PIC Math: A Modified SEIR Model for the Spread of Ebola in Western Africa

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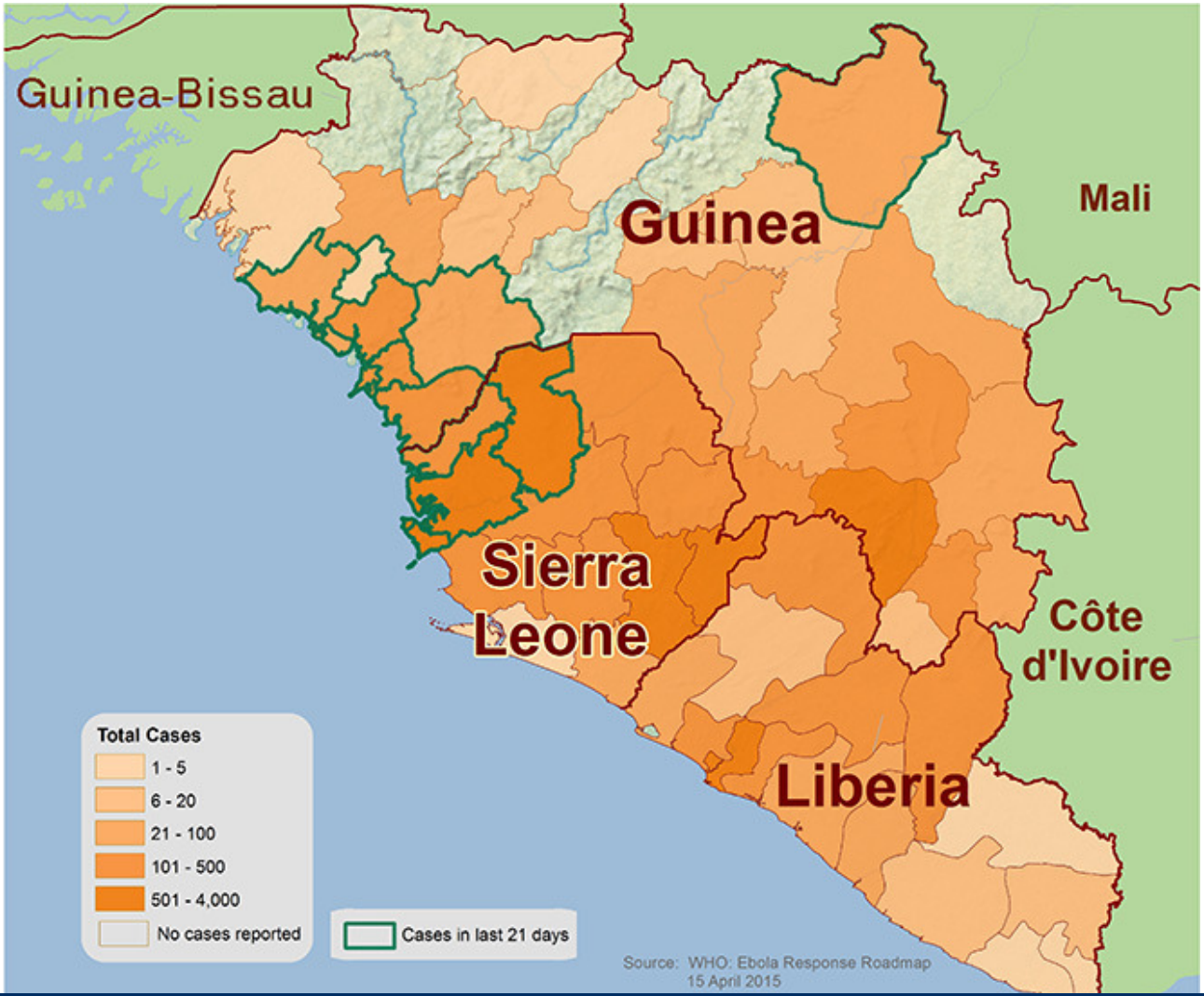
Background - Cumulative Ebola Outbreak Distribution Map

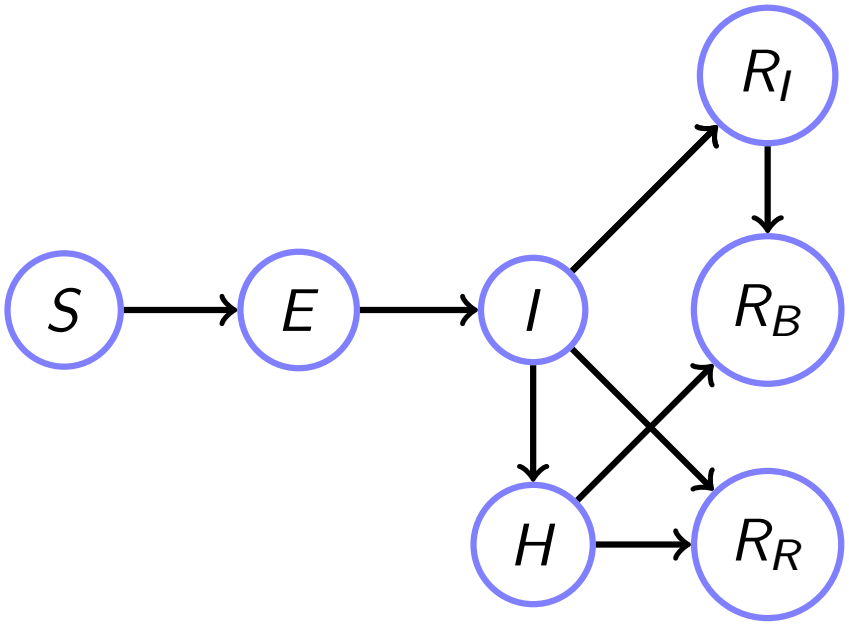


Goal: Mitigate the spread of Ebola in West Africa by determining the optimal placement of a treatment center

