

Multiple Scattering: Interaction of Time-Harmonic Waves with N Obstacles

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CORRECTIONS AND ADDITIONS

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Corrections

p. 34. Example 2.5. Insert $(-1)^{n-1}$ on right-hand side, giving

$$\left(\frac{\partial}{\partial x} \pm i\frac{\partial}{\partial y}\right)^n \log r = (-2)^{n-1}(n-1)! \frac{e^{\pm in\theta}}{r^n}, \quad n = 1, 2, \dots$$

Error found by A. J. Yuffa.

p. 80. There is a short paragraph about the axisymmetric addition theorem:

The special case of Theorem 3.15 in which $z_0 = j_0$ was published by Clebsch in 1863; see [M1298, p. 363]. According to Watson, the special case with $z_0 = y_0$ is due to Gegenbauer: it is formula (3) on p. 365 of [M1298] with $\nu = \frac{1}{2}$.

This is not quite right. Replace by:

Theorem 3.15 was proved by Clebsch [105, p. 227] in 1863.

Gegenbauer gave a more general result. Email exchanges with A. F. Rawle encouraged me to look at Clebsch's paper.

p. 81. Missing $(-1)^l$ in (3.67). It should read

$$B_l^{(n)} \equiv B_l^{n,0} = \frac{(-1)^l (2n-2l)!}{2^n l! (n-l)! (n-2l)!}. \quad (3.67)$$

This agrees with $B_l^{n,m}$ defined by (3.27).

p. 133. The formula (4.45) is correct but the derivation is not. For a proof, see [112, Theorem 2.16]. The same criticism applies to formula (4.10) on p. 124.

p. 209. Below (6.91), the specification “where H has the following properties ... for some $Q \in B_e$ ” is inadequate. Change to:

where sufficient conditions on H are: for every $q \in S$, $H(P; q)$ is an outgoing wavefunction for all $P \in B_e$ whereas $H(P; q)$ must have singularities at some points $P \in B$.

p. 213. Remove overbars in (6.108), giving

$$[U, V] = \int_{\Omega_a} \left(U \frac{\partial V}{\partial r} - V \frac{\partial U}{\partial r} \right) ds = -[V, U]. \quad (6.108)$$

Error found by M. Ganesh.

p. 269. Theorem 7.4. The second property, (7.68), holds for lossless scatterers. The proof sketched assumes that

$$\int_S \left(u \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial u}{\partial n} \right) ds = 0.$$

p. 275. Remove hat from $\hat{\psi}_\nu^\mu$ in last displayed equation at bottom of page, giving

$$\psi_n^m(\mathbf{r}_j^P) = \sum_{\nu,\mu} \hat{S}_{n\nu}^{m\mu}(-\mathbf{b}_j) \psi_\nu^\mu(\mathbf{r}_P), \quad j = 1, 2.$$

Error found by K. Spalding.

p. 303. At the end of the displayed equation below (8.72), replace $h_2(kR_{rj})$ by $h_2(kR_{nj})$.

p. 324. Two lines below (A.8), replace “positive” by “non-negative”.

p. 362. Reference 72. It should be F.G. Bass, not F. Bass. ([71] is correct.)

p. 363. Reference 103. Remove space: should be unidirectional, not uni directional.

p. 368. Reference 212. It should be pages T14–T25, not T11–T25.

p. 378. Reference 436. Change [241] to [241].

p. 379. Reference 459. It should be volume 14, not 18.

p. 398. Reference 896. Remove space: should be electromagnetic, not electro magnetic.

p. 406. Reference 1067. It should be C: p. 202, not C: pp. 202, 202.

p. 407. Reference 1089. Hyphen missing: should be A.-R. Sebak.

p. 411. Reference 1176. Remove one *PCPS*.

Citation index. Achenbach [2]–[7] (not [3]–[7]); Banerjee [60] (omitted); Gumerov [447]–[450] (not [447]–[449]); Hasheminejad [484]–[486] (not [484, 485, 486]); Rodríguez-Zúñiga (not Rodríguez-Zúñiga).

Several extraneous square brackets, such as Achenbach 623], [1275, which should be 623, 1275. Similarly: Barber 685, 1074; Mackowski 867, 868; Mal 768, 1351; Martin 730, 750; Pao 1251, 1260; Sabina 1042, 1120; Truell 1185, 1297.

1.1. What is ‘multiple scattering’?

[Just above §1.1.1, following on from “process.”] Stokes gave an earlier description, in a letter to J. Herschel in 1856, and quoted by James [219, p. 90]:

Suppose a number of ships moored together, or simply placed near each other, on a perfectly still ocean, the assemblage extending in two directions, and not being merely a line. Imagine a series of waves now to be propagated along the surface of the ocean. These would set the ships swinging in an irregular manner, and would then pass on. The ships would of course act as a sort of breakwater, so that as the series passed on the water would be comparatively calm behind them. The ships, however, having been set swinging, would in turn become centres of disturbance, and waves would spread from them in all directions.

1.1.1.2. Independent scattering

Bottom of p. 2. There have been further attempts to quantify when single scattering and independent scattering are reasonable assumptions. See, for example, [355, 352].

1.1.3. Self-consistent methods

Second paragraph below (1.7), after “see [M687, §III]”, insert: See also [M416, eqn (253)].

1.2. Narrowing the scope

[Beginning of §1.2, p.10]:

The phenomenon of multiple scattering is frequently encountered and is conventionally treated in a great variety of ways. (Goldberger & Watson [M416, p. 750])

[End of first paragraph]: Introductory overviews include [335, 336]. For electromagnetic scattering, see [353].

There is a 1968 book by Ivanov, with a 557-page English translation in 1970 [214]; the Russian author is Evgeniĭ Ivanov, cited as both E. A. Ivanov and Ye. A. Ivanov. The book describes methods based on separation of variables and addition theorems (see Chapter 4) for two scatterers: circles, ellipses, spheres, spheroids and circular discs.

1.3.3. Boundary conditions

[Top of p. 14, last sentence in §1.3.2, amended]:

For more information, including uniqueness theorems, see [M657] and [504].

1.5.3. [Elastic] transmission problems

“For more information on [fluid-solid interactions]...”, add [199, 209]

1.5.7. Flexural waves in plates [New section]

For thin elastic plates, the simplest approximation is known as Kirchhoff theory, in which flexural waves are modelled using $(\nabla^4 - k^4)w = 0$, where $w(x, y)$ is the out-of-plane displacement of the plate, ∇^2 is the two-dimensional Laplacian and k is a wavenumber. For background, see Kitahara’s book [M619]. Scattering problems can be formulated: the scatterers could be holes or inclusions of various kinds. There are more refined theories (such as Mindlin plates) but most work on scattering problems has used Kirchhoff theory.

2.4. Wavefunctions

[p. 35, below (2.11)]: For an application of \mathcal{T}_m , see [117, eqn (3.10)].

2.9. Elliptical wavefunctions

[End of §2.9, p. 49, just before §2.9.1]: See also [59].

2.9.1. Addition theorem [for elliptical wavefunctions]

[p. 53, just before §2.10]: See also [214, Chapter 4, §5].

2.11. Multipoles for water waves

[p. 55, below (2.81)]: “For more information on multipole potentials ...” Add [140].

3.1. Introduction [to 3D addition theorems]

For an overview, covering scalar and vector additions theorems, see [191].

3.3. Legendre’s addition theorem

[p. 66, end of §3.3]: Putting $\hat{\mathbf{r}}_1 = \hat{\mathbf{r}}_2 = \hat{\mathbf{r}}$, say, in (3.20), and using $P_n(1) = 1$, we obtain

$$\sum_{m=-n}^n |Y_n^m(\hat{\mathbf{r}})|^2 = \frac{2n+1}{4\pi}.$$

The fact that the left-hand side of this formula is independent of $\hat{\mathbf{r}}$ is sometimes known as Unsöld’s theorem [487, p. 379]; see also [389, eqn (21–16)].

3.5. Hobson’s theorem

[p. 68, below (3.29)]: For a lengthy proof of (3.29), see [532, Appendix A].

3.6. Wavefunctions and the operator \mathcal{Y}_n^m

[p. 71, 5 lines from bottom, after “... $\mathcal{P}_n^m u = i^n \mathcal{Y}_n^m u$.”] In detail, using (A.2),

$$\mathcal{P}_n^0 = P_n(i\mathcal{D}_1^0) = P_n\left(\frac{1}{ik} \frac{\partial}{\partial z}\right),$$

whence $\mathcal{P}_n^0 z_0(kr) = i^n z_n(kr) P_n(\cos \theta)$. In fact, this formula is stated (without proof) in a 1912 paper by Sudhansukumar (S.K.) Banerji [35, p. 8].

