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Maurice Jaswon and boundary element methods

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ABSTRACT

In the direct boundary integral equation method, boundary-value problems are reduced to integral equations by an application of Green's theorem to the unknown function and a fundamental solution (Green's function). Discretization of the integral equation then leads to a boundary element method. This approach was pioneered by Jaswon and his students in the early 1960s. Jaswon's work is reviewed together with his influence on later workers.

Contents

1. Introduction

Professor Maurice Jaswon, who died on 24th November 2011, aged 89, was a pioneer in advocating the use of direct boundary integral equations, and in the use of boundary elements for their numerical treatment. We shall review his contributions and discuss his influence.

For biographical information, see the obituary in the London *Daily Telegraph* [1]. For a history of boundary integral methods, see [3].

2. Early years: metal physics

The early part of Jaswon's research career was in metal physics and crystallography. He began as a PhD student in the Department of Metallurgy at the University of Birmingham, publishing his first paper in 1947 [J01].[1] His thesis [J02] was written under Alan Cottrell's guidance. They have one much-cited joint paper [J04] introducing what is now known as the Cottrell–Jaswon mechanism for the drag on a moving dislocation. Indeed, Jaswon would return to the theory of dislocations throughout his career, right up to his last published paper in 2001 [J45].

In 1949, Jaswon became a lecturer in the Department of Mathematics, Imperial College, London. He remained there until 1967 when he moved to City University London.

At Imperial College, Jaswon continued his researches in physical metallurgy, dislocations and crystallography. He published three books during this period [J09J18J25] and he supervised many PhD students.

One project concerned modifications to the Peierls–Nabarro equation [5, p. 61]. This nonlinear integrodifferential equation can be written as

$$\int_{-\infty}^{\infty} \frac{u'(\xi)}{x-\xi}\,\mathrm{d}\xi = \lambda\sin(\pi u), \quad -\infty < x < \infty, \tag{1}$$

where λ is a constant. It is to be solved for $u(x)$ subject to the conditions $u(x) \to \pm 1$ as $x \to \pm\infty$. Surprisingly, the exact solution is known: $u(x) = (2/\pi)\arctan(\lambda x)$. Foreman, Jaswon and Wood

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[1] A citation of the form [Jxx] indicates one of Jaswon's publications listed separately at the end of the paper. Citations without the prefix J are to the list "References"

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[J07] investigated modifications of the sinusoidal force-law on the right-hand side of (1), obtaining better agreement with experiments.

Since his first paper [J01], Jaswon had been interested in martensitic transformations [J03J14]. "Usually the transformed region has the shape of a thin lenticular plate, of approximately elliptic cross-section normal to its plane, and may be regarded as approximately rigid compared with the surrounding austenitic matrix" [J20, p. 678]. Jaswon wanted to calculate the stresses around these regions or *inclusions*. He began with analytical methods but soon moved to computational methods based on integral equations.

In 1961, Jaswon published a paper with his student, Bhargava (Fig. 1), on two-dimensional inclusion problems [J20]. They began with "an account of Eshelby's point-force method for solving elastic inclusion problems" [8,9]. "The main limitation of Eshelby's solution concerns the formidable nature of the point-force integrals. Only for problems exhibiting spherical or circular symmetry can these be handled analytically. It was in an effort to overcome this practical limitation, at least for two-dimensional systems, that the complex variable approach was initiated" [J20, p. 671]. Thus, they formulated the plane-strain elasticity problem in terms of biharmonic functions which were then represented using analytic functions of a complex variable. After writing down point-force solutions (taken from [10]), "a continuous distribution of forces round a contour" was introduced. Explicit solutions for elliptical inclusions were obtained.

3. Boundary integral equations and boundary elements

Jaswon and two of his students, Alan Ponter and George Symm, published three papers in 1963 in the *Proceedings of the Royal Society* on boundary integral equations. The first, by Jaswon and Ponter [J22], is on the torsion problem. This can be reduced to an interior Dirichlet problem for Laplace's equation, $\nabla^2 u = 0$, in two dimensions.

Fig. 1. Jaswon (left) and R.D. Bhargava.