Model Assumptions

- Infected individuals can move to three different compartments: Removed and infectious, removed and buried, removed and recovered
- Individuals who have died from the disease but who have not yet been buried can still infect susceptible individuals
- Individuals who recover from the disease are no longer susceptible
- Once hospitalized, infected individuals can still infect others. However, those who die in the hospital are buried immediately, and thus cannot infect others once dead
- Hospitals have unlimited space, but there is some delay in hospitalization

State Diagram



The Model

$$
\begin{cases}
\dfrac{dS}{dt} = \alpha S - \beta_1 SI - \beta_2 SR_I - \beta_3 SH \\
\dfrac{dE}{dt} = \beta_1 SI + \beta_2 SR_I + \beta_3 SH - \delta E \\
\dfrac{dI}{dt} = \delta E - \gamma_1 I - \psi I \\
\dfrac{dH}{dt} = \psi I - \gamma_2 H \\
\dfrac{dR_I}{dt} = \rho_1 \gamma_1 I - \omega R_I \\
\dfrac{dR_B}{dt} = \omega R_I + \rho_2 \gamma_2 H \\
\dfrac{dR_R}{dt} = (1-\rho_1)\gamma_1 I + (1-\rho_2)\gamma_2 H
\end{cases}
$$

Parameter Fitting

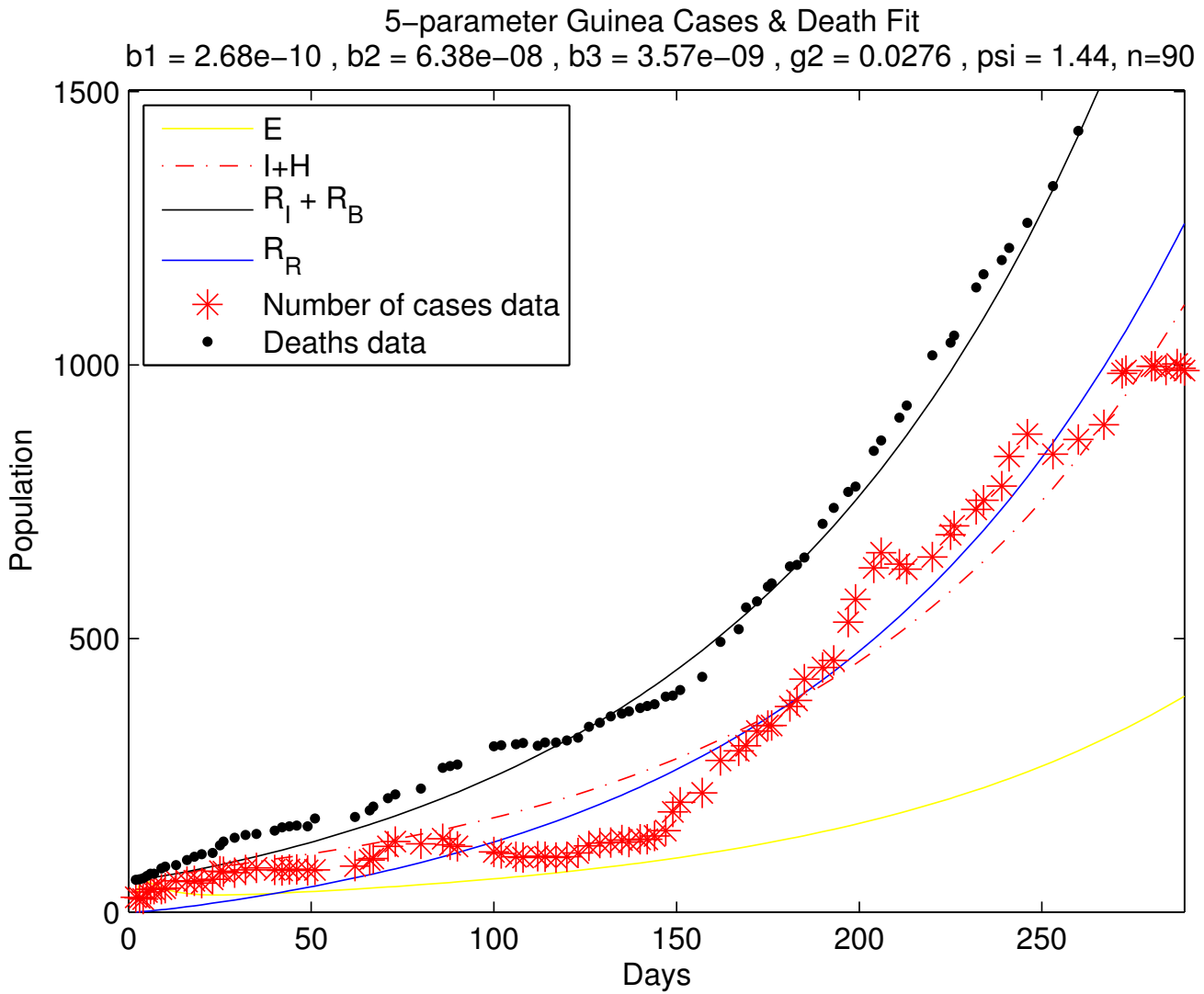
Data: country time series data compiled from WHO reports up to December, 2014 [9].