3.12. Addition theorem for $h_n^{(1)} Y_n^m$

[p. 90, 5 lines below (3.87), after “§1.2.”]: See also [214, Chapter 3, §7].

[Top of p. 91, line 5, after “For direct proofs ...”]: add [397].

In [137], the authors find $\widehat{S}_{n\nu}^{m\mu}$ by letting $r_1 \rightarrow \infty$ in (3.85), using (3.90), $h_n^{(1)}(w) \sim i^{-n-1} w^{-1} e^{iw}$ as $w \rightarrow \infty$. This gives

$$\sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} \widehat{S}_{n\nu}^{m\mu}(\mathbf{b}) i^{-\nu} Y_{\nu}^{\mu}(\hat{\mathbf{r}}_1) = \lim_{r_1 \rightarrow \infty} \left(i^{-n} \frac{r_1}{r_2} e^{ik(r_2-r_1)} Y_n^m(\hat{\mathbf{r}}_2) \right).$$

But $r_2 = |\mathbf{r}_1 + \mathbf{b}| \sim r_1 + \mathbf{b} \cdot \hat{\mathbf{r}}_1$ and $\hat{\mathbf{r}}_2 = \mathbf{r}_2/r_2 \sim \hat{\mathbf{r}}_1$ as $r_1 \rightarrow \infty$ so that the limit is $i^{-n} e^{ik\mathbf{b} \cdot \hat{\mathbf{r}}_1} Y_n^m(\hat{\mathbf{r}}_1)$. Use of orthogonality, (3.9), then gives (3.79).

3.13.1. Computation of the separation matrices

Bottom of p. 93, “See, for example”: add [436, 527]. For a short review, see [447].

Top of p. 94: For more on recursive methods, see [241].

3.14. Two-centre expansions

p. 96. Theorem 3.30 can be found in [523, eqn (2)], together with related expansions. For an earlier study, see [M919, §5].

3.17. Vector spherical wavefunctions

Bottom of p. 107, “For alternative proofs...”, add: [478], [186, Appendix A3], [M196, Appendix D], [51, §1.8], [99, 137, 541, 317], [258, §7.4.2]. Wittmann [M1332] has defined and exploited a vector analogue of the operator \mathcal{Y}_n^m (Definition 3.9).

3.18. Multipoles for water waves

[p. 110, above Theorem 3.40]: “For more information on three-dimensional multipole ...” Add [140].

Chapter 4

For [single] spheres and circular cylinders the solutions are well known, and even the scattering problem has answers which must suffer the contempt bred by familiarity.

(Morse & Feshbach [M884, p. 1513])

4.5.1. Literature [on acoustic Závíška]

For two cylinders ... , add [211, 138, 357] and [214, Chapter 6]

For more than two cylinders ... from the 1970s: [197]; ... from the 1980s: [32, 47]; ... from the 1990s: [143, 321]; ... from the 2000s: [509, 507, 486, 427, 31, 314]; ... from the 2010s: [372, 24, 434, 491, 126, 351, 437, 371]; ... since 2020: [113, 544, 543].

For circles centred at the vertices of concentric regular polygons, see [220].

Chamberlain [80] has combined Závíška’s method with a mild-slope equation inside each cylinder so as to model the scattering of water waves by various circular structures. Random water waves are considered in [224].

Závíška’s method has been used to study the interaction of SH waves with tunnels. The earliest paper in this context is by Lee & Trifunac from 1979 [272]. They consider one circular tunnel near the flat boundary of a half-plane, which is equivalent to two circles in a full space. For more tunnels (three in a full space and two in a half-plane), see [537]. Extension to lined tunnels has been made. Each tunnel is circular with a concentric lining in which a different Helmholtz equation holds. There is a Neumann condition on the inner boundary and transmission conditions at the outer boundary. For two lined tunnels in a half-plane, see [34].

[Top of p. 130, discussion of Grote & Kirsch [M444]]. For applications, numerical analysis and generalizations, see [4, 5, 107, 255, 342].

Circular scatterers in a layered medium have been considered in many papers, mainly in the context of electromagnetic waves. A typical problem is two half-spaces with waves in both but circular cylinders in one. For example, see [52, 271, 128, 382, 161, 391, 268, 269, 270]. For cylinders in both half-spaces, see [392]. There are analogous treatments for problems involving a slab sandwiched between two half-spaces. For example, see [266, 159, 160, 162, 263]. The paper by Lai *et al.* [263] also gives results for non-circular cylinders using a T -matrix method.

[End of §4.5.1, p. 130, on orders-of scattering]. For other applications, see [142, 303, 467, 528].

4.5.3. Numerical solution of the system (4.27)

[Top of p. 132, end of first paragraph]. For another investigation into the choice of M , see [390]. For errors incurred by truncating the infinite series in Graf’s addition theorem, see [347] and [69, Lemma 9].

Zhang & Li [545] combined (4.27), Theorem 2.7 and a two-dimensional form of the Fast Multipole Method (Section 6.14). Results for up to 2200 cylinders are given. Amirkulova & Norris [15] have exploited the algebraic structure of (4.27) when the cylinders are identical and arranged in a rectangular array, leading to a fast recursive algorithm. Results for up to 1000 cylinders are given when the distance between the centres of adjacent circular cross-sections is $2.02a$.

For additional papers dedicated to numerical aspects, see [22, 471, 417, 190]. For a detailed numerical analysis, see [157].

4.5.4. Three-dimensional problems [New section]

Suppose we have a configuration of parallel cylinders, with generators parallel to the z -axis. Thus far, we have considered incident fields that do not depend on z , but three-dimensional incident fields can be of

interest.

The simplest situations occur when we have a plane wave at oblique incidence,

$$u_{\text{inc}}(x, y, z) = \exp \{i(\ell_1 x + \ell_2 y + \ell_3 z)\} \quad \text{with} \quad k^2 = \ell_1^2 + \ell_2^2 + \ell_3^2.$$

Then, assuming the boundary conditions do not depend on z , we can write

$$u_{\text{sc}}(x, y, z) = v(x, y) e^{i\ell_3 z},$$

where v solves the two-dimensional Helmholtz equation in a cross-sectional plane, $(\nabla^2 + \ell^2)v = 0$, with $\ell^2 = \ell_1^2 + \ell_2^2$, together with boundary and radiation conditions. For circular cylinders, the two-dimensional boundary-value problem for v can be solved by multipole methods. (For non-circular cylinders, v can be calculated using methods described later in the book.) Some of the papers cited in Section 4.5.1 use Závřiska's method to find v for plane waves at oblique incidence; see, for example, [M1367] and [M700]. See also [267].

Similar methods can be developed for cylinders in a waveguide, with infinite plane walls at $z = \pm h$, say. See [482, 63, 379] and references therein. For other axisymmetric structures, see [215]. There are also related water-wave problems for vertical cylinders and water of constant finite depth (Section 4.11). For example, in [41], the authors develop a hybrid method, combining an indirect integral equation method (Section 5.9) with modal expansions.

When the incident wave is not plane, the natural approach is to use a Fourier transform in the z -direction. This decomposes the problem into a superposition of cross-sectional problems. For incident waves generated by a point source, we can effect the decomposition using Weyrich's formula ([M278], §2.9.4). See also [M133], eqn (2.9). For a point source and one circular cylinder, see the 1955 paper by Wait [505]. Wait has discussed analogous electromagnetic problems in his book [506]. For acoustic problems with many circular cylinders perpendicular to an impenetrable plane, see [492]. For an acoustic point source and an elastic cylinder, see [294]. For an analogous problem with elastic waves generated by a point force near a cylindrical cavity, see [53]. For two cylinders, see [M38] and [M1164].

For a point source in a waveguide, with infinite plane walls at $z = \pm h$, modal decompositions can be used, with propagating and evanescent modes. (See Section 4.11.1 for such a decomposition in the context of water waves.) Results for acoustic scattering by four penetrable cylinders are given in [63].

4.6.2 Literature [on scattering by one sphere]

[p. 134, end of first paragraph]: Rayleigh's 1872 paper on scattering by a sphere [461] treats several problems: scattering of a plane wave by a rigid sphere, by a fluid-filled sphere (transmission problem) and by a movable sphere. He also indicates how to proceed when the incident waves are generated by a point source.

[Top of p. 135]: "For further results, see [M561]": add [155, 432]

4.7.1. Literature [on multipole method for two spheres]

[p. 137, first paragraph]: In his 1912 paper, Banerji [35] attempted an orders-of-scattering calculation for a vibrating sphere in the presence of a fixed rigid sphere. Sircar extended this approach in 1929, replacing the rigid sphere by a rigid spheroid [453].