After some discussion of the torsion problem itself, Jaswon and Ponter start; we quote from [J22, p. 239], including their section heading:

> *Integral equation formulation*: Green's formula, adapted to two-dimensional potential theory, appears as
>
> $$\frac{1}{2\pi}\int_L w'(\mathbf{q}) \ln|\mathbf{q}-\mathbf{P}| \, dq - \frac{1}{2\pi}\int_L w(\mathbf{q}) \ln'|\mathbf{q}-\mathbf{P}| \, dq = w(\mathbf{P}), \tag{2}$$
>
> where $\mathbf{P}$ is a vector variable specifying points within [the bounded region] D, $\mathbf{q}$ is a vector variable specifying points on [the boundary] L and dq denotes the arc differential at $\mathbf{q}$; $\ln'|\mathbf{q}-\mathbf{P}|$ denotes the inward normal derivative of $\ln|\mathbf{q}-\mathbf{P}|$ at $\mathbf{q}$ keeping $\mathbf{P}$ fixed, and $w'(\mathbf{q})$ stands for the inward normal derivative of w at $\mathbf{q}$. This formula means that a harmonic function w within D, exhibiting boundary values $w(\mathbf{q})$ and boundary normal derivatives $w'(\mathbf{q})$, may be represented throughout D by the left-hand side of (2). For a point $\mathbf{P}$ outside D, the left-hand side gives zero identically. For a point $\mathbf{p}$ on L itself, it gives $\frac{1}{2}w(\mathbf{p})$ owing to the jump in the double-layer integral on crossing L, from which we see that $w(\mathbf{q})$, $w'(\mathbf{q})$ satisfies the linear functional relation
>
> $$\int_L w(\mathbf{q}) \ln'|\mathbf{q}-\mathbf{p}| \, dq + \pi w(\mathbf{p}) = \int_L w'(\mathbf{q}) \ln|\mathbf{q}-\mathbf{p}| \, dq. \tag{3}$$
>
> This important relation, which we term Green's boundary formula, will be recognised as the real-variable analogue of Plemelj's well-known complex-variable formula. Its importance stems from the fact that we may view it as a compatibility relation between $w(\mathbf{q})$ and $w'(\mathbf{q})$ which, under appropriate circumstances, ensures that they both appertain to the same harmonic function w. Thus, given $w(\mathbf{q})$ on L (Dirichlet problem), the formula (3) becomes a Fredholm equation of the first kind for $w'(\mathbf{q})$. Conversely, given $w'(\mathbf{q})$ on L (Neumann problem), it becomes a Fredholm equation of the second kind for $w(\mathbf{q})$.

We have given this lengthy quotation because it shows that Jaswon already had the essence of the direct boundary integral equation method. No references are given. By way of comparison, Jaswon and Ponter [J22, p. 240] observe that the "conventional formulation of the Neumann problem [7] proceeds by writing

$$w(\mathbf{P}) = \int_L \sigma(\mathbf{q}) \ln|\mathbf{q}-\mathbf{P}| \, dq, \tag{4}$$

and noting that

$$w'(\mathbf{p}) = \int_L \sigma(\mathbf{q}) \ln|\mathbf{q}-\mathbf{p}|' \, dq + \pi\sigma(\mathbf{p}), \tag{5}$$

where $\ln|\mathbf{q}-\mathbf{p}|'$ denotes the inward normal derivative of $\ln|\mathbf{q}-\mathbf{p}|$ at $\mathbf{p}$ keeping $\mathbf{q}$ fixed. This formulation, in which (5) appears as Fredholm equation of the second kind for $\sigma(\mathbf{q})$ is inferior to the preceding in not directly yielding the wanted boundary function $w(\mathbf{q})$." Thus, the classical "indirect" method was recognised as being less attractive when the goal is to calculate the missing boundary data (although the left-hand side of (5) is simpler than the right-hand side of (3)).

(Jaswon's preferred notation for the two normal derivatives of the logarithmic fundamental solution, namely, $\ln'|\mathbf{q}-\mathbf{p}|$ for the normal derivative at $\mathbf{q}$ and $\ln|\mathbf{q}-\mathbf{p}|'$ for the normal derivative at $\mathbf{p}$, did not become popular.)