fminsearch: Finds the minimum of unconstrained multivariable function using derivative-free method.

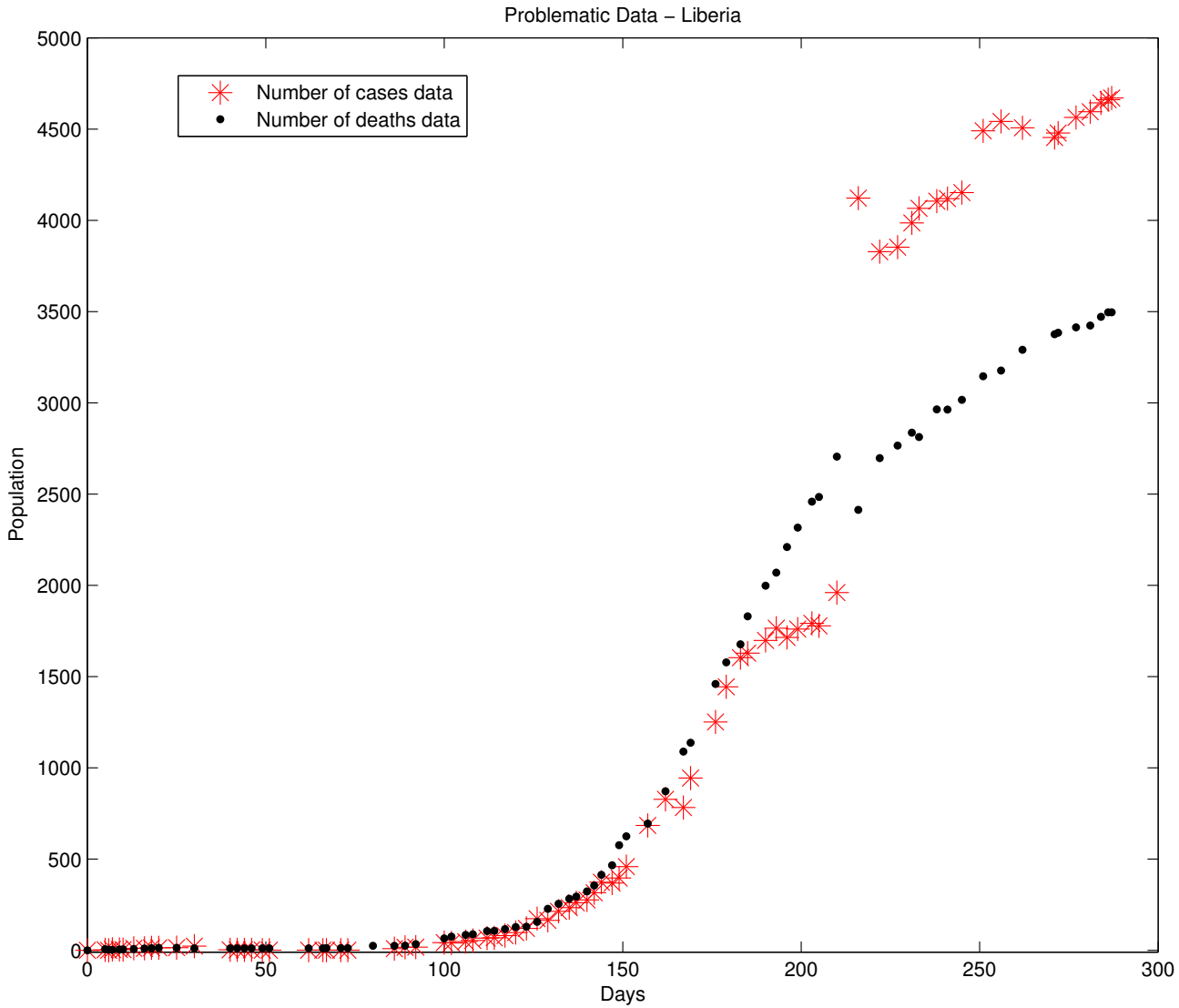
Error function: Sums of Squares