[p. 137, 2nd paragraph, with discussion of Marnevskaia]: At about the same time, Ivanov [214, Chapter 7, §3] made an extensive study of two-sphere problems.

In [419], the authors consider scattering by two spheres when one of the spheres is small ($ka \ll 1$). In [407], the authors consider a radiation problem for two spheres with emphasis on computing the (nonlinear) radiation forces. For acoustic scattering by two spheres, with many numerical results and associated software, see [418, Chapter 4].

[The last paragraph, from "In [880]" to "for 162 cylinders are given" has been absorbed into the (new) Section 4.13; see below.]

4.8.3. Literature [on multipole method for N spheres]

p. 139. For scattering by a few hard spheres, see [13]. For computational aspects, with results for 100 spheres and $ka = \frac{1}{10}\pi$, see [242].

4.9. Electromagnetic waves [by separation of variables]

[End of first paragraph on p. 140, after “by Mie [M849, M717].”]: See also the book edited by Hergert & Wriedt [194].

[3rd paragraph on p. 140, after “see also [M117, §4.2].”]: add [51, §5.2]. [End of 3rd paragraph, add] For a very detailed study of scattering by a layered sphere, see [446]. For anisotropic spheres, see [258, Chapter 8].

“Many papers on electromagnetic scattering by clusters of spheres . . .” (p. 141): add [179, 21, 185, 421, 135, 354, 55, 296, 58, 295, 40],

Also, add: For a good overview, including computational aspects, see [317]. For a ‘tutorial’, see [158]. For scattering of waveguide modes by several spheres inside a tube (or optical fibre), see [102].

[New paragraph, end of §4.9]:

Electromagnetic scattering by spheres near the boundary of a (penetrable) dielectric half-space has been studied by many authors, perhaps starting with the paper by Bobbert & Vlioger [46] on one sphere. For multiple spheres, see [51, Chapter 6], [319]. For scattering by a sphere near a cylinder, using an orders-of-scattering approach, see [429].

4.10. Elastic waves [by separation of variables]

[p. 141, 4th paragraph]: “Elastic waves exterior to single circular cylinders . . .”: add [306], [490, §2.2]. [End of paragraph]: “. . . Veksler [M1261]”: add [490, §2.1]. For acoustic scattering by a radially inhomogeneous elastic sphere, see [474]. For layered spheres, see [496, 497].

Bottom of p. 141, “The analogous problems for incident shear waves”: add [217], [490, §2.3].

Top of p. 142, “Movable rigid spheres are considered in [M1333, M943]”: add [216]. Same paragraph, add: For a review, see [30].

3rd paragraph on p. 142, after “Petrashen and his students”: add [253].

[After “see also [M421].”, middle of 4th paragraph on p. 142]: Avilés & Sánchez-Sesma [32] gave results for scattering of P -waves and S -waves by a row of eight rigid movable cylinders.

[End of 5th paragraph]: “later review [M459].” Add: see also [97].

Moore & Guan [359] have considered elastodynamic problems for two lined tunnels in an unbounded elastic space. They treated the problem using an orders-of-scattering approach. This approach was used earlier for two circular cavities by Sancar & Pao [425], with extensive numerical results and comparisons with experiments in a companion paper by Sancar & Sachse [426]. The twin-lined-tunnel problem with three-dimensional loadings has also been studied: Kuo *et al.* [259] use orders-of-scattering whereas Yuan *et al.* [533, 534, 535, 536] give a complete analysis. Transient plane-strain problems with two identical circular cavities have been solved by Chen & Zhang [92] and by Itou [213]. They combine symmetry, separated solutions and Fourier integrals (addition theorems are not used); the resulting method is complicated and special. The method of [M103] has been extended to plane-strain problems [462], to flexural waves in plates [515] and to Mindlin plates [244].

For scattering by hexagonal arrangements of circular inclusions with imperfect interfaces, see [514]. For two circular defects where surface elasticity is included, see [510, 381].

[p. 143, end of second paragraph]: Doyle has developed [M292] into a full paper [134] on scattering of elastic waves by spheres.

“For axisymmetric . . . see [M484].” For axisymmetric scattering by two spheres (representing bubbles), see [489]. “For scattering by one poro-elastic sphere, see [M93, M1380]”: add [304]. For Závíska-like studies of circular cavities in an unbounded poroelastic space, see [189, 66].

For two elliptical cavities in an unbounded poroelastic space, there are two papers [513, 556] in which circular-cylindrical wavefunctions are used. This is expected to be valid for low eccentricities, otherwise the Rayleigh hypothesis will be relevant.

4.11. Water waves [by multipoles]

[p. 146, end of 2nd paragraph]: Multipole methods for oblique incidence on several horizontal parallel circular cylinders, submerged in water of finite depth, have been used by Shen *et al.* [442]; similar methods have been used for two-layer fluids [154].

[p. 146, end of 3rd paragraph]: See also [85, §5.9]. For N movable spheres in water of constant finite depth, see [286]. For several porous hemispheres (known as ‘reef balls’) on the bottom of finite-depth water, see [287].

4.11.2. Matched eigenfunction expansions

[First paragraph, after “another expansion of some kind.”] In [1], the authors try to represent the enclosed bodies using the ‘method of fundamental solutions’, as described in Section 7.3.2.

[Bottom of p. 147 and first paragraph at top of p. 148, with insertions]: “McIver [M828] ... pair of immersed rectangular cylinders”; see also [530]. “For a pair of thin vertical barriers, see [M982].” See also [445]; this paper includes comparisons with experiments. “For a vertical stack of aligned horizontal plates, see [M1278].” For a pair of truncated vertical circular cylinders, see [554]. There is an earlier paper by Silva *et al.* [451] in which evanescent modes are neglected. For several truncated vertical circular cylinders, see [448, 100, 520, 538, 86, 87, 548] and [85, §2.9]. For several *submerged* truncated vertical circular cylinders, see [338]. For a pair of truncated vertical elliptical cylinders, using Mathieu functions (defined in Section 4.12.1), see [84]. For several floating elastic plates, see [251, 250]. For floating elastic discs, see [358, 549]. For several thin vertical porous barriers, see [236, 237]. For horizontal porous barriers, see [101].

[New paragraph, remainder of first paragraph on p. 148]: “In [M927, M815] ... orders-of-scattering ... bodies ... are given.” The combination of matched eigenfunction expansions with an orders-of-scattering approach has been revived and applied in the context of arrays of wave-energy devices [252, 542].

[End of §4.11.2, p. 148]: after “scattering by N rectangular grooves.” Interest in problems of this kind has increased; see [511, 33, 292, 293, 525]. See also [147, Chapter 1]. For scattering by N holes drilled through a thick screen, see [385].

[End of §4.11.2, p. 148, new paragraph]: In [302], the authors use (4.67) for a vertical cylinder of non-circular cross-section. Their direct application of the boundary condition presupposes that the Rayleigh hypothesis is valid; see p. 249 and [328]. This limitation on the geometry is not noted. Nevertheless, extension to multiple vertical truncated cylinders has been made [551, 550].

4.12.1. Elliptic cylinders

[Middle of p. 149]: after “and [M1139].” In his book on Mathieu functions, McLachlan [340, p. 3] pointed out that Sieger had ‘published an important paper on the diffraction of electromagnetic waves by an elliptical cylinder’ in 1908 [449]; see also [340, §XIX].

Then, insert, giving “see [M1321, M1375], [85, Chapter 4] and references therein.”

[Bottom of p. 149]: Scattering by two elliptic cylinders was first analysed by Ivanov [214, Chapter 8, §3].

Nigsch [374] has given results for time-dependent scattering by three elliptical cylinders. Results for time-harmonic scattering by a pair of identical elliptical cylinders were given by Chatjigeorgiou & Mavrakos [89] and by Kleshchev & Kuznetsova [248]. For arrays of ellipses, see [88] and [85, §4.6]. For a combination of circular and elliptical cylinders, see [83].

4.12.2. Spheroids

[End of 2nd paragraph] ... and [M1, Chapter 21]. See also [214, Chapter 5] and [289, Chapter 2].

[3rd paragraph] See also [420, 6]. For scattering by a penetrable spheroid, see [529, 254]. For acoustic scattering by an elastic spheroid, see [450]. For interaction of water waves with one spheroid, see [90] and [85, §§5.7 & 5.8].

[4th paragraph] ... see also [M710], [M870, §6.1] and [289, 210]. [Then, at end] For scattering of elastic waves by a spheroidal inclusion, see [227].

