

Multiple Scattering: Interaction of Time-Harmonic Waves with N Obstacles

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CORRECTIONS AND ADDITIONS

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Corrections

p. 34. Example 2.5. Insert $(-1)^{n-1}$ on right-hand side, giving

$$\left(\frac{\partial}{\partial x} \pm i \frac{\partial}{\partial y}\right)^n \log r = (-2)^{n-1} (n-1)! \frac{e^{\pm in\theta}}{r^n}, \quad n = 1, 2, \dots$$

Error found by A. J. Yuffa.

p. 80. There is a short paragraph about the axisymmetric addition theorem:

The special case of Theorem 3.15 in which $z_0 = j_0$ was published by Clebsch in 1863; see [M1298, p. 363]. According to Watson, the special case with $z_0 = y_0$ is due to Gegenbauer: it is formula (3) on p. 365 of [M1298] with $\nu = \frac{1}{2}$.

This is not quite right. Replace by:

Theorem 3.15 was proved by Clebsch [77, p. 227] in 1863.

Gegenbauer gave a more general result. Email exchanges with A. F. Rawle encouraged me to look at Clebsch's paper.

p. 81. Missing $(-1)^l$ in (3.67). It should read

$$B_l^{(n)} \equiv B_l^{n,0} = \frac{(-1)^l (2n-2l)!}{2^n l! (n-l)! (n-2l)!}. \quad (3.67)$$

This agrees with $B_l^{n,m}$ defined by (3.27).

p. 133. The formula (4.45) is correct but the derivation is not. For a proof, see [80, Theorem 2.16]. The same criticism applies to the formula (4.10) on p. 124.

p. 213. Remove overbars in (6.108), giving

$$[U, V] = \int_{\Omega_a} \left(U \frac{\partial V}{\partial r} - V \frac{\partial U}{\partial r} \right) ds = -[V, U]. \quad (6.108)$$

Error found by M. Ganesh.

p. 269. Theorem 7.4. The second property, (7.68), holds for lossless scatterers. The proof sketched assumes that

$$\int_S \left(u \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial u}{\partial n} \right) ds = 0.$$

p. 275. Remove hat from $\hat{\psi}_\nu^\mu$ in last displayed equation at bottom of page, giving

$$\psi_n^m(\mathbf{r}_j^P) = \sum_{\nu, \mu} \hat{S}_{n\nu}^{m\mu}(-\mathbf{b}_j) \psi_\nu^\mu(\mathbf{r}_P), \quad j = 1, 2.$$

Error found by K. Spalding.

- p. 303. At the end of the displayed equation below (8.72), replace $h_2(kR_{rj})$ by $h_2(kR_{nj})$.
- p. 362. Reference 72. It should be F.G. Bass, not F. Bass. ([71] is correct.)
- p. 368. Reference 212. It should be pages T14–T25, not T11–T25.
- p. 379. Reference 459. It should be volume **14**, not **18**.
- p. 411. Reference 1176. Remove one *PCPS*.

1.2. Narrowing the scope

[End of first paragraph]: Introductory overviews include [230, 231]. For electromagnetic scattering, see [240].

There is a 1968 book by Ivanov, with a 557-page English translation in 1970 [144]; the Russian author is Evgeniĭ Ivanov, cited as both E. A. Ivanov and Ye. A. Ivanov. The book describes methods based on separation of variables and addition theorems (see Chapter 4) for two scatterers: circles, ellipses, spheres, spheroids and circular discs.

1.5.3. [Elastic] transmission problems

“For more information on [fluid-solid interactions]...”, add [132, 140]

2.9.1. Addition theorem [for elliptical wavefunctions]

[p.53, just before §2.10]: See also [144, Chapter 4, §5].

3.1. Introduction [to 3D addition theorems]

For an overview, covering scalar and vector additions theorems, see [127].

3.12. Addition theorem for $h_n^{(1)}Y_n^m$

Top of p. 91: For alternative proofs of the scalar addition theorems, see [98]. See also [144, Chapter 3, §7].

3.13.1. Computation of the separation matrices

Top of p. 94: For more on recursive methods, see [162].

3.17. Vector spherical wavefunctions

Bottom of p. 107, “For alternative proofs...”, add: [38, §1.8], [71, 98, 348, 220]

4.5.1. Literature [on acoustic Závíška]

For two cylinders... , add [141, 99] and [144, Chapter 6]

For more than two cylinders ...from the 1970s: [131] ...from the 1980s: [26, 35] ...from the 1990s: [103, 223] ...from the 2000s: [333, 332, 322, 288, 25, 218] ...since 2010: [253, 18, 290, 324, 90, 239]

For circles centred at the vertices of concentric regular polygons, see [148].

Chamberlain [60] has combined Závíška’s method with a mild-slope equation inside each cylinder so as to model the scattering of water waves by various circular structures. Random water waves are considered in [149].

