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Corrigendum

Corrigendum to "On angular-spectrum representations for scattering by infinite rough surfaces" [Wave Motion 24 (1996) 421–433] ☆

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In [1], we used energy arguments to deduce various results for the problem of acoustic scattering of a plane wave by an infinite one-dimensional rough surface, S, defined by $z=s(x)$ with $-h\le s(x)\le 0$ for all x. Some of the results must be modified as was pointed out by Kazandjian [3, this volume].

Let $\mathbb{S}_r = S_r \cup H_r \cup T_r$ be a closed curve, where $S_r=\{(x, z)\colon z=s(x),\ |x|\le r\}$ is a truncated rough surface, H_r a semicircle (centred at the origin) of radius r in $z\ge 0$, and T_r consists of two line segments at $x=\pm r$. In [1], we considered the energy flux through $\mathbb{S}_r$, and deduced various consequences of assumed representations for the reflected wave field. However, it was implicitly assumed that the energy flux through T_r was negligible compared to that through H_r. (In fact, in a later paper [2] concerned with the derivation of boundary integral equations for reflection of a plane wave by a two-dimensional rough surface, we showed that T_r can give a significant contribution.) This contribution is given by

$$E(T_r) = \mathrm{Im}\int_{s(-r)}^{0} \left(u_{\mathrm{tot}} \frac{\partial u^*_{\mathrm{tot}}}{\partial x} \right)\bigg|_{x=-r} \mathrm{d}z - \mathrm{Im}\int_{s(r)}^{0} \left(u_{\mathrm{tot}} \frac{\partial u^*_{\mathrm{tot}}}{\partial x} \right)\bigg|_{x=r} \mathrm{d}z = E_- - E_+,$$

say, where $u_{\mathrm{tot}}(x, z)$ is the total field.

Assuming that u_{tot} and $\partial u_{\mathrm{tot}}/\partial x$ are bounded on T_r (as is reasonable), we see that $E(T_r)$ is also bounded as $r\to\infty$, and so Theorem 1 of [1] is correct. Similarly, (34) in [1] is correct.

The remaining deductions are valid if $E(T_r)\to 0$ as $r\to\infty$. This is an additional assumption. It will be true if $s(x)\to 0$ as $x\to\pm\infty$, or if $s(x)\to s_0$, a constant, as $x\to\pm\infty$. It will also be true if $E_-=E_+ + \mathrm{o}(1)$ as $r\to\infty$, which means that the energy fluxes from left to right, say, through the two pieces of T_r are asymptotically equal for large r.

Note that, when S is a *periodic* surface, we can deduce that $E(T_r)=0$, but only for certain *discrete* values of r, namely $r=Nd$, where d is the period and N any positive integer. For such surfaces, Kazandjian [3] has obtained $T_3+E(T_r)=0$ (below (18) in [3]), where T_3 is defined by (13) in [3]. This equation holds for all r. From it, we can observe that if $E(T_r)\to 0$ as $r\to\infty$, then it follows that $T_3\to 0$ too; the deductions following (36) in [1] would then be valid.

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In general, we may expect that

$$E(T_r) = 4\pi\mathcal{A} + \text{oscillatory terms} + o(1) \quad \text{as} \quad r \to \infty, \tag{1}$$

where $\mathcal{A}$ is a constant. An asymptotic energy balance would then require $\mathcal{A}$ on the right-hand side of (35) in [1] instead of zero. Without further knowledge on the form of the oscillatory terms in (1) (if any), we cannot determine how they will contribute to the overall energy balance.

One example, suggested to the authors by Simon Chandler-Wilde, concerns the 'smoothed step': $s(x)=0$ for $x<-a$, $s(x)=-h$ for $x>a$, with $s(x)$ being smooth and monotonically decreasing between $x=-a$ and $x=a$. Thus $E_-=0$ (because $s(-r)=0$). Far to the right, we expect that u_{tot} is given approximately by reflection from a flat surface at $z=-h$, so that

$$u_{\text{tot}} \simeq e^{ikx\sin\theta_i}(e^{-ikz\cos\theta_i} + e^{ik(2h+z)\cos\theta_i}),$$

where θ_i is the angle of incidence. Direct calculation then gives

$$E_+ = -\sin\theta_i[2kh + \mathbb{S}(2kh, \cos\theta_i)],$$

with $\mathbb{S}$ defined by (33) in [1]. In fact, if we choose h so that $kh\cos\theta_i=\pi$, we see that

$$u_{\text{tot}} \simeq 2e^{ikx\sin\theta_i}\cos(kz\cos\theta_i)$$

satisfies the Neumann boundary condition exactly on the flat parts of S. (The boundary condition on the finite transitional region between $x=-a$ and $x=a$, S_0 say, could be satisfied too by appending a suitable source distribution on S_0, using the exact Green's function for S, corresponding to a line source at a point on S_0; this Green's function is $O((kr)^{-1/2})$ as $kr\to\infty$, and so cannot contribute to E_+.) Then,

$$E(T_r) = -E_+ = 2kh\sin\theta_i.$$

References

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