[5th paragraph, with insertions] “Addition theorems for spheroidal wavefunctions are available ...” Also [214, Chapter 5, §5]. Ivanov [214, Chapter 9] has given results for two co-axial spheroids including the special case of two circular discs.

[Last paragraph, with insertions] “Addition theorems for vector spheroidal wavefunctions are also available [M1106, M253].” Also: [120, 205]. “They have been used extensively for scattering by two perfectly conducting spheroids [M1107, M252], by two dielectric parallel spheroids [M234] and by spheroids that are oriented arbitrarily [M232, M233, M215].” Also: [116, 370]. “All of these results are reviewed in Chapter 4 of [M866].”

4.12.3. Ellipsoids

[Top of p. 151] “Low-frequency scattering ... [M262, Chapter 8]” Add “and in [122, Chapter 14].” For the scattering of water waves by one submerged ellipsoid, see [85, §6.16].

4.13. Expansion-collocation methods [New section]

The scattered field exterior to N circular cylinders can be represented as (4.24). Truncating this expansion gives the approximation

$$u_{sc}(\mathbf{r}) \simeq \sum_{j=1}^N \sum_{m=-M}^M c_m^j \psi_m(\mathbf{r}_j)$$

containing $Q = (2M + 1)N$ coefficients. To determine these, we may apply the boundary condition at a total of P points on the cylinders, with $P \geq Q$; when $P > Q$, the overdetermined system may be solved in a least-squares sense. This basic method is usually known as ‘point-matching’; there is an extensive literature on its implementation and limitations [M76, M625, M213]. Note that for multiple-scattering problems, point-matching yields a numerical method that avoids addition theorems but one whose conditioning should be monitored carefully. For some generalisations of this method, see Section 7.3.2.

The basic method, as described above, was used in [M103], where results for 162 cylinders are given. See also [278, 281, 458].

For scattering by two spheres, using a truncated form of (4.51), see [M880]. For application to a few hard spheres, see [274].

For elliptic cylinders, outgoing elliptical wavefunctions $\psi_{\sigma n}$ (see Definition 2.15) can be used. Lee [273] gave numerical results for scattering by two and three ellipses. For the degenerate case of thin strips, see [109]. For several spheroids, using outgoing spheroidal wavefunctions, see [279, 280].

5.4.4. Spectral properties [New section]

When S is a single circle of radius a , we can take $x(t) = a \cos 2\pi t$ and $y(t) = a \sin 2\pi t$. Then, all the boundary integral operators defined above have eigenfunctions $e^{2\pi i n t}$ and their eigenvalues can be calculated explicitly in terms of Bessel and Hankel functions. Of course, this follows because separation of variables is available. These spectral properties can be used to estimate condition numbers, for example. For detailed calculations, see Kitahara’s book [M619, Chapter 4], [M659], [256], [M23] and [498]; these four papers also give analogous results for one sphere. For one ellipse, using Mathieu functions, see [44].

Antoine & Thierry [25, 470] have studied the spectral properties of the single-layer operator when S consists of N circles.

5.9.1. Literature [on indirect method]

[Bottom of p. 171]: Prior to Kupradze, in 1910, Poincaré used (5.50) and derived (5.51); see [399, p. 181 and §4]. He was interested in electromagnetic scattering by a sphere, with the incident field generated by

a point dipole, this being a contemporary problem of direct relevance to the understanding of long-distance radio wave propagation. For an analysis of Poincaré’s long paper [399], see [400].

[End of 2nd paragraph on p. 172, “the indirect method has also been used in”]: add [350, 349, 39]. Domínguez *et al.* [133] have developed an effective method for solving the two-dimensional form of (5.54); they give numerical results for a pair of ellipses. For scattering by thousands of three-dimensional sound-hard obstacles, using (5.51), see [168].

[End of 3rd paragraph on p. 172]: Single-layer potentials have also been used for two-dimensional mixture problems, with a combination of soft, hard and penetrable cylinders [412].

5.10.4. Literature [on direct method]

[First paragraph on p. 178, line 9, insert giving]: “Linton & Evans [M726]; see also [175].”

[End of first paragraph on p. 178]: For mixture problems, see [504].

6.2. Transmission problems [by integral equations]

[Top of p. 180]: “Examples of other direct methods ... M470” add [94, 136]. For an application to a transmission problem where the interior Helmholtz equation is replaced by Poisson’s equation, see [360].

The computational performance of several formulations, both direct and indirect, has been evaluated [494], with detailed results for scattering by four spheres.

[New paragraphs, end of §6.2, p. 180]: When an obstacle has internal interfaces, essentially two situations can arise. The simpler is when the interfaces are disjoint as with an onion: for concentric circles or spheres, separated solutions can be written down for each layer, and then patched together using the transmission conditions across each interface. When the geometry is more complicated, more-or-less standard boundary integral formulations can be developed. For scattering by several layered ellipses, see [439].

Suppose now that interfaces meet. For example, consider a circular scatterer with a single internal interface along a diameter; call this a ‘Janus circle’ [508]. The two points at the ends of the diameter (known as ‘triple points’ or ‘junctions’) can be reached from three different subdomains (the two halves of the scatterer and the exterior) and this makes it more difficult to construct well-conditioned systems of boundary integral equations; see [504, 406, 68, 144, 181, 103, 104, 222, 193] and references therein. In particular, numerical results for scattering by a Janus circle are available [222, 193]. The difficulties at junction points have been avoided [395] by using simple boundary element methods (with piecewise-constant approximations and collocation at element midpoints). In fact, there are several earlier papers by Ström & Zheng [460, 552, 553], [M1155, M1376] on electromagnetic scattering by objects with interior interfaces (such as a ball composed of two dissimilar halves); they use null-field methods (see Chapter 7). For acoustic scattering by Janus spheres, see [418, Chapter 3].

6.3.3. The Lippmann–Schwinger equation

[New paragraph, end of §6.3.3, p. 186]: The Lippmann–Schwinger equation (6.26) was derived by a direct method; the unknown w is a physical quantity, defined above (6.26). There is an indirect form, derived in [14, eqn (1.7)]. Start by writing

$$U(P) = \int_B G(P, Q) \varphi(Q) dV_Q$$

where U is defined by (6.6) and φ is to be found. This gives a representation for $p_{sc}(P)$ as a volume potential when $P \in B_e$. When $P \in B$, $U = p_0 - p_{inc}$ and we require $(\nabla^2 + k_0^2)p_0 = 0$ in B . Imposing this gives

$$\begin{aligned} 0 &= (\nabla^2 + k_0^2)p_{inc} + (\nabla^2 + k_0^2) \int_B G(P, Q) \varphi(Q) dV_Q \\ &= (k_0^2 - k^2)p_{inc} + 2\varphi(P) + (k_0^2 - k^2) \int_B G(P, Q) \varphi(Q) dV_Q \end{aligned}$$

using (6.8) and $(\nabla^2 + k^2)p_{\text{inc}} = 0$ in B . Rearranging gives

$$\varphi(P) - \frac{1}{2}k^2\{1 - n(P)\} \int_B G(P, Q) \varphi(Q) dV_Q = k^2\{1 - n(P)\}p_{\text{inc}}(P).$$

This should be compared with (6.26), where $\{1 - n(Q)\}$ appears under the integral sign.

6.3.4. An alternative equation [for inhomogeneous obstacles]

Equations similar to (6.28), involving both volume and surface integrals, were derived in two dimensions by Jin *et al.* [225]; see also [503, §3.2.5]. For three-dimensional electromagnetic problems, see [502] and [503, §3.1.7].

6.4.6. Electromagnetic waves [by integral equations]

[p. 195, end of 4th paragraph, on books]: add [503].

For applications of the EFIE and the MFIE to scattering by several polygonal cylinders, see [313].

For applications of (6.34) to three-dimensional multiple scattering problems, see [166].

For applications of the EFIE to three-dimensional multiple scattering problems, see [170].

[6th paragraph, on transmission problems]: add [423, 81] to “see also [M617, M471]”. For more complicated configurations of scatterers, see [531, 247].

For scattering by a dielectric sphere near a thin square plate, see [28].

For applications of volume integral equations to scattering by many spheroids, see [484, 38]. For a $20 \times 20 \times 20$ array of spheres, see [326]. For other shapes, see [93]. For reviews of the work done by Tsang and his collaborators, see [483, 481].

For inhomogeneous scatterers, boundary integral equations for the exterior region can be coupled to finite-element discretisations inside the scatterers [118].

6.5.5. Elastic waves [by integral equations]

[p. 201, end of 2nd paragraph]: The idea of using two scalar potentials for plane-strain elastodynamic problems has been revived by Lai & Li [264]. For example, they give results for scattering by 1000 fixed rigid obstacles.