Záviška's method has been used to study the interaction of SH waves with tunnels. The earliest paper in this context is by Lee & Trifunac from 1979 [187]. They consider one circular tunnel near the flat boundary of a half-plane, which is equivalent to two circles in a full space. For more tunnels (three in a full space and two in a half-plane), see [346]. Extension to lined tunnels has been made. Each tunnel is circular with a concentric lining in which a different Helmholtz equation holds. There is a Neumann condition on the inner boundary and transmission conditions at the outer boundary. For two lined tunnels in a half-plane, see [28].

[Top of p. 130, discussion of Grote & Kirsch [M444]]. For applications, numerical analysis and generalizations, see [3, 4, 78, 173].

Circular scatterers in a layered medium have been considered in many papers, mainly in the context of electromagnetic waves. A typical problem is two half-spaces with waves in both but circular cylinders in one. For example, see [39, 186, 91, 260, 111, 265, 183, 184, 185]. For cylinders in both half-spaces, see [266]. There are analogous treatments for problems involving a slab sandwiched between two half-spaces. For example, see [181, 109, 110, 112].

[End of §4.5.1, p. 130, on orders-of scattering]. For other applications, see [102, 211, 343].

4.5.3. Numerical solution of the system (4.27)

[Top of p. 132, end of first paragraph]. For another investigation into the choice of M , see [264].

Amirkulova & Norris [11] have exploited the algebraic structure of (4.27) when the cylinders are identical and arranged in a rectangular array, leading to a fast recursive algorithm. Results for up to 1000 cylinders are given when the distance between the centres of adjacent circular cross-sections is $2.02a$. For additional papers dedicated to numerical aspects, see [16, 314].

4.5.4. Three-dimensional problems [New section]

Suppose we have a configuration of parallel cylinders, with generators parallel to the z -axis. Thus far, we have considered incident fields that do not depend on z , but three-dimensional incident fields can be of interest.

The simplest situations occur when we have a plane wave at oblique incidence,

$$u_{\text{inc}}(x, y, z) = \exp \{i(\ell_1 x + \ell_2 y + \ell_3 z)\} \quad \text{with} \quad k^2 = \ell_1^2 + \ell_2^2 + \ell_3^2.$$

Then, assuming the boundary conditions do not depend on z , we can write

$$u_{\text{sc}}(x, y, z) = v(x, y) e^{i\ell_3 z},$$

where v solves the two-dimensional Helmholtz equation in a cross-sectional plane, $(\nabla^2 + \ell^2)v = 0$, with $\ell^2 = \ell_1^2 + \ell_2^2$, together with boundary and radiation conditions. For circular cylinders, the two-dimensional boundary-value problem for v can be solved by multipole methods. (For non-circular cylinders, v can be calculated using methods described later in the book.) Some of the papers cited in Section 4.5.1 use Záviška's method to find v for plane waves at oblique incidence; see, for example, [M1367] and [M700]. See also [182].

Similar methods can be developed for cylinders in a waveguide, with infinite plane walls at $z = \pm h$, say. See [319, 45, 258] and references therein. There are also related water-wave problems for vertical cylinders and water of constant finite depth; see Section 4.11.

When the incident wave is not plane, the natural approach is to use a Fourier transform in the z -direction. This decomposes the problem into a superposition of cross-sectional problems. For incident waves generated by a point source, we can effect the decomposition using Weyrich's formula [DeSanto [M278], §2.9.4]. See also [Boström *et al.* [M133], eqn (2.9)]. For a point source and one circular cylinder, see the 1955 paper by Wait [330]. Wait has discussed analogous electromagnetic problems in his book [331]. For an acoustic point source and an elastic cylinder, see [203]. For an analogous problem with elastic waves generated by a point force near a cylindrical cavity, see Boström & Burden [40]. We are not aware of studies involving two or more cylinders.

For a point source in a waveguide, with infinite plane walls at $z = \pm h$, modal decompositions can be used, with propagating and evanescent modes. (See Section 4.11.1 for such a decomposition in the context of water waves.) Results for acoustic scattering by four penetrable cylinders are given in [45].

4.6.2 Literature [on scattering by one sphere]

[p. 134, end of first paragraph]: Rayleigh’s 1872 paper on scattering by a sphere [309] treats several problems: scattering of a plane wave by a rigid sphere, by a fluid-filled sphere (transmission problem) and by a movable sphere. He also indicates how to proceed when the incident waves are generated by a point source.

4.7.1. Literature [on multipole method for two spheres]

[p. 137, 2nd paragraph, with discussion of Marnevskaia]: At about the same time, Ivanov [144, Chapter 7, §3] made an extensive study of two-sphere problems.

In [282], the authors consider scattering by two spheres when one of the spheres is small ($ka \ll 1$).

[Top of p. 138]: For application to a few hard spheres, see [189]. [After “for 162 cylinders are given.”]: See also [193].