Jaswon and Ponter gave some numerical results, obtained using a straightforward method. They divided L into n arcs, L_i, $i = 1, 2, \ldots, n$. Let $w(\mathbf{q}_i) = w_i$ and $w'(\mathbf{q}_i) = w_i'$ where $\mathbf{q}_i$ is the midpoint of L_i. "Our fundamental approximation is to assume that $w(\mathbf{q})$, $w'(\mathbf{q})$ remain constant within each interval $[L_i]$ so that (3) becomes"

$$\sum_{i=1}^{n} w_i \int_{L_i} \ln' |\mathbf{q}-\mathbf{q}_j| \, \mathrm{d}q + \pi w_j = \sum_{i=1}^{n} w_i' \int_{L_i} \ln |\mathbf{q}-\mathbf{q}_j| \, \mathrm{d}q \tag{6}$$

for $j=1,2,\ldots,n$. There is a discussion of how to compute the remaining integrals over L_i. Simpson's rule is used when $i \neq j$, using the locations of the endpoints of each L_i (denoted by $\mathbf{q}_{i\pm 1/2}$). When $i=j$, the logarithmic integral on the right-hand side of (6) is evaluated analytically having first approximated L_i by two straight lines, joining the endpoints $\mathbf{q}_{i\pm 1/2}$ to the midpoint $\mathbf{q}_i$. The integral on the left-hand side of (6) when $i=j$ is evaluated by a now-familiar observation: as $w(\mathbf{P})=1$ solves $\nabla^2 w=0$ in D, (6) gives

$$\sum_{i=1}^{n} \int_{L_i} \ln' |\mathbf{q}-\mathbf{q}_j| \, \mathrm{d}q + \pi = 0, \quad j=1,2,\ldots,n, \tag{7}$$

which allows the integral with $i=j$ to be expressed as a sum of all the off-diagonal integrals with $i \neq j$.

The numerical results are all for Dirichlet problems, which means that Fredholm integral equations of the first kind are being solved. "Generally speaking, the accuracy increases with n until a limiting value is reached, specific to each contour L, above which inherent ill-conditioning develops unless the numerical techniques become considerably more refined" [J22, p. 242]. Nevertheless, results are given for a variety of cross-sections D, including ellipses, triangles, squares and notched circles. For non-smooth L, the collocation nodes, $\mathbf{q}_i$, are not taken at corners. The authors conclude by noting that "the integral equation method compares favourably, both in scope and effectiveness, with any analytical or numerical method so far devised. An examination of inhomogeneous torsion and plastic torsion on similar lines will be taken up in subsequent papers"; see [14,15].

Later in 1963, Jaswon [J23] and Symm [18] published a pair of consecutive papers, with Jaswon covering theoretical issues and Symm concentrating on numerical procedures. Jaswon starts by investigating the first-kind equation

$$\int_L \sigma(\mathbf{q}) \ln |\mathbf{p}-\mathbf{q}| \, \mathrm{d}q = f(\mathbf{p}), \quad \mathbf{p} \in L, \tag{8}$$

"with a view to preparing the ground for [its] exploitation in the numerical solution of difficult boundary-value problems". He shows that the homogeneous form of (8) (with $f \equiv 0$) can have non-trivial solutions for a certain scaled version of L (called a Γ-contour in [J31]). He then turns to consequences of Green's boundary formula, recalling (2) and (3), and he notes that these formulas can be generalised to three dimensions. Next, he observes that "Green's boundary formula (3) provides a fresh means of attacking the mixed boundary-value problems of potential theory", in which w is prescribed on a part of L and w' is prescribed on the rest of L. He also allows the possibility of more complicated boundary conditions where $\alpha w+\beta w'$ is prescribed (α and β are given functions, defined on L).

Symm's paper [18] applies the numerical methods described by Jaswon and Ponter [J22] to a range of harmonic problems (including a mixed boundary-value problem), using both direct and indirect formulations. His subsequent paper [19] has led to (8) being known as "Symm's integral equation".

The last part of Jaswon's paper [J23] concerns two-dimensional problems governed by the biharmonic equation, $\nabla^4 \chi=0$, motivated by elasticity problems. Jaswon writes $\chi(x,y)=r^2\phi+\psi$, where $r^2=x^2+y^2$, $\nabla^2\phi=0$ and $\nabla^2\psi=0$. Then, he proposes to use formulas such as (3), one for ϕ and one for ψ. In two later papers [J26J28], this approach was abandoned in favour of single-layer representations, (4). For problems involving corners, such as a V-shaped notch in a plate, ill-conditioning was observed: "the trouble can to a large extent be eliminated by 'rounding off' the corner" [J26, p. 316]. John Willis, another one of Jaswon's students in the early 1960s, began his PhD studies with the V-shaped notch problem. Willis managed to resolve the ill-conditioning but his approach was not as simple as Jaswon would have liked. Jaswon suggested to Willis that he switch to analytic work: his first paper extended [J20] to anisotropic inclusions [20].