$$e := \sum \sqrt{(R_{data} - (R_I + R_B))^2 + (I_{data} - (I + H))^2}$$

Parameter Fitting



Parameter Fitting



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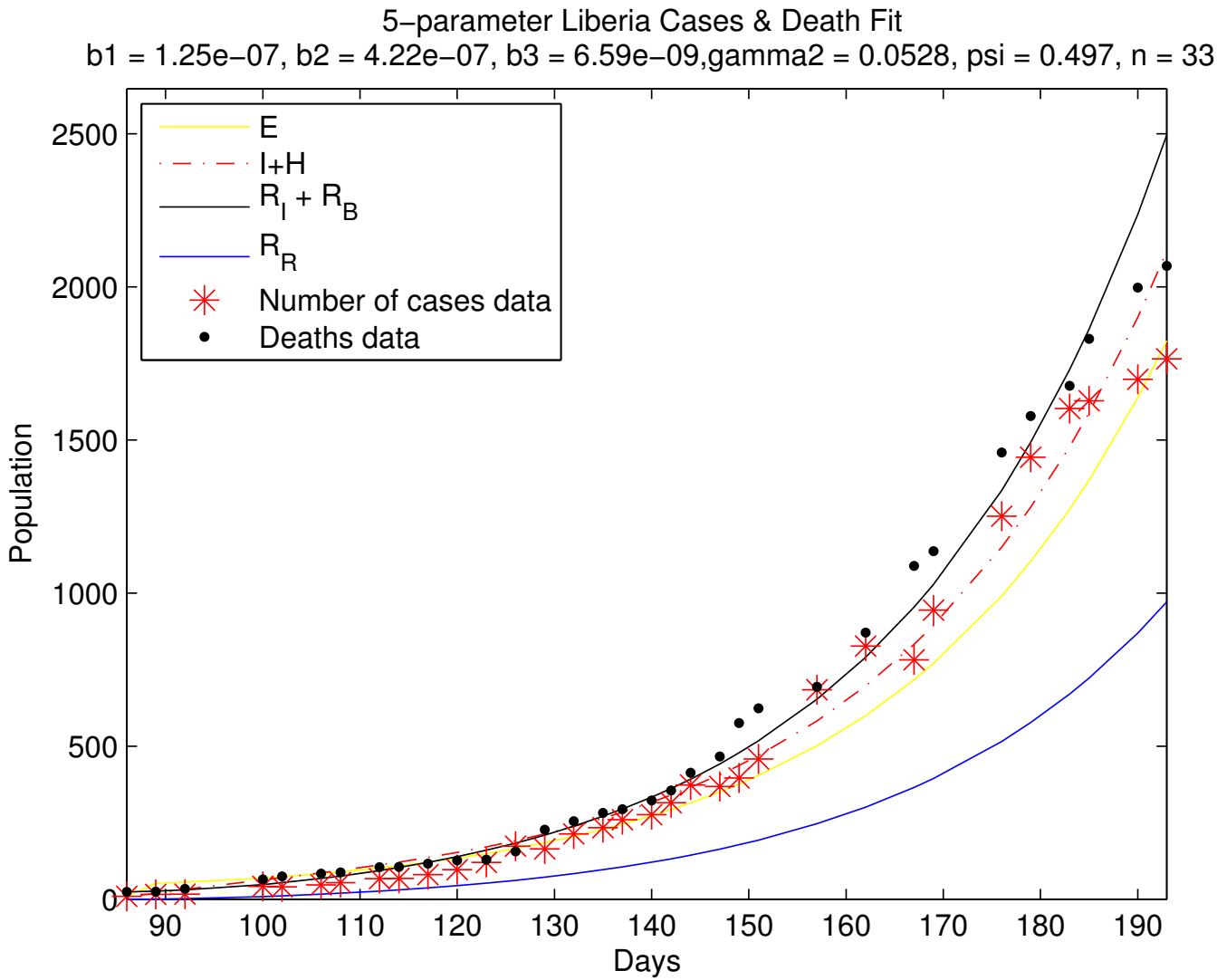