4.8.3. Literature [on multipole method for N spheres]

p. 139. For computational aspects, with results for 100 spheres and $ka = \frac{1}{10}\pi$, see [163].

4.9. Electromagnetic waves [by separation of variables]

[End of first paragraph on p. 140, after “by Mie [M849, M717].”]: See also the book edited by Hergert & Wriedt [128]. For a very detailed study of scattering by a layered sphere, see [298].

[3rd paragraph on p. 140, after “see also [M117, §4.2].”]: add [38, §5.2].

“Many papers on electromagnetic scattering by clusters of spheres . . .” (p. 141): add [15, 284, 96, 241, 205, 204]. Also, add: For a good overview, including computational aspects, see [220]. For scattering of waveguide modes by several spheres inside a tube (or optical fibre), see [74].

[New paragraph, end of §4.9]:

Electromagnetic scattering by spheres near the boundary of a (penetrable) dielectric half-space has been studied by many authors, perhaps starting with the paper by Bobbert & Vlieger [34] on one sphere. For multiple spheres, see [38, Chapter 6], [222].

4.10. Elastic waves [by separation of variables]

[p. 141, 4th paragraph]: “Elastic waves exterior to single circular cylinder. . .”: add [214]. [End of paragraph]: For acoustic scattering by a radially inhomogeneous elastic sphere, see [316].

Bottom of p. 141, “The analogous problems for incident shear waves”: add [146].

Top of p. 142, “Movable rigid spheres are considered in [M1333, M943]”: add [145]. Same paragraph, add: For a review, see [24].

3rd paragraph on p. 142, after “Petrashen and his students”: add [171].

[After “see also [M421].”, middle of 4th paragraph on p. 142]: Avilés & Sánchez-Sesma [26] gave results for scattering of P -waves and S -waves by a row of eight rigid movable cylinders.

[End of 5th paragraph]: “later review [M459].” Add: see also [69].

Moore & Guan [243] have considered elastodynamic problems for two lined tunnels in an unbounded elastic space. They treated the problem using an orders-of-scattering approach. This approach was used earlier for two circular cavities by Sancar & Pao [286], with extensive numerical results and comparisons with experiments in a companion paper by Sancar & Sachse [287]. The twin-lined-tunnel problem has also been considered by Kuo *et al.* [176]. They combine separated solutions with a numerical scheme for computing the interaction effects (so that addition theorems are not used). Transient plane-strain problems with two identical circular cavities have been solved by Chen & Zhang [65] and by Itou [143]. They combine symmetry, separated solutions and Fourier integrals (again, addition theorems are not used); the resulting method is complicated and special. The method of [M103] is extended to plane-strain problems in [310].

For scattering by hexagonal arrangements of circular inclusions with imperfect interfaces, see [337]. For two circular defects where surface elasticity is included, see [334, 259].

[p. 143, end of second paragraph]: Doyle has developed [M292] into a full paper [95] on scattering of elastic waves by spheres.

“For scattering by one poro-elastic sphere, see [M93, M1380]”: add [212]. For Závíska-like studies of circular cavities in an unbounded poroelastic space, see [126, 48].

For two elliptical cavities in an unbounded poroelastic space, there are two papers [336, 352] in which circular-cylindrical wavefunctions are used. This is expected to be valid for low eccentricities, otherwise the Rayleigh hypothesis will be relevant.

4.11. Water waves [by multipoles]

Multipole methods for oblique incidence on several horizontal parallel circular cylinders, submerged in water of finite depth, have been used by Shen *et al.* [295]; similar methods have been used for two-layer fluids [108].

4.11.2. Matched eigenfunction expansions

[First paragraph, after “another expansion of some kind.”] In [1], the authors try to represent the enclosed bodies using the ‘method of fundamental solutions’, as described in Section 7.3.2.

[Bottom of p. 147 to top of p. 148]. After “McIver [M828] ... pair of immersed rectangular cylinders”, add: see also [345]. Then, after “For a pair of thin vertical barriers, see [M982]”, add: See also [297]; this paper includes comparisons with experiments. For a pair of truncated vertical circular cylinders, see [351]. There is an earlier paper by Silva *et al.* [301] in which evanescent modes are neglected. For several truncated vertical circular cylinders, see [299, 72, 340]. For a pair of truncated vertical elliptical cylinders, using Mathieu functions (defined in Section 4.12.1), see [62].

After “In [M927, M815] ... orders-of-scattering ... bodies”, add: for applications to an array of free-floating oscillating water columns, see [170].

For several floating elastic plates, see [169, 168]. For several thin vertical porous barriers, see [158, 159]. For horizontal porous barriers, see [73].

[End of §4.11.2, p. 148]: after “scattering by N rectangular grooves.” Interest in problems of this kind has increased; see [27, 201, 202, 342].

4.12.1. Elliptic cylinders

[Bottom of p. 149]: Scattering by two elliptic cylinders was first analysed by Ivanov [144, Chapter 8, §3].