Alan Ponter has described his experiences as one of Jaswon's PhD students:

> John Willis and I began in the autumn of 1961. George Symm was already Jaswon's student, a year ahead, and Bhargava had just finished and returned to India. Maurice gave John and I a series of talks about potential theory and integral equations. The main text was Kellogg's book [11]. The plan was that I should work on potential problems associated with elasticity problems. George was working on biharmonic problems and John should look at crack-like problems. The motivation for such an assault on the numerical solution of integral equations was the availability of a Ferranti Mercury computer, run jointly for all the London University colleges. It occupied two large rooms in a house in Gordon Square, Bloomsbury. The Mercury was a valve computer. It had a high-speed store able to hold 256 numbers to six significant figures. It had a slow store on a massive rotating drum that could hold a further 16k numbers. Programs were written in Autocode, and were typed onto eight-holed telex tape on standard machines in the Department. The original plan was to transmit the programs to Bloomsbury where the resulting tape would be run on the Mercury and the results then sent back to Imperial. However, the failure rate was high and program development was desperately slow. Consequently, I took to riding my bicycle through central London between Imperial and Bloomsbury (about three miles) in a daily routine, which allowed me to have two short runs on the computer each day instead of one.
>
> Maurice wanted the method to be as simple as possible and defined the approximations I should use. By the end of 1961, I had obtained good results for the torsion of an oval. Working through the following months, I produced the results for the first paper [J22]. Maurice was very keen to get them published. I suggested we improve the numerical method but Maurice wanted to demonstrate the method in its simplest form. The paper was submitted in October 1962. By this time, George was producing results, and his paper [18], together with Maurice's [J23], were finished not long afterwards. The reason our paper was published first was due, I suppose, to my fixed-wheel sports bicycle.

Much of the work described above found its way into the 1977 book by Jaswon and Symm [J31]. It was the first book on the numerical solution of boundary integral equations [3].

It is striking that Jaswon recognised that one strength of direct boundary integral formulations is that they do not depend, essentially, on the number of spatial dimensions. However, he chose to tackle elasticity problems using biharmonic functions, thus limiting his scope to two-dimensional static problems. Perhaps this choice was due to his successful simplification of Eshelby's method for two-dimensional inclusion problems, described above [J20].

4. America

Jaswon spent the academic year 1963–1964 as Visiting Professor of Engineering at Brown University (Fig. 2). He taught a graduate course on crystallography that led to a book [J25]. The book is dedicated to the memory of his young son, David, who was hit by a car and killed [1].

Marc Richman has recalled Jaswon's visit:

> When Maurice came to Brown, I had just joined the faculty. He came because of the fantastic group of people working in Materials and Solid Mechanics. William Prager and Dan Drucker in Solid Mechanics, John Gilman and Joseph Gurland in Materials, as well as Harry Kolsky and Charles Elbaum in Applied Mathematics. Brown had just received a large grant from the Advanced Research Projects Agency and was setting up an interdisciplinary research and teaching group.
>
> Maurice contributed greatly to both research and graduate education in Materials. At Brown we were trying to tie together the continuum approach and the microscopic approach. Maurice was able to bring world class expertise in the latter and interface with world class experts in the former.
>
> I was just a newly arrived assistant professor who had studied physical metallurgy at MIT and mechanics and dislocation theory with Egon Orowan at that school. I benefited greatly from my interactions with Maurice and was proud to be asked to coauthor the paper in *Applied Mechanics Reviews* [J24].

Ponter spent the first half of 1964 in the Mechanics Department, University of Iowa, where he gave a course on integral equation methods. "During the time I gave the course, Maurice was flying through Chicago and I met up with him in the departure lounge with his wife. Maurice passed over to me a paper containing the Somigliana equations [6] and, either then or earlier, a report of a US government laboratory with solutions for sonar transmitter problems (Helmholtz equation). At the time, I regarded this sonar work as the only solutions comparable with our own work." The 1960 paper by Roland de Wit [6] does not contain any integral equations. It is likely that the sonar report was by George Chertock at the David Taylor Basin, Washington, DC; his 1964 paper [4] uses a direct boundary integral equation for the exterior Neumann problem (a Fredholm equation of the second kind) specialised to an axisymmetric boundary.