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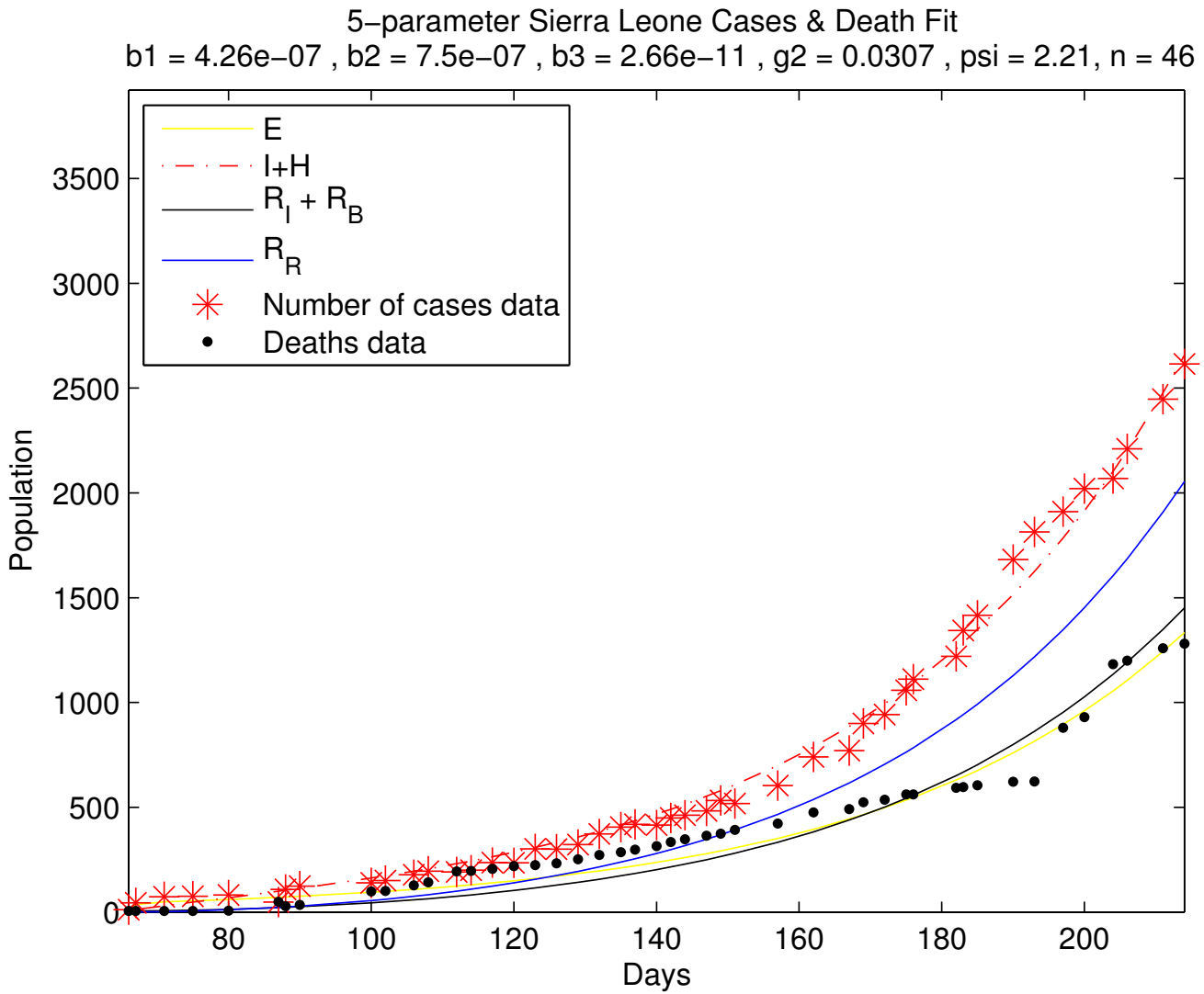
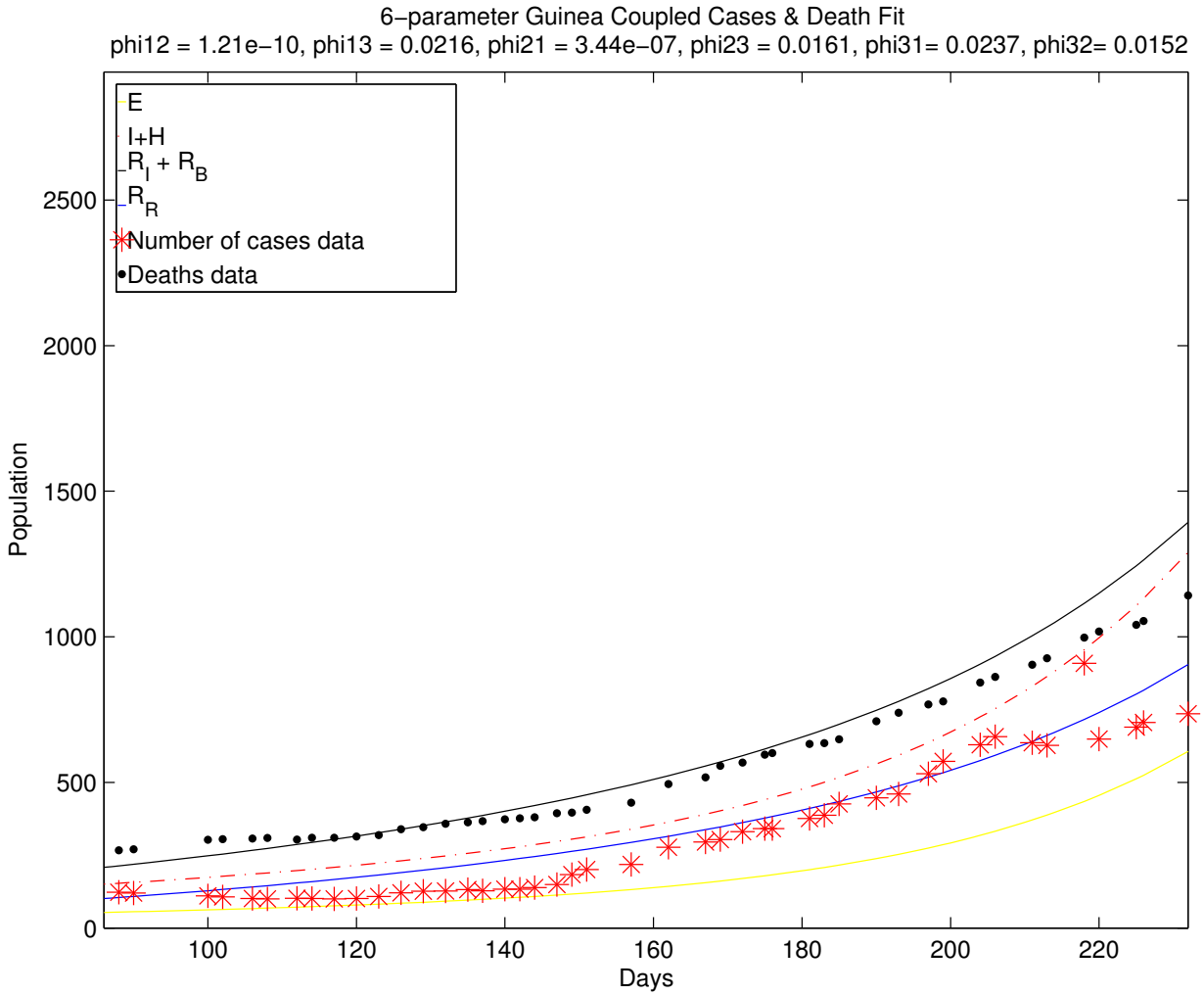