Nigsch [255] has given results for time-dependent scattering by three elliptical cylinders. Results for time-harmonic scattering by a pair of identical elliptical cylinders were given by Chatjigeorgiou & Mavrakos [63] and by Kleshchev & Kuznetsova [166]; see [61] for a combination of circular and elliptical cylinders.

4.12.2. Spheroids

[End of 2nd paragraph] ... and [M1, Chapter 21]. See also [144, Chapter 5] and [198, Chapter 2].

[3rd paragraph] See also [283, 5]. For scattering by a penetrable spheroid, see [344, 172]. For acoustic scattering by an elastic spheroid, see [300].

[4th paragraph] ... see also [M710], [M870, §6.1] and [198].

[5th paragraph, with insertions] “Addition theorems for spheroidal wavefunctions are available ...” Also [144, Chapter 5, §5]. Ivanov [144, Chapter 9] has given results for two co-axial spheroids including the special case of two circular discs.

[Last paragraph, with insertions] “Addition theorems for vector spheroidal wavefunctions are also available [M1106, M253].” Also: [85]. “They have been used extensively for scattering by two perfectly conducting spheroids [M1107, M252], by two dielectric parallel spheroids [M234] and by spheroids that are oriented arbitrarily [M232, M233, M215].” Also: [82, 252]. “All of these results are reviewed in Chapter 4 of [M866].”

4.12.3. Ellipsoids

[Top of p. 151] “Low-frequency scattering . . . [M262, Chapter 8]” Add “and in [87, Chapter 14].”

5.4.4. Spectral properties [New section]

When S is a single circle of radius a , we can take $x(t) = a \cos 2\pi t$ and $y(t) = a \sin 2\pi t$. Then, all the boundary integral operators defined above have eigenfunctions $e^{2\pi i n t}$ and their eigenvalues can be calculated explicitly in terms of Bessel and Hankel functions. Of course, this follows because separation of variables is available. These spectral properties can be used to estimate condition numbers, for example. For detailed calculations, see Kitahara’s book [M619, Chapter 4], [M659], [174], [M23] and [326]; these four papers also give analogous results for one sphere. For one ellipse, using Mathieu functions, see [33].

Antoine & Thierry [19, 313] have studied the spectral properties of the single-layer operator when S consists of N circles.

5.9.1. Literature [on indirect method]

[End of 2nd paragraph on p. 172, “the indirect method has also been used in”]: add [238, 237]. Domínguez *et al.* [94] have developed an effective method for solving the two-dimensional form of (5.54); they give numerical results for a pair of ellipses.

6.2. Transmission problems [by integral equations]

[Top of p. 180]: Examples of other direct methods . . . [97]. For an application to a transmission problem where the interior Helmholtz equation is replaced by Poisson’s equation, see [244].

[New paragraphs, end of §6.2, p. 180]: When an obstacle has internal interfaces, essentially two situations can arise. The simpler is when the interfaces are disjoint as with an onion: for concentric circles or spheres, separated solutions can be written down for each layer, and then patched together using the transmission conditions across each interface. When the geometry is more complicated, more-or-less standard boundary integral formulations can be developed. For scattering by several layered ellipses, see [292].

Suppose now that interfaces meet. For example, consider a circular scatterer with a single internal interface along a diameter. The two points at the ends of the diameter (known as ‘triple points’ or ‘junctions’) can be reached from three different subdomains (the two halves of the scatterer and the exterior) and this makes it more difficult to construct well-conditioned systems of boundary integral equations; see [50, 104, 123, 75, 76] and references therein. The difficulties at junction points have been avoided [268] by using simple boundary element methods (with piecewise-constant approximations and collocation at element midpoints). In fact, there are several earlier papers by Ström & Zheng [308, 349, 350], [M1155, M1376] on electromagnetic scattering by objects with interior interfaces (such as a ball composed of two dissimilar halves); they use null-field methods (see Chapter 7).

6.3.4. An alternative equation [for inhomogeneous obstacles]

Equations similar to (6.28), involving both volume and surface integrals, were derived in two dimensions by Jin *et al.* [150]; see also [329, §3.2.5]. For three-dimensional electromagnetic problems, see [328] and [329, §3.1.7].

6.4.6. Electromagnetic waves [by integral equations]

[p. 195, end of 4th paragraph, on books]: add [329].

For applications of the EFIE and the MFIE to scattering by several polygonal cylinders, see [217].

For applications of (6.34) to three-dimensional multiple scattering problems, see [115].

For applications of the EFIE to three-dimensional multiple scattering problems, see [117].

For scattering by a dielectric sphere near a thin square plate, see [22].

For applications of volume integral equations to scattering by many spheroids, see [321, 30]. For other shapes, see [66]. For a review of the work done by Tsang and his collaborators, see [320].

For inhomogeneous scatterers, boundary integral equations for the exterior region can be coupled to finite-element discretisations inside the scatterers [83].

