal parameters as unknowns are then solved using well established numerical techniques. The unknown function at any point on the surface can then be obtained via interpolation using prespecified shape functions and the known nodal values. The most attractive feature of this method when compared to other numerical techniques such as the finite element method (FEM), is that whereas FEM solves for the unknowns throughout the region of interest, the BEM solves for unknown only at the boundaries, thus reducing the computational effort.

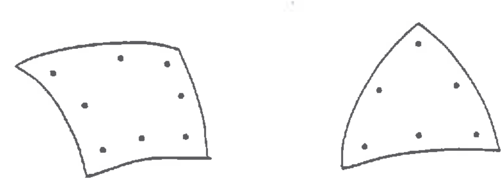

Figure 2. Typical nonconforming surface elements [Liu92]

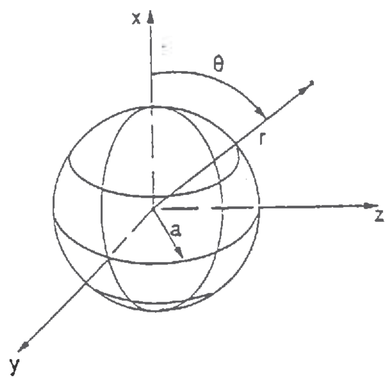

Figure 3. Discretization of a spherical scatterer[Liu92]

4. Conclusion

The BEM has shown to be a very effctive method in solving scattering problems due to acoustic waves and elastic waves. The major advantage of the boundary element method is the computational efficiency offered by the technique, since unknowns are solved only on the surface of the scatterer as opposed to solving for unknown current values numerically throughout the volume. Presently, the numerical simulation is under development and the results will be presented at the conference.

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