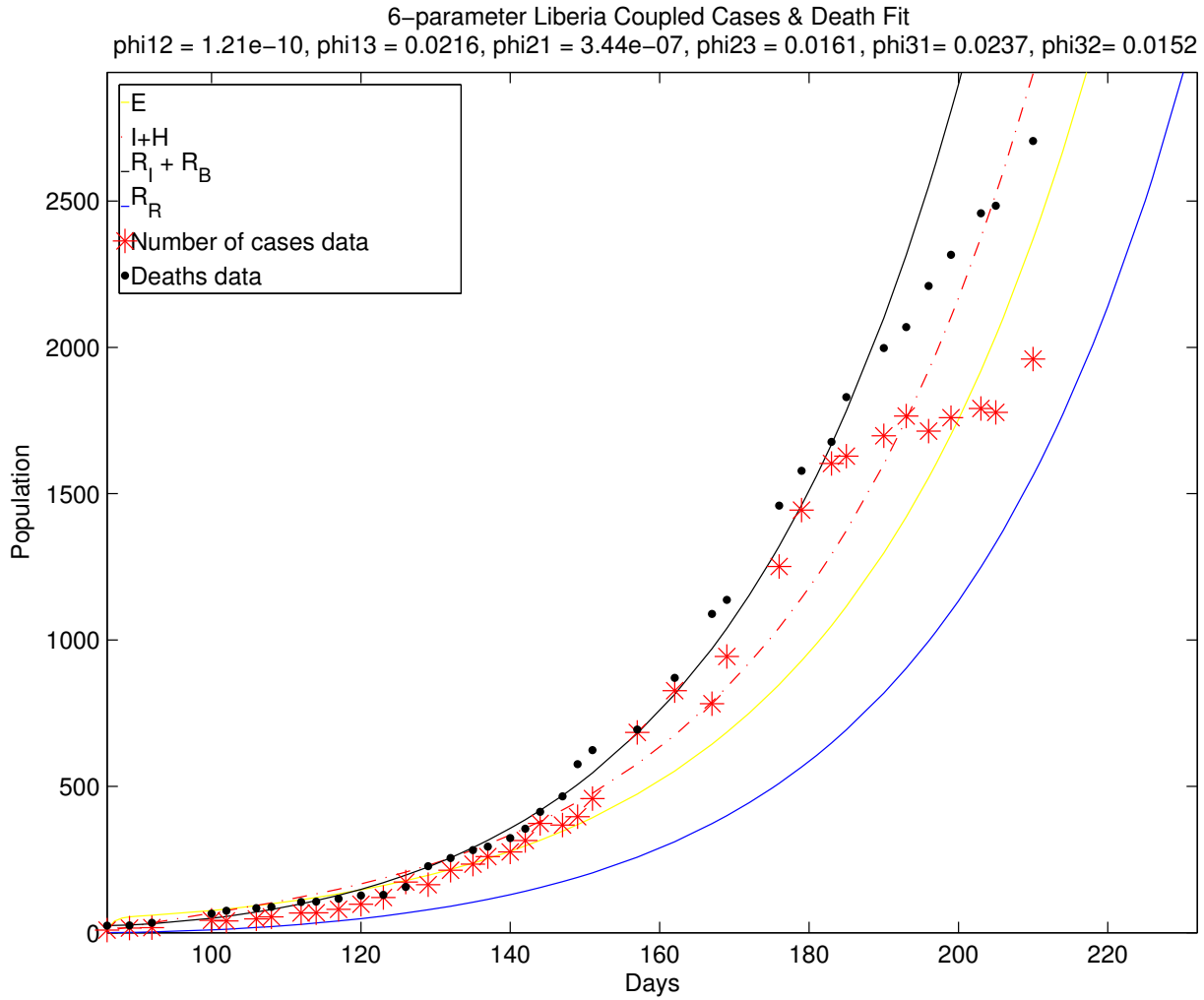
Adding Transportation Between Countries

$$
\begin{cases}
\dfrac{dS_i}{dt} = \alpha S_i - \beta_1 S_i I_i - \beta_2 S_i R_{I,i} - \beta_3 S_i H_i \\
\dfrac{dE_i}{dt} = \beta_1 S_i I_i + \beta_2 S_i R_{I,i} + \beta_3 S_i H_i - \delta E_i - \phi_{ij} E_i + \phi_{ji} E_j \\
\dfrac{dI_i}{dt} = \delta E_i - \gamma_1 I_i - \psi I_i \\
\dfrac{dH_i}{dt} = \psi I_i - \gamma_2 H_i \\
\dfrac{dR_{I,i}}{dt} = \rho_1 \gamma_1 I_i - \omega R_{I,i} \\