6.5.5. Elastic waves [by integral equations]

[p. 202, 2nd paragraph]: Numerical results for two-dimensional problems have been presented in [263]. [End of paragraph, on 3D transmission problems]: add [44].

[p. 202, end of 3rd paragraph]: “Orthotropic cylinders are considered in [M698]” and [180].

[p. 202, 4th paragraph on half-space problems]: for the indirect method [using \mathbf{G}], see [M1052, M752]: add [281]; for the direct method, see [M1018, M593]: add [262].

Sheng *et al.* [296] have used \mathbf{G} for tunnels in a half-space. They used a hybrid method, coupling finite elements around the tunnels with a boundary integral representation for the surrounding exterior domain.

Chaillat & Bonnet [54] have shown how the computation of \mathbf{G}^H can be improved so that a version of the Fast Multipole Method is effective.

[p. 202, 5th paragraph on anisotropy]: For inhomogeneous scatterers, using volume integral equations, see [36].

For a collection of elastodynamic fundamental solutions, see Kausel’s book [160].

Flexural waves in thin elastic plates can be modelled using $(\nabla^4 - k^4)w = 0$, where $w(x, y)$ is the out-of-plane displacement of the plate, ∇^2 is the two-dimensional Laplacian and k is a wavenumber. For background, see Kitahara’s book [M619]. For scattering by a hole of arbitrary shape in an infinite plate, see [303]. For several circular holes and inclusions, see [190, 191, 192].

[Top of p. 203]: After “several reviews”: add [224]

6.6.3. Water waves [by integral equations]

[End of first paragraph on p. 205]: For two flat *elastic* strips, see [129].

[End of 4th paragraph on p. 205]: For applications of (6.81) to several wave-energy devices, see [106].

[Near the end of 5th paragraph on p. 205]: After “floating buoys”, add: For an application of *WAMIT* to 32 floating axisymmetric wave-energy devices, see [105]. Then “For other applications of the direct method to multibody problems, see [M303, M510]”: add [353, 311, 124].

[After citation of Chakrabarti [M181], bottom of p. 205]: Chen & Mahrenholtz [68] solved (6.83) and related integral equations for two floating cylinders in finite-depth water. [End of same paragraph] . . . between two ships, see [M837, M591]: add [197].

Porter [271] started with the hypersingular integral equation obtained from the normal derivative of (6.82), for two-dimensional problems and submerged cylinders. He regularised the equation by using the Cauchy–Riemann equations, leading to a Fredholm integral equation of the first kind with a symmetric weakly-singular kernel. He applied this method in a search for trapped modes above a pair of elliptical cylinders [272].

[End of last-but-one paragraph in §6.6.3, p. 206, after citations of [M506, M527, M28]]: Cho *et al.* [73] used integral equations based on a simple fundamental solution in a finite region containing two submerged inclined plates, and then matched these with eigenfunction expansions (as described in Section 4.11.2).

[End of last paragraph in §6.6.3, p. 206, after citation of [M304]]: Leonard *et al.* [195] used finite elements, truncating the computational domain with simple outgoing-wave conditions, and gave results for two half-immersed circular cylinders. See also [289, 251] for similar approaches. More recently, several authors [306, 199, 200] have used the ‘scaled boundary finite element method’ for two or three vertical cylinders in water of finite depth.

6.7. Cracks and other thin scatterers

[p. 207. End of paragraph about use of Fourier transforms]: For scattering by parallel finite cracks in a thin elastic plate over water, see [273].

6.7.1. Two-dimensional [crack] problems

[Top of p. 208] ...acoustic scattering by several straight rigid strips [M443, M1124]: add [142]. Murai *et al.* [246] gave results for scattering by up to 72 identical parallel rigid strips using a piecewise-constant approximation for $u(t)$ on each strip. For two coplanar sound-soft strips, see [291]. Hewett *et al.* [130] have developed a hybrid analytical-numerical method for scattering by N coplanar sound-soft strips.

“For the interaction of water waves with thin plates ...M397”]: add [119].

Analogous problems arise when water waves are scattered by thin vertical rectangular barriers in a three-dimensional ocean of constant finite depth; this is a special case of the formulation described in §1.6.4, where each vertical cylinder has no interior. Such problems have been solved using hypersingular integral equations and Chebyshev expansions in the context of wave-energy devices; see [279] for one device and [278] for several.

6.7.2. Three-dimensional [crack] problems

[Top of p. 209]. For two coplanar rectangular cracks, see [210]. For scattering by a penny-shaped crack near a rigid movable disc or a spherical inclusion, see [250, 247]. Earlier, Mykhas'kiv & Khai had derived the governing integral equations for N plane cracks and then given results for two or three coplanar penny-shaped cracks [248, 249].

For scattering by two coplanar discs, using special integral-equation methods, see [79, 315].

6.8. Modified integral equations: general remarks

“There is an extensive literature ...for reviews, see ...[M959, Chapter 6]”: add [226, 92]. Note that [92] includes numerical results for scattering by four sound-hard circles.