[p. 201, end of 3rd paragraph on indirect methods]: Cavity problems have been treated by solving the hypersingular equation $\mathbf{N}\mathbf{f} = \mathbf{g}$ [36].

[p. 202, 2nd paragraph]: Numerical results for two-dimensional problems have been presented in [388].

[End of paragraph, on 3D transmission problems]: add [62].

[p. 202, end of 3rd paragraph]: “Orthotropic cylinders are considered in [M698]” and [265].

[p. 202, 4th paragraph on half-space problems]: for the indirect method [using \mathbf{G}], see [M1052, M752]: add [416]; for the direct method, see [M1018, M593]: add [387].

For several rigid discs attached to a half-space, see [479].

Sheng *et al.* [444] have used \mathbf{G} for tunnels in a half-space. They used a hybrid method, coupling finite elements around the tunnels with a boundary integral representation for the surrounding exterior domain.

Chaillat & Bonnet [72] have shown how the computation of \mathbf{G}^{H} can be improved so that a version of the Fast Multipole Method is effective.

[p. 202, 5th paragraph on anisotropy]: For inhomogeneous scatterers, using volume integral equations, see [48, 49].

For collections of elastodynamic fundamental solutions, see [127, 383] and Kausel’s book [238].

[Top of p. 203]: After “several reviews”: add [54, 322]

For flexural waves in a thin elastic plate, we may use Kirchhoff theory. The governing partial differential equation is $(\nabla^4 - k^4)w = 0$, where $w(x, y)$ is the out-of-plane displacement and ∇^4 is the two-dimensional biharmonic operator. The relevant fundamental solution can be taken as

$$G(P, Q) = H_0^{(1)}(kR) - H_0^{(1)}(ikR) = 1 + O(R^2 \log R) \quad \text{as } R \rightarrow 0.$$

The fact that G is finite at $R = 0$ can be exploited to good effect. For G itself, see [M619, p. 211], [228, eqn (7.66)], [376, eqn (28)], [149, eqn (2.3)] and [330, eqn (11)], for example. For boundary integral equations, see Kitahara’s book [M619]. For scattering of flexural waves by a hole of arbitrary shape in an infinite plate, see [454]. For several circular holes and inclusions, see [275, 276, 277].

In [405], the authors attach the ends of N parallel Kirchhoff plates to the boundary of an elastic half-space; plane-strain waves interact with the plates.

6.6.3. Water waves [by integral equations]

[End of first paragraph on p. 205]: For two flat *elastic* strips, see [195].

[End of 4th paragraph on p. 205]: For applications of (6.81) to several wave-energy devices, see [148].

[Near the end of 5th paragraph on p. 205]: After “floating buoys”, add: For applications of *WAMIT* to arrays of floating axisymmetric wave-energy devices, see [145, 311]. Then “For other applications of the direct method to multibody problems, see [M303, M510]”: add [557, 464, 182, 546, 114].

[After citation of Chakrabarti [M181], bottom of p. 205]: Chen & Mahrenholtz [96] solved (6.83) and related integral equations for two floating cylinders in finite-depth water. [End of same paragraph] . . . between two ships, see [M837, M591]: add [285, 555].

Porter [401] started with the hypersingular integral equation obtained from the normal derivative of (6.82), for two-dimensional problems and submerged cylinders. He regularised the equation by using the Cauchy–Riemann equations, leading to a Fredholm integral equation of the first kind with a symmetric weakly-singular kernel. He applied this method in a search for trapped modes above a pair of elliptical cylinders [402].

[End of last-but-one paragraph in §6.6.3, p. 206, after citations of [M506, M527, M28]]: Cho *et al.* [101] used integral equations based on a simple fundamental solution in a finite region containing two submerged inclined plates, and then matched these with eigenfunction expansions (as described in Section 4.11.2). For another application, see [480].

[End of last paragraph in §6.6.3, p. 206, after citation of [M304]]: Leonard *et al.* [282] used finite elements, truncating the computational domain with simple outgoing-wave conditions, and gave results for two half-immersed circular cylinders. See also [428, 369] for similar approaches. More recently, several authors [457, 290, 291] have used the ‘scaled boundary finite element method’ for two or three vertical cylinders in water of finite depth.

For a review of integral-equation methods with a focus on the simulation of wave-energy devices, see [384].

6.7. Cracks and other thin scatterers

[p. 207. End of paragraph about use of Fourier transforms]: For scattering by parallel finite cracks in a thin elastic plate over water, see [403].

6.7.1. Two-dimensional [crack] problems

[Top of p. 208] Revised paragraph, with insertions:

If there are N screens, one obtains a system of N coupled hypersingular integral equations; the hypersingularity can be treated using Chebyshev expansions. This and other methods have been used for acoustic scattering by several straight rigid strips [212], [M443, M1124, M1364], [183, 378]. Murai *et al.* [364] gave results for scattering by up to 72 identical parallel rigid strips using a piecewise-constant approximation for $u(t)$ on each strip. For two coplanar sound-soft strips, see [438]. Chebyshev expansions have been used for up to 28 curved sound-soft strips [223]. Hewett *et al.* [196] have developed a hybrid analytical-numerical method for scattering by N coplanar sound-soft strips. For sound-hard strips, see [110].

For scattering of elastic waves by several straight cracks, see [M545, M519], [540], [M1373, M1056, M548, M259]. For several straight coplanar strips, see [M1364]. For acoustic scattering by several elastic strips, see [108]. “For the interaction of water waves with thin plates . . . M397”: add [172, 422].

Analogous problems arise when water waves are scattered by thin vertical rectangular barriers in a three-dimensional ocean of constant finite depth; this is a special case of the formulation described in §1.6.4, where each vertical cylinder has no interior. Such problems have been solved using hypersingular integral equations and Chebyshev expansions in the context of wave-energy devices; see [414] for one device and [413, 431, 430] for several. The last of these discusses how machine learning may be used to optimise the layout of arrays of devices; see also [542].

For two scatterers, one can contemplate using an orders-of-scattering approach. This has been pursued by Meguid & Wang for two straight strips [343] and for two straight cracks [344].

Acoustic scattering by strips on which the boundary condition involves a fractional derivative (thus interpolating between sound-soft and sound-hard conditions) has been studied [495, 465]. A numerical method can be developed in which Gegenbauer polynomials are used instead of Chebyshev polynomials; this method has been used for scattering by two parallel strips [465].

6.7.2. Three-dimensional [crack] problems

[Top of p. 209]. For two coplanar rectangular cracks, see [301]. For scattering by a penny-shaped crack near a rigid movable disc or a spherical inclusion, see [368, 365]. Earlier, Mykhas'kiv & Khai had derived the governing integral equations for N plane cracks and then given results for two or three coplanar penny-shaped cracks [366, 367].

For scattering by two coplanar discs, using special integral-equation methods, see [111, 472, 325].

6.8. Modified integral equations: general remarks

“There is an extensive literature ... for reviews, see ... [M959, Chapter 6]”: add [324, 129]. Note that [129] includes numerical results for scattering by four sound-hard circles.

6.9. Modified fundamental solutions

[p. 210, new paragraph]: The analysis in this section makes use of the bilinear expansion for $G(\mathbf{r}_P, \mathbf{r}_Q)$, as given in (6.102); this is an infinite series. It is natural to ask if G can be approximated in the separated form

$$G(\mathbf{r}_P, \mathbf{r}_Q) \simeq \sum_{m=0}^M f_m(\mathbf{r}_P) g_m(\mathbf{r}_Q),$$

where M and the functions f_m and g_m are to be chosen. For investigations in this direction, see [146].

6.10.1. A combined-layer method

Numerical solutions of (6.121) and (6.122) for two three-dimensional obstacles have been presented by Ganesh & Hawkins [164]. Subsequently, they [167], Bremer *et al.* [57] and Hao *et al.* [187] solved (6.122) for many soft obstacles.

6.10.2. The method of Burton and Miller

p. 218. “Buffa & Hiptmair [M158] first multiply (5.59) by a regularising operator ... Dirichlet problem.” This idea has been developed in several subsequent papers; see, for example, [23, 61].

[Top of p. 219] “However, we are not aware of any published numerical results for multiple-scattering problems, based on (6.124).” In 2007, Shen & Liu [440] combined (6.124) with a fast multipole method (see §6.14) for scattering by many objects; see also [522] for scattering by a cube containing 27 spherical inclusions.

“The choice of η has been investigated in [M23].” For a review, see [323].