\dfrac{dR_{B,i}}{dt} = \omega R_{I,i} + \rho_2 \gamma_2 H_i \\
\dfrac{dR_{R,i}}{dt} = (1-\rho_1)\gamma_1 I_i + (1-\rho_2)\gamma_2 H_i
\end{cases}
$$

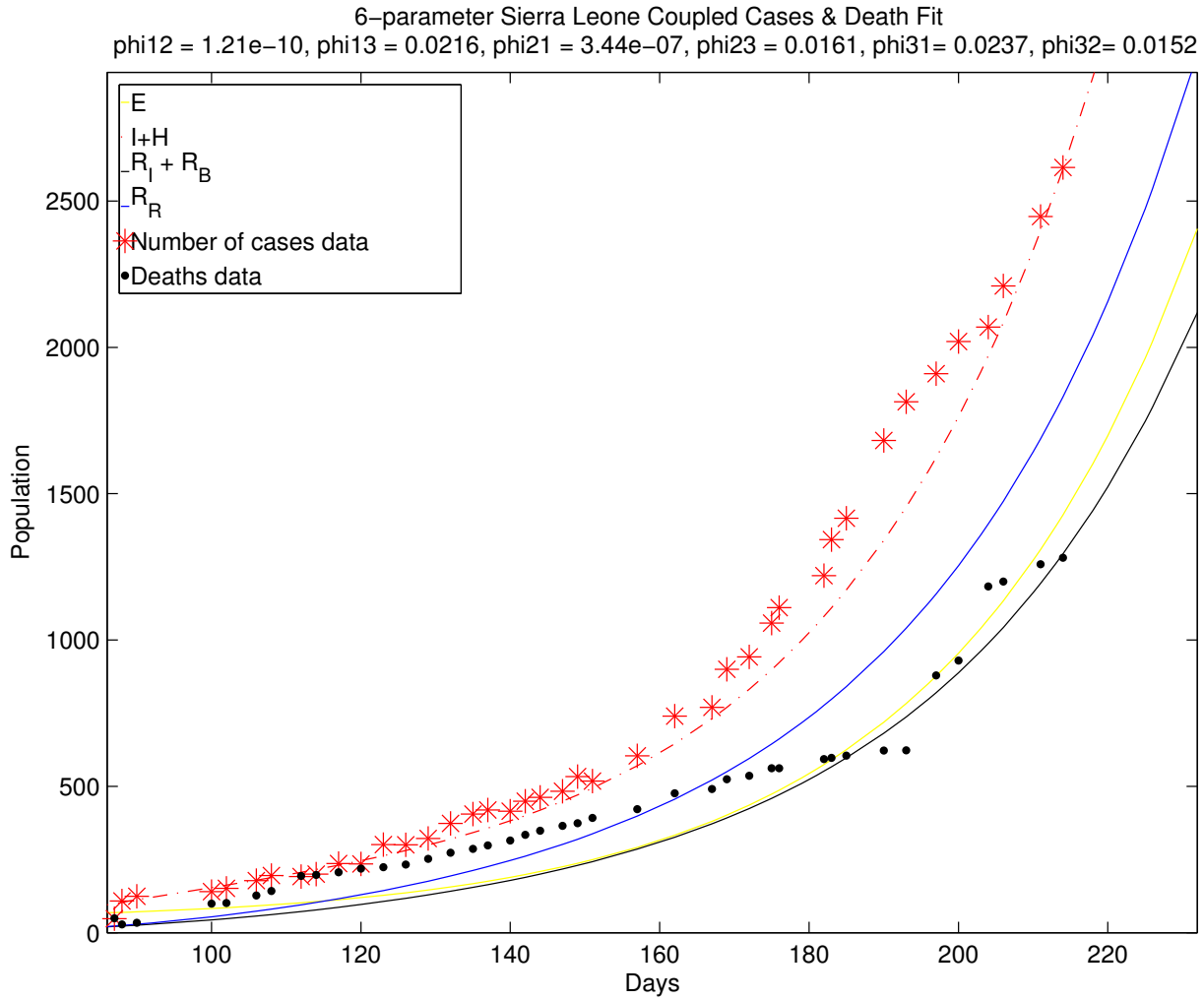
Coupled Parameter Fitting



Coupled Parameter Fitting



Coupled Parameter Fitting



Coupling Method and Results

- Created coupled (21 compartment) model, with initial conditions "stitched" together
- Used previously fit parameter values for each country, then fit transportation coefficients to the same data as before