6.10.1. A combined-layer method

Numerical solutions of (6.121) and (6.122) for two three-dimensional obstacles have been presented by Ganesh & Hawkins [114]. Subsequently, they [116] and Bremer *et al.* [42] solved (6.122) for thousands of soft obstacles.

6.10.2. The method of Burton and Miller

p. 218. “Buffa & Hiptmair [M158] first multiply (5.59) by a regularising operator ...Dirichlet problem.” This idea has been developed in several subsequent papers; see, for example, [17, 43].

[Top of p. 219] “However, we are not aware of any published numerical results for multiple-scattering problems, based on (6.124).” In 2007, Shen & Liu [293] combined (6.124) with a fast multipole method (see §6.14) for scattering by many objects; see also [341] for scattering by a cube containing 27 spherical inclusions.

“The choice of η has been investigated in [M23].” For a review, see [225].

[End] “in particular, [M404] contains results for scattering by two circular cylinders.” The method used in [M404] is designed for high-frequency problems and it uses an iterative orders-of-scattering approach. It has been analysed in detail [101] and it has been extended to three dimensions [14] with results given for scattering by two soft ellipsoids.

6.11. Augmentation methods

[p. 221 “We know of two ... [M287, M1353].” For an application to three two-dimensional scatterers, see [267].

6.11.1. Extensions [Augmentation methods]

[There is an unnumbered equation on p. 221]:

$$u(P) = \int_{S \cup S'} \mu(q) G(P, q) ds_q, \quad P \in B_e.$$

This representation has been used by Bremer [41] for two-dimensional problems involving many non-smooth star-like scatterers.

6.12.2.1. Partitioning

Multiplying (6.137) by A_{11}^{-1} and (6.138) by A_{22}^{-1} gives an equivalent pair of equations. Thierry [312] has investigated the numerical consequences of doing this for N scatterers: one can say that the system has been *preconditioned*. He gives some numerical results for $N = 30$.

6.12.3. Generalised Born series for two scatterers

[Bottom of p. 227]: “In [M290] ... elastic half-space.” For an application to scattering by two coplanar penny-shaped cracks, see [165].

“Geuzaine *et al.* [M404] ... $ka = 1000$.” For two ellipsoids, see [14].

[End of §6.12.3, p. 228]: Balabane’s approach has been developed and used when there is a combination of soft scatterers, hard scatterers and scatterers with an impedance (Robin) boundary condition [335].

There are alternative interpretations. For the exterior Neumann problem with two obstacles, we can write $u(P) = u_1(P) + u_2(P)$, where

$$2u_j(P) = \int_{S_j} \left(f_j(q)G(P, q) - u(q) \frac{\partial}{\partial n_q} G(P, q) \right) ds_q,$$

valid for all points P outside S_j (including inside the other obstacle). Thus u_j is a radiating wavefunction outside S_j . To use this formula, we need to find, or approximate, $u(q)$ for $q \in S_j$. For work in this direction, using variants of the ‘on-surface radiation condition’ of Kriegsmann *et al.* [175], see [2, 10].

6.13.5. Extensions [of Twersky’s method]

Kyurkchan has developed a variant of Twersky’s method in several papers, some with collaborators; see, for example, [178, 177] and [179, §3.5]. Results are given for scattering by two obstacles.

6.14. Fast multipole methods

[Bottom of p. 235]: Chapter 6 of Liu’s book [213] is concerned with acoustic scattering.

6.14.2. Literature [on FMM]

[Bottom of p. 241]: For applications of FMM to scattering of two-dimensional elastic waves by many circular inclusions, see [285]. For three-dimensional elastic waves, see the review [53]. For applications to multiple scattering of water waves, see [37].

7.1. Introduction [to null-field and T -matrix methods]

[End of first paragraph, p. 242]: “For electromagnetic problems, see . . . [M873]” and the book [93].

7.3.1. Literature [on Kupradze’s method]

[End of section, p.244]. For a later review, see [179].

7.3.2. Generalized multipole techniques

[End of first paragraph, top of p. 245]: Lee [188] has used outgoing elliptical wavefunctions $\psi_{\sigma n}$ (see Definition 2.15) for χ_m and given numerical results for scattering by two and three ellipses. For several spheroids, using outgoing spheroidal wavefunctions, see [194].

MFS has been used for multiple scattering problems arising in computer graphics, where the aim is to give realistic sounds in virtual environments [147, 234].

[New paragraph, end of §7.3.2, p. 246]: Another related numerical technique is known as the *Trefftz method* [70]. In two dimensions, surround the scatterers by a circle, C . Outside C , approximate u_{sc} using a (finite) series of outgoing cylindrical wavefunctions, ψ_n . Break the region between C and S into a finite number of simply-connected subregions, B_m , $m = 1, 2, \dots, M$. In each B_m , approximate u_{sc} using a (finite) series of wavefunctions; these could be plane waves in various directions or regular cylindrical wavefunctions. Finally, determine the coefficients in each expansion by imposing continuity conditions across each interface and the boundary condition on S . Variants of this approach have been used for multiple scattering problems. For four squares, see [307]. For five circles, see [325]. Trefftz methods have also been coupled with boundary element methods [23].