Fig. 2. Maurice Jaswon, circa 1964.

In May 1964, Jaswon visited the University of Illinois in Urbana, where he met Frank Rizzo, a graduate student of Marvin Stippes. Rizzo recalls as follows [17]:

> Marvin was very fond of classical work such as to be found in Kellogg's *Potential Theory* [11] and Love's treatise [12] on elasticity theory…He wanted his students to be acquainted with the fundamental contributions of the early Italian elasticians such as Betti, Lauricella, and Somigliana…In spring 1963 Marvin suggested that "something with integral equations" might be fruitful for me. He pointed me in the direction of Lovitt [13], Fredholm [as cited on [12, p. 266]], a recent paper by Jaswon [and Ponter] [J22] and promptly left to go on sabbatical for a year.
>
> …That work on potential theory was the model, motivation, and springboard for everything I did that year for elasticity theory and, indeed, for any other topic that I have subsequently worked on…In retrospect, all three of those papers [J22J2318] represent at once the birth and quintessence of what has become known as the "direct" boundary element or boundary integral equation method for problems of every description.

Rizzo knew that the analogue of the formula (2) in elasticity theory is the Somigliana formula (the three-dimensional version is in Section 169 of Love's book [12]); it gives an expression for the displacement vector $\mathbf{u}(\mathbf{P})$ in the region D in terms of displacements and tractions on the boundary L. Letting $\mathbf{P} \to \mathbf{p} \in L$ would lead to the analogue of (3) which Rizzo could then try to solve numerically [17]:

> I saw what I wanted to do, but I was worried. Why hadn't Jaswon already done what I intended to do since he was clearly interested in elasticity also?…[I asked him when he visited Urbana. His answer] was simple—he hadn't thought of it!

Eventually, Rizzo's first paper was published [16]. In it, he derives the two-dimensional elastostatic analogue of (3), a vector equation containing Cauchy principal-value integrals, and he gives some numerical results. Jaswon persevered with vector potential theory; see chapter 5 of [J31] and some of his later papers [J33J39].

Later in 1964, Jaswon returned to London and Rizzo became Assistant Professor of Civil Engineering at the University of Washington, Seattle.

Jaswon returned to the US during 1965–1966 as Chairman of the Department of Engineering Mechanics, University of Kentucky, and Director of its Institute for Theoretical and Applied Mechanics. Rizzo spent the first quarter of 1966 at the Institute, and then moved to Kentucky permanently in the fall of 1966 as Assistant Professor of Engineering Mechanics. Two final quotes from Rizzo [17]:

> It is curious, and to my mind regrettable, that while Maurice, Tom [Cruse], and I were all so very interested in the same things in the middle to late sixties, no two of us were physically in the same place for very long…Jaswon, whose appointment at Kentucky had originally made me aware of opportunities there, had exited for England not long after I moved from Seattle. He has been in England ever since.

5. City University London

Jaswon returned to London. In 1967, he accepted the position of Professor and Head of the Department of Mathematics at City University London. For the next 20 years, he directed his efforts at building the department, improving its teaching and research activities. He described his activities as follows: ''primarily responsible for guiding the development of the Department from polytechnic towards university status, i.e., staff were encouraged to achieve PhD's and to engage with research students; actuarial science unit successfully integrated into the Department; and hosted the annual British Theoretical Mechanics Colloquium in 1981''.

Maurice Jaswon retired in 1987. Throughout the 80s and 90s, he was a regular participant at conferences on boundary element methods. ''Maurice will always be remembered by the Boundary Elements community for his patience and tolerance. He was generous with his time and knowledge and has the unique qualities that define an 'academic gentleman''' [2].

6. Discussion

Looking back at Jaswon's contributions in the context of boundary integral equations, we see that he clearly saw the advantages of the direct method over the indirect method. He did not flinch from using integral equations off the first kind, despite the extensive literature of classical potential theory in which second-kind equations are used exclusively. The classical indirect approach does have its virtues, of course, especially if one is interested in mathematical questions such as proving that solutions to boundary-value problems exist.

Jaswon missed the natural step of using Somigliana's formula for elasticity problems, and he was reluctant to go beyond piecewise-constant approximations. Nevertheless, with his students, he showed that many potential and elastic boundary-value problems could be solved numerically using what are now called boundary element methods.

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