[End] “in particular, [M404] contains results for scattering by two circular cylinders.” The method used in [M404] is designed for high-frequency problems and it uses an iterative orders-of-scattering approach. It has been analysed in detail [141] and it has been extended to three dimensions [20] with results given for scattering by two soft ellipsoids. Gibbs *et al.* [174] consider scattering by soft cylinders, one of which is a large convex polygon; they also start from (6.125), and they give results for one small triangle near a large triangle.

6.11. Augmentation methods

[p. 221 “...has been implemented numerically [M1158].” In [246], the authors add one null-field equation (that with $n = m = 0$ in the unnumbered equation at the top of p. 221) to (5.58) (see [246, eqn (19)]) and give some numerical results.

[p. 221 “We know of two ... [M287, M1353].” For an application to three two-dimensional scatterers, see [393].

Woodworth & Yaghjian [521] augmented using (6.127) evaluated for $P \in S'$, with S' being inside but close to S , giving what they call ‘dual-surface integral equations’.

6.11.1. Extensions [Augmentation methods]

[There is an unnumbered equation on p. 221]:

$$u(P) = \int_{S \cup S'} \mu(q) G(P, q) ds_q, \quad P \in B_e.$$

This representation has been used by Bremer [56] for two-dimensional problems involving many non-smooth star-like scatterers.

[Bottom of p. 221]: “For more details ... see [M694]” add [232].

6.12.1. The exact Green’s function

[Bottom of p. 223, after “Boley [M112].”]: For numerical solutions of (6.135), see [466, eqn (21)] and [73, eqn (13)] (where G^E is called a ‘tailored Green’s function’).

6.12.2. Multiple scattering by two obstacles

[p. 224, just before §6.12.2.1] Maystre [337, eqn (44)] characterises each scatterer using an operator S_j leading to a formulation that has similarities with others discussed in §6.12.

6.12.2.1. Partitioning

Multiplying (6.137) by A_{11}^{-1} and (6.138) by A_{22}^{-1} gives an equivalent pair of equations. Thierry [469] has investigated the numerical consequences of doing this for N scatterers: one can say that the system has been *preconditioned*. He gives some numerical results for $N = 30$.

6.12.3. Generalised Born series for two scatterers

[Bottom of p. 227]: “In [M290] ... elastic half-space.” For an application to scattering by two coplanar penny-shaped cracks, see [245].

“Geuzaine *et al.* [M404] ... $ka = 1000$.” For two ellipsoids, see [20].

[End of §6.12.3, p. 228]: Balabane’s general approach, combining N single-scatterer problems with some kind of iterative process, has been developed by several groups [173, 500, 512, 526, 499].

There are alternative interpretations. For the exterior Neumann problem with two obstacles, we can write $u(P) = u_1(P) + u_2(P)$, where

$$2u_j(P) = \int_{S_j} \left(f_j(q)G(P, q) - u(q) \frac{\partial}{\partial n_q} G(P, q) \right) ds_q,$$

valid for all points P outside S_j (including inside the other obstacle). Thus u_j is a radiating wavefunction outside S_j . To use this formula, we need to find, or approximate, $u(q)$ for $q \in S_j$. For work in this direction, using variants of the ‘on-surface radiation condition’ of Kriegsmann *et al.* [257], see [2, 12, 3].

Another way to break a multiple scattering problem into pieces is to use a *domain decomposition method* (DDM) [132]. In some variants, the domains are the scatterers themselves [547, 118]. For a DDM that Schwarz might recognise, see [394], where a two-dimensional cluster is enclosed by a square which is itself broken into smaller square subdomains, each containing a number of scatterers. Numerical results for up to 40,960 scatterers are given.

6.13.5. Extensions [of Twersky’s method]

Kyurkchan has developed a variant of Twersky’s method in several papers, some with collaborators; see, for example, [261, 260] and [262, §3.5]. Results are given for scattering by two obstacles.

6.13.6. Weyl’s integral representation [new section]

Friedman & Russek [M366, eqn (11)] used a three-dimensional analogue of Sommerfeld’s integral representation due to Weyl [518], namely

$$h_0(kr) = \frac{e^{ikr}}{ikr} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_0^{\pi/2-i\infty} e^{ikr \cdot \hat{s}} \sin \alpha \, d\alpha \, d\beta; \quad (1)$$

see also [M1149, p. 578, eqn (26)]. In (1), $\hat{s} = (\sin \alpha \cos \beta, \sin \alpha \sin \beta, \cos \alpha)$ is a vector in which α and β may be complex; although \hat{s} is not a unit vector in general, we do have $\hat{s} \cdot \hat{s} = 1$. Although not stated, (1) is valid for $z > 0$. To check this, we compare with a formula derived by Clemmow [106, eqn (2.120)],

$$\frac{i e^{-ikr}}{kr} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ik(\ell x + my \mp z\sqrt{1-\ell^2-m^2})} \frac{d\ell \, dm}{\sqrt{1-\ell^2-m^2}}$$

where the upper (lower) sign is taken for $z > 0$ ($z < 0$). Noting that Clemmow uses $e^{+i\omega t}$, we obtain the two-dimensional Fourier integral,

$$\frac{e^{ikr}}{ikr} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ik(\ell x + my + z\sqrt{1-\ell^2-m^2})} \frac{d\ell \, dm}{\sqrt{1-\ell^2-m^2}}, \quad z > 0;$$

see also [M177, pp. 338–339] and [M1332, eqn (18)]. Use plane polar coordinates for the double integral, putting $\ell = \tau \cos \beta$ and $m = \tau \sin \beta$. This gives

$$\frac{e^{ikr}}{ikr} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_0^{\infty} e^{ik(\ell x + my + z\sqrt{1-\tau^2})} \frac{\tau \, d\tau \, d\beta}{\sqrt{1-\tau^2}}, \quad z > 0,$$

with $\sqrt{1-\tau^2} = i\sqrt{\tau^2-1}$ when $\tau > 1$, giving exponential decay with increasing z . Next, put $\tau = \sin \alpha$. This gives $\ell x + my + z\sqrt{1-\tau^2} = \mathbf{r} \cdot \hat{s}$, as in (1). The interval $\tau = 0$ to $\tau = 1$ becomes $\alpha = 0$ to $\alpha = \pi/2$. For $\tau > 1$, put $\alpha = A + iB$. Then $i\sqrt{1-\tau^2} = i \cos \alpha = i \cos A \cosh B + \sin A \sinh B$. Hence, for exponential decay with z , we can take $A = \pi/2$ with B going to $-\infty$. This leads precisely to (1), with a continuous contour going from $\alpha = 0$ to $\alpha = \pi/2 - i\infty$, valid for $z > 0$; see also [518, Fig. 1].

Let us apply the operator \mathcal{Y}_n^m to (1). From Theorems 3.10 and 3.11, we have $\mathcal{Y}_n^m \{h_0\} = h_n Y_n^m$ and $\mathcal{Y}_n^m \{e^{ikr \cdot \hat{s}}\} = (-i)^n e^{ikr \cdot \hat{s}} Y_n^m(\hat{s})$. Hence

$$\psi_n^m(\mathbf{r}) = h_n(kr) Y_n^m(\hat{\mathbf{r}}) = \frac{(-i)^n}{2\pi} \int_{-\pi}^{\pi} \int_0^{\pi/2-i\infty} Y_n^m(\hat{s}) e^{ikr \cdot \hat{s}} \sin \alpha \, d\alpha \, d\beta, \quad (2)$$

which agrees with [M366, eqn (15)]; see also [M1332, eqn (19)].

For the addition theorem, we assume that $z = \mathbf{r} \cdot \hat{\mathbf{z}} > 0$ and $\mathbf{b} \cdot \hat{\mathbf{z}} > 0$. Thus

$$\psi_n^m(\mathbf{r} + \mathbf{b}) = \frac{(-i)^n}{2\pi} \int_{-\pi}^{\pi} \int_0^{\pi/2-i\infty} Y_n^m(\hat{\mathbf{s}}) e^{ik(\mathbf{r}+\mathbf{b})\cdot\hat{\mathbf{s}}} \sin \alpha \, d\alpha \, d\beta. \quad (3)$$

Using the plane-wave expansion $e^{ik\mathbf{r}\cdot\hat{\mathbf{s}}} = 4\pi \sum_{\nu,\mu} i^\nu \hat{\psi}_\nu^\mu(\mathbf{r}) \overline{Y_\nu^\mu(\hat{\mathbf{s}})}$, we obtain (3.84), in which

$$S_{n\nu}^{m\mu}(\mathbf{b}) = 2(-1)^\mu i^{\nu-n} \int_{-\pi}^{\pi} \int_0^{\pi/2-i\infty} e^{ik\mathbf{b}\cdot\hat{\mathbf{s}}} Y_n^m(\hat{\mathbf{s}}) Y_\nu^{-\mu}(\hat{\mathbf{s}}) \sin \alpha \, d\alpha \, d\beta;$$

here, we have used (3.7). This formula for $S_{n\nu}^{m\mu}$ reduces to (3.87) after use of Theorem 3.20 and (2).