7.8.1. Regular wavefunctions

Waterman [338] has refined his approach for spheroids and other quadric surfaces. For electromagnetic scattering by spheroids, see [305]. For fluid-solid problems, see [208], where Waterman’s ideas are combined with representations used by Doicu *et al.* [M285].

7.8.4. Numerical experience and convergence

[p. 265, before §7.8.4.1]: For more examples of poor conditioning when computing the Q -matrix using regular wavefunctions, see [304]. For static problems, see [107].

7.8.6. The scattered field

[Bottom of p. 267]: Harness & Ditkowski [125] have given an iterative method for computing the coefficients c_n^m without a direct solution of the null-field equations.

7.9.4. Using a BIE to calculate the T -matrix

[p. 272] “. . . this connection between boundary integral equations and T -matrix methods has not been exploited.” In 2013, Gimbutas & Greengard [121] did this in the context of electromagnetic scattering (although they use the terminology ‘scattering matrix’ for the T -matrix), and then gave results for scattering by 200 ellipsoids. See also [164].

Other authors have constructed the T -matrix in a similar way (using regular spherical wavefunctions as incident fields) except they used other methods to solve the associated boundary-value problems [221, 254, 294]. In [113], the authors construct approximations to the T -matrix using many plane waves as incident fields.

7.9.5. The far field

[End of section, p. 273]: The elastodynamic analogues of (7.77) and (7.80) are in [86] and [67], respectively.

7.9.6. Literature [on T -matrix for one obstacle]

The T -matrix for a perturbed circular cylinder was constructed by Givoli [122].

Hu *et al.* [135] consider electromagnetic scattering by axisymmetric objects. They build matrices akin to T -matrices by scattering plane waves in various directions, and then use them for scattering by a few objects. Similar methods, where ‘characteristic basis functions’ are constructed at the discrete level, were developed by Mittra and his co-workers for large-scale computational electromagnetics [216], and they have been used for various multiple scattering problems [136, 84].

[p. 274, end of line 2]: For a careful discussion of averaging over random orientations, see [242].

[p. 274, paragraph starting “For an elastic half-space”]: See also [M132] and [207].

The T -matrix for one defect in a thin plate (using Kirchhoff theory) has been studied [229, 261, 46, 47].

7.10.3. Discussion [of T -matrix for two obstacles]

The key equations are (7.102) and (7.103). It turns out that they are in a 1972 paper by Wilton & Mittra [339, eqn (28)], where they are derived in the form

$$\beta_I a_I + H_{II} a_{II} = -E_I, \quad H_I a_I + \beta_{II} a_{II} = -E_{II}.$$

Here, a_I and a_{II} are vectors of coefficients (equivalent to \mathbf{c}_1 and \mathbf{c}_2), E_I and E_{II} are analogous vectors for the incident field, the matrices H_I and H_{II} are recognised as separation matrices and the inverses of the matrices β_I and β_{II} can be recognised as individual T -matrices. As the authors note, their equations ‘include all the interactions between the two scatterers and [are] therefore exact’ [339, p. 315]. Numerical results for scattering by a pair of square cylinders are given.

7.11.1. T -matrix for N obstacles: discussion and literature

[p. 282, paragraph starting “The two-dimensional version”]: For applications in the context of homogenization theory, see [317, 318].

[p. 282, paragraph starting “The electromagnetic version”]: For applications to scattering by an array of thin-wire structures, see [347]. For scattering by inhomogeneous objects, see [227].

[After citations of Cai & Williams [M170, M171], bottom of p. 282]: Liu & Cai [215] have extended the method of [M169] to scattering of elastic waves by spherical inclusions. For defects in a thin plate, see [46].

[Top of p. 283, end of first paragraph]: For a review and comparison of several recursive algorithms, including those in [M203, M1048], see [235].

[End of next paragraph]: After “see, for example, [M607, . . . , M575]”, add [154, 233]. The method of Kagemoto & Yue [M576] has been extended to deep water [269]. Their notion of a T -matrix has also been used in conjunction with the mild-slope equation so as to calculate how water waves over varying bathymetry interact with several rigid obstacles [236].

8.2.7. Literature [on small scatterers]

[p. 297, first paragraph]: “Standard low-frequency theory gives an approximation for a fixed geometry (a/b fixed) and long waves ($kb \ll 1$).” Garnaud & Mei [118] have developed a multiple-scales analysis for such problems in the context of water waves.