The proof given here assumes we can use the integral representation (3), which required the conditions $z = \mathbf{r} \cdot \hat{\mathbf{z}} > 0$ and $\mathbf{b} \cdot \hat{\mathbf{z}} > 0$. But it is clear that the final result, (3.84), cannot require these conditions.

6.14. Fast multipole methods

[Bottom of p. 235]: Chapter 6 of Liu’s book [305] is concerned with acoustic scattering.

6.14.2. Literature [on FMM]

[Bottom of p. 240, after “...M255, M256”]: In particular, the electromagnetic version of (6.195) can be found in [M1130, eqn (10)] (in discrete form); for an application to scattering by 56 unmanned aerial vehicles, see [524].

[Bottom of p. 241]: For applications of FMM to scattering of two-dimensional elastic waves by many circular inclusions, see [424]. For three-dimensional elastic waves, see the review [71]. For applications to multiple scattering of water waves, see [50]. For acoustic scattering by finite periodic arrays, see [221].

7.1. Introduction [to null-field and T -matrix methods]

[End of first paragraph, p. 242]: After “For electromagnetic problems, see ... [M873].” add: See also [130] and [463, Chapter 3].

7.3.1. Literature [on Kupradze’s method]

[End of section, p. 244]. For a later review, see [262].

7.3.2. Generalized multipole techniques

MFS has been used for multiple scattering problems arising in computer graphics, where the aim is to give realistic sounds in virtual environments [218, 345].

[Top of p. 246] For a combination of GMT with a fast multipole method (FMM, see §6.14), with applications to electromagnetic problems, see [45].

[Line 5, amend:] ... rigid wall); for comparisons with Foldy-type approximations (Section 8.3), see [239].

[New paragraph, end of §7.3.2, p. 246]: Another related numerical technique is known as the *Trefftz method* [98]. In two dimensions, surround the scatterers by a circle, C . Outside C , approximate u_{sc} using a (finite) series of outgoing cylindrical wavefunctions, ψ_n . Break the region between C and S into a finite number of simply-connected subregions, B_m , $m = 1, 2, \dots, M$. In each B_m , approximate u_{sc} using a (finite) series of wavefunctions; these could be plane waves in various directions or regular cylindrical wavefunctions. Finally, determine the coefficients in each expansion by imposing continuity conditions across each interface and the boundary condition on S . Variants of this approach have been used for multiple scattering problems. For four squares, see [459]. For five circles, see [493]. Trefftz methods have also been coupled with boundary element methods [29].

7.8.1. Regular wavefunctions

Waterman [516] has refined his approach for spheroids and other quadric surfaces. For electromagnetic scattering by spheroids, see [456]. For fluid-solid problems, see [299], where Waterman’s ideas are combined with representations used by Doicu *et al.* [M285].

7.8.4. Numerical experience and convergence

[p. 265, before §7.8.4.1]: For more examples of poor conditioning when computing the Q -matrix using regular wavefunctions, see [455]. For static problems, see [153].

7.8.6. The scattered field

[Bottom of p. 267]: Harness & Ditkowski [188] have given an iterative method for computing the coefficients c_n^m without a direct solution of the null-field equations.

7.9.4. Using a BIE to calculate the T -matrix

[p. 272] “. . . this connection between boundary integral equations and T -matrix methods has not been exploited.” In 2013, Gimbutas & Greengard [177] did this in the context of electromagnetic scattering (although they use the terminology ‘scattering matrix’ for the T -matrix), and then gave results for scattering by 200 ellipsoids. See also [243].

Other authors have constructed the T -matrix in a similar way (using regular spherical wavefunctions as incident fields) except they used other methods to solve the associated boundary-value problems [318, 373, 441, 123]. Ganesh & Hawkins [165, 169] went one step further and computed the corresponding far-field patterns, f_N^M , say. Then using $d_n^m = \delta_{nN}\delta_{mM}$ (see above (7.74)) in (4.46) gives

$$f_N^M(\hat{\mathbf{r}}) = \sum_{n,m} (-i)^n Y_n^m(\hat{\mathbf{r}}) T_{nN}^{mM} \quad \text{and} \quad T_{nN}^{mM} = i^n \int_{\Omega} f_N^M \overline{Y_n^m} \, d\Omega.$$

In [163], the authors construct approximations to the T -matrix using many plane waves as incident fields. This idea had been used earlier in [341] for analogous hydrodynamic problems. For electromagnetic problems, see [205].

7.9.5. The far field

[End of section, p. 273]: The elastodynamic analogues of (7.77) and (7.80) are in [121] and [95], respectively.

7.9.6. Literature [on T -matrix for one obstacle]

The T -matrix for a perturbed circular cylinder was constructed by Givoli [178].

Methods for computing the T -matrix recursively have been developed, using invariant imbedding and matrix Riccati equations; for a review, see [131].

Hu *et al.* [203] consider electromagnetic scattering by axisymmetric objects. They build matrices akin to T -matrices by scattering plane waves in various directions, and then use them for scattering by a few objects. Similar methods, where ‘characteristic basis functions’ are constructed at the discrete level, were developed by Mittra and his co-workers for large-scale computational electromagnetics [312], and they have been used for various multiple scattering problems [204, 119].

[p. 274, end of line 2]: For a careful discussion of averaging over random orientations, see [356].

[p. 274, paragraph starting “For an elastic half-space”]: See also [M132] and [298].

The T -matrix for one defect in a thin plate (using Kirchhoff theory) has been studied [334, 386, 64, 65, 475].

7.10.3. Discussion [of T -matrix for two obstacles]

The key equations are (7.102) and (7.103). It turns out that they are in a 1972 paper by Wilton & Mittra [519, eqn (28)], where they are derived in the form

$$\beta_I a_I + H_{II} a_{II} = -E_I, \quad H_I a_I + \beta_{II} a_{II} = -E_{II}.$$

Here, a_I and a_{II} are vectors of coefficients (equivalent to \mathbf{c}_1 and \mathbf{c}_2), E_I and E_{II} are analogous vectors for the incident field, the matrices H_I and H_{II} are recognised as separation matrices and the inverses of the matrices β_I and β_{II} can be recognised as individual T -matrices. As the authors note, their equations ‘include all the interactions between the two scatterers and [are] therefore exact’ [519, p. 315]. Numerical results for scattering by a pair of square cylinders are given.

The formalism in section 7.10 uses two T -matrices, whereas the formalism in section 7.10 uses partitioned boundary integral equations. A hybrid approach can be developed, using a T -matrix for one scatterer and an integral equation for the other. This development is given for electromagnetic problems in [310].

7.10.4. Literature [on T -matrix for two obstacles]

[Top of p. 281, after first paragraph]: The T -matrix is not very convenient if one wants to compute the near field. Methods for doing this, using various re-expansions, have been developed [468]. For an extreme case, Martin [332] has shown how to compute the electromagnetic field between a pair of coaxial discs, with a small gap between the discs.

7.11.1. T -matrix for N obstacles: discussion and literature

[p. 282, paragraph starting “The two-dimensional version”]: For applications in the context of homogenization theory, see [476, 477].

[p. 282, paragraph starting “The electromagnetic version”]: For a good overview, see [258, Chapter 9]. For applications to scattering by an array of thin-wire structures, see [539]. For scattering by closely spaced spheroids, see [468, 435]. For scattering by inhomogeneous objects, see [327]. The method used in [9] is essentially a T -matrix method.

[After citations of Cai & Williams [M170, M171], bottom of p. 282]: Liu & Cai [307] have extended the method of [M169] to scattering of elastic waves by spherical inclusions. For defects in a thin plate, see [64, 475].

[Top of p. 283, end of first paragraph]: For a review and comparison of several recursive algorithms, including those in [M203, M1048], see [346].

[End of next paragraph]: After “see, for example, [M607, . . . , M575]”, add [231, 341, 150, 151, 473]. The method of Kagimoto & Yue [M576] has been extended to deep water [396]. Their notion of a T -matrix has also been used in conjunction with the mild-slope equation so as to calculate how water waves over varying bathymetry interact with several rigid obstacles [348].

8.2.2. Three dimensions: sound-hard obstacles

[p. 288, end of paragraph containing (8.25)]: See also [198, §2.8.3].

[Below (8.26), correct “polarisability” to “polarizability”. Then, end of paragraph, add]: It is also known that $\mathbf{X}(1)$ vanishes (see [M262], eqn (5.27)) so that the scattering is isotropic when there is no density mismatch across S ; this fact was known to Rayleigh [M1010, §335]. In this very special case, we can use the Lippmann–Schwinger equation (§6.3.3), and Foldy’s method (§8.6.3) is available. We observe that the transmission problem with $\gamma = 1$ (see (1.26)) is atypical in that the far-field scattering is isotropic (as for sound-soft scatterers) but, in the far field, $|u_{sc}|^2$ is ‘proportional to ω^4 in the low-frequency limit [as for sound-hard scatterers]. This dependence is well known as the signature of *Rayleigh scattering*’ [443, p. 57] but the reader should keep in mind the underlying assumption that $\gamma = 1$.

8.2.7. Literature [on small scatterers]

[Bottom of p. 296]: Bispherical coordinates have also been used to study sub-wavelength resonances of two close-to-touching spheres; this is relevant to a pair of bubbles (transmission problem) [16]. Extensions to other geometries have been made [288].

[p. 297, first paragraph]: “Standard low-frequency theory gives an approximation for a fixed geometry (a/b fixed) and long waves ($kb \ll 1$).” Garnaud & Mei [171] have developed a multiple-scales analysis for such problems in the context of water waves.

[End of first paragraph]: The ‘small-body’ approximation ($a \rightarrow 0$ with k and minimum spacing held fixed) has been developed in a long series of papers by Ammari and his collaborators. For example, Ammari *et al.* [17, eqn (23)] give the following generalisation of (8.26) for N small scatterers:

$$f(\hat{\mathbf{r}}; \hat{\boldsymbol{\alpha}}) \sim -\frac{ik^3}{4\pi} \sum_{j=1}^N \{ \hat{\mathbf{r}} \cdot \mathbf{X}_j(\gamma) \cdot \hat{\boldsymbol{\alpha}} + [1 - \gamma(k_0/k)^2] |B_j| \} \exp(ik[\hat{\boldsymbol{\alpha}} - \hat{\mathbf{r}}] \cdot \mathbf{b}_j).$$

Here $|B_j|$ is the volume of the scatterer B_j located at \mathbf{b}_j , $j = 1, 2, \dots, N$, and \mathbf{X}_j is the polarizability tensor for B_j *in isolation*. This formula for f says that the far-field contributions from each scatterer are summed with a suitable phase correction to take account of a change of origin, as given by (4.47): at leading order, there is no multiple scattering. For related acoustic results, see [26, 42]. For electromagnetic scattering, see [501, 18].

[End of section, p. 297]: Alker [10] has used matched asymptotic expansions for two-dimensional water-wave problems, with results for two docks and two immersed cylinders.

8.3. Foldy’s method

[Beginning of section 8.3, p. 297]:

The point interaction approximation is a natural tool to analyze a variety of interesting problems in the continuum or homogenization limit. In the physical literature it goes back to Foldy’s paper [M354] on sound propagation in a bubbly liquid and perhaps earlier. In almost all papers that followed Foldy’s, the point interaction approximation is not treated as an important approximation in itself and averaging is carried out over the [scatterer] center locations. The closure problem that arises is then treated in a variety of ways depending on other parameters in the problem. [See Section 8.6.] . . . But averaging is not necessary. The continuum limit holds for deterministic sequences [of scatterer locations] subject to some conditions that hold for “most” realizations in the random case. The closure difficulties are thus avoided for many problems.

(Figari *et al.* [156, p. 47])

[Top of p. 299, after first paragraph]: The Monte Carlo approach of Fikioris [M345] was implemented by Hahn [184]. An earlier study was made by Bruno & Novarini in 1982 [60]. They solved the Foldy system (8.49) [60, eqn (20)] (although Foldy is not cited) for scattering by a row of N bubbles, with random spacings between adjacent bubbles.

The idea of approximating small spherical bubbles by point sources is natural; for historical remarks, see [329]. For two-dimensional analogues, see [517].

The problem of scattering by N isotropic point scatterers in the vicinity of a large scatterer has been treated by combining Foldy’s method with a boundary integral equation [206, 208, 207, 200, 201].

For other applications and developments of Foldy’s method, see [485, 415, 125, 488, 37, 249, 70, 283, 297, 433], [308, §4] and [192, Chapter 26]. An iterative variant of Foldy’s method has been analysed [69].

Bendali *et al.* [43] have given a detailed analysis using matched asymptotic expansions; the small parameter is ka , with N and the geometrical configuration of the scatterers held fixed. They recover the Foldy scheme at leading order and then go further.

Foldy-type methods can be developed for flexural waves in plates. In the simplest situations, the plate is ‘pinned’ at a configuration of points, and incident waves are scattered by these points. As the basic fundamental solution, $G(P, Q) = H_0^{(1)}(kR) - H_0^{(1)}(ikR)$, is finite at $R = 0$, collocation at the pinned points

is permitted, leading to a simple numerical scheme. This approach was introduced by Evans & Porter in 2007 [149]. Since then, many other investigations have appeared [404, 380, 82, 362, 330, 361, 333].

8.3.2. Ramm’s method

[End of section, p. 302.] Ramm has revived his method in numerous papers; see, for example, [409]. He has also extended it to electromagnetic problems [408, 410, 411]. Similar methods, starting from (8.63), have been developed by Argatov & Sabina [26, 27]. For a more detailed analysis of Ramm’s method, see [76, 7, 78] with extensions to elastodynamic problems [77, 11]. We also mention an earlier paper [363] in which scattering by N small obstacles is reduced to solving an $N \times N$ linear system.

Ammari & Zhang [19] have used a variant of Ramm’s method for bubbly liquids. They start with the single-layer representation for u_{sc} and then use it for the transmission problem with N small spherical bubbles. Under certain circumstances, the limit $N \rightarrow \infty$ can be taken; bubbles can resonate so that the limiting process is not straightforward.

8.4. Point scatterers

[Bottom of p. 305, after “... the approximation (8.79).”] Challa & Sini [75] have used (8.82) for direct and inverse problems. They have made similar applications to elastodynamic and electromagnetic problems; see also [202, 74].

The standard book on point scatterers is [8]. Its bibliography contains 747 items. Apart from a few books such as Abramowitz & Stegun [M1] and Gradshteyn & Ryzhik [M427], the only items in common are three of Foldy’s papers, [M354, M460]. Foldy’s seminal paper is described in one sentence [8, p. 353]: ‘Multiple scattering of waves by randomly distributed point scatterers has been discussed in [M354]’.

8.5.2. Two-dimensional water waves [wide-spacing approximations]

[p. 308, end of first paragraph, after “For detailed derivations... [M731, §6.3].”] For N bodies, see [M329] and [236, 237]. Kagemoto [229] has given an extended theory for two bodies in which evanescent modes are taken into account; see also [230].

[End of second paragraph]: McIver [339] has extended the wide-spacing approximation within a formal procedure of matched asymptotic expansions.

8.5.4. Three-dimensional water waves [wide-spacing approx]

p. 312, after “It was exploited ... used in [M827].” Add: [452].

8.6.3. Foldy’s approximation [random problems]

[p. 316, second paragraph, after citing M492, M493]: add [309]

8.6.6. Finite-size effects [random problems]

[p. 319, end of first paragraph, after citing [M492, M493, M586]: A two-dimensional version of (8.117) was obtained by Noskov [377, eqn (32)].

[p. 319, end of second paragraph]: For a review of some applications of (8.118) with $Q = 0$, see [320].

[End of §8.6.6]: The literature on formulas such as (8.118), (8.121) and (8.122), including generalisations and applications, has grown. For a new proof of the Lloyd–Berry formula (8.121), see [300]. For further results on scattering by random arrangements of spheres, see [79, 315, 67, 398, 316], [490, §2.4].

For two-dimensional acoustic problems, see [124, 331, 240, 375].

For plane-strain elastodynamics, see [152, 115, 91]. For waves in plates, see [386]. For elastodynamic problems involving spheres, see [315] and [139]; the second of these includes comparisons with experiments.

Gower *et al.* [180] have obtained generalisations for media comprising random arrangements of spheres of two (or more) sizes.

8.6.7. Effective field methods and effective medium methods

The book by Kanaun & Levin [235] reviews effective field methods and effective medium methods, with an emphasis on elastodynamic problems. For example, [235, Chapter 7] considers random configurations of spherical inclusions; see also [M583] and [233, 234].

Appendix B. Integrating a product of three spherical harmonics

[p. 326, end of 2nd paragraph]: For numerical computation of $3-j$ symbols, see [226] and references therein.

Appendix C. Rotation matrices

[At the end, p. 333]: Further computational algorithms have been developed [186, Appendix A2], [176, 284].

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