[End of first paragraph]: The ‘small-body’ approximation ($a \rightarrow 0$ with k and minimum spacing held fixed) has been developed in a long series of papers by Ammari and his collaborators. For example, Ammari

et al. [12, eqn (23)] give the following generalisation of (8.26) for N small scatterers:

$$f(\hat{\mathbf{r}}; \hat{\boldsymbol{\alpha}}) \sim -\frac{ik^3}{4\pi} \sum_{j=1}^N \{ \hat{\mathbf{r}} \cdot \mathbf{X}_j(\gamma) \cdot \hat{\boldsymbol{\alpha}} + [1 - \gamma(k_0/k)^2] |B_j| \} \exp(ik[\hat{\boldsymbol{\alpha}} - \hat{\mathbf{r}}] \cdot \mathbf{b}_j).$$

Here $|B_j|$ is the volume of the scatterer B_j located at \mathbf{b}_j , $j = 1, 2, \dots, N$, and \mathbf{X}_j is the polarisability tensor for B_j *in isolation*. This formula for f says that the far-field contributions from each scatterer are summed with a suitable phase correction to take account of a change of origin, as given by (4.47): at leading order, there is no multiple scattering. For related acoustic results, see [20, 31]. For electromagnetic scattering, see [327, 13].

[End of section, p. 297]: Alker [8] has used matched asymptotic expansions for two-dimensional water-wave problems, with results for two docks and two immersed cylinders.

8.3. Foldy’s method

[Top of p. 299, after first paragraph]: An iterative variant of Foldy’s method has been analysed [51].

The problem of scattering by N isotropic point scatterers in the vicinity of a large scatterer has been treated by combining Foldy’s method with a boundary integral equation [137, 139, 138, 133].

For other applications and developments of Foldy’s method, see [280, 89, 323, 29, 167, 52, 206].

Bendali *et al.* [32] have given a detailed analysis using matched asymptotic expansions; the small parameter is ka , with N and the geometrical configuration of the scatterers held fixed. They recover the Foldy scheme at leading order and then go further.

8.3.2. Ramm’s method

[End of section, p. 302.] Ramm has revived his method in numerous papers; see, for example, [275]. He has also extended it to electromagnetic problems [274, 276, 277]. Similar methods, starting from (8.63), have been developed by Argatov & Sabina [20, 21]. For a more detailed analysis of Ramm’s method, see [57, 6, 59] with extensions to elastodynamic problems [58, 9]. We also mention an earlier paper [245] in which scattering by N small obstacles is reduced to solving an $N \times N$ linear system.

8.4. Point scatterers

[Bottom of p. 305, after “. . . the approximation (8.79).”] Challa & Sini [56] have used (8.82) for direct and inverse problems. They have made similar applications to elastodynamic and electromagnetic problems; see also [134, 55].

The standard book on point scatterers is [7]. Its bibliography contains 747 items. Apart from a few books such as Abramowitz & Stegun [M1] and Gradshteyn & Ryzhik [M427], the only items in common are three of Foldy’s papers, [M354, M460]. Foldy’s seminal paper is described in one sentence [7, p. 353]: ‘Multiple scattering of waves by randomly distributed point scatterers has been discussed in [M354]’.

8.5.2. Two-dimensional water waves [wide-spacing approximations]

[p. 308, end of first paragraph, after “For detailed derivations. . . [M731, §6.3].”] For N bodies, see [M329] and [158, 159]. Kagemoto [152] has given an extended theory for two bodies in which evanescent modes are taken into account; see also [153].

[End of second paragraph]: McIver [232] has extended the wide-spacing approximation within a formal procedure of matched asymptotic expansions.

8.5.4. Three-dimensional water waves [wide-spacing approx]

p. 312, after “It was exploited . . . used in [M827].” Add: [302].

8.6.6. Finite-size effects [random problems]

[p. 319, end of first paragraph, after citing [M492, M493, M586]: A two-dimensional version of (8.117) was obtained by Noskov [257, eqn (32)].

[End of §8.6.6]: The literature on formulas such as (8.118), (8.121) and (8.122), including generalisations and applications, has grown. For a new proof of the Lloyd–Berry formula (8.121), see [209]. For further results on scattering by random arrangements of spheres, see [219, 49, 270].

For two-dimensional acoustic problems, see [88, 228, 161, 256].

For plane-strain elastodynamics, see [81, 64]. For waves in plates, see [261]. For elastodynamic problems involving spheres, see [219] and [100]; the second of these includes comparisons with experiments.

8.6.7. Effective field methods and effective medium methods

The book by Kanaun & Levin [157] reviews effective field methods and effective medium methods, with an emphasis on elastodynamic problems. For example, [157, Chapter 7] considers random configurations of spherical inclusions; see also [M583] and [155, 156]

Appendix B. Integrating a product of three spherical harmonics

[p. 326, end of 2nd paragraph]: For numerical computation of $3-j$ symbols, see [151] and references therein.

Appendix C. Rotation matrices

[At the end, p. 333]: Further computational algorithms have been developed [120, 196].

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