- **Error without coupling:** 142
- **Error with coupling:** 14

Placement Based on Parameter Values

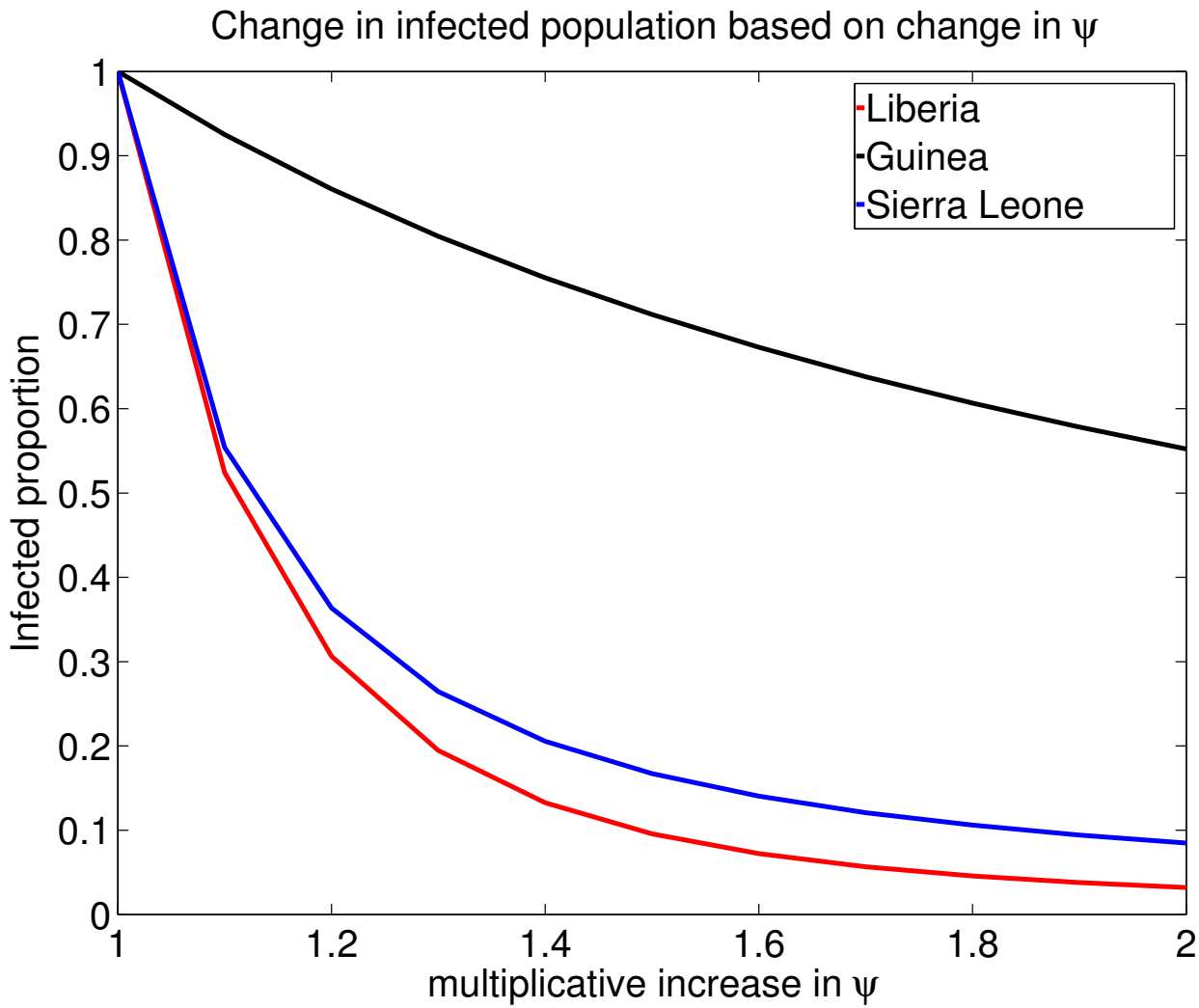
	Guinea	Liberia	Sierra Leone
(hospitalization rate)$^{-1}$ ψ	1.44	0.497	2.21
death rate (not hospitalized) ρ_1	0.649	0.77	0.451
death rate (hospitalized) ρ_2	0.59	0.7	0.41

Placement Based on Sensitivity Analysis

Relative change in population steady state given a change in a parameter value

$\frac{\partial \bar{I}}{\partial \psi}\Delta\psi$	
Guinea	-336.882
Liberia	-84.6367
Sierra Leone	-0.467521

Placement Based on Minimizing Infectious Population



Future Work

- Increase resolution — provinces rather than countries
- Assume hospitals have a finite capacity (logistic transfer rate)
- Observe epidemic by placing "patient zero" in one country and watching it spread with the coupled model
- Gather recent data and update our model while the epidemic is ongoing

Questions?

Variable Definitions

Populations:

S = Susceptible Population

E = Exposed Population (infected but not yet showing symptoms)

I = Infectious Population

H = Hospitalized Population

R_I = Removed and Infectious Population (dead but not yet buried)

R_B = Removed and Buried

R_R = Removed and Recovered

Variable Definitions

Parameters:

$\alpha =$ population growth constant[3]

$\beta_1 =$ transmission rate between infected and susceptible

$\beta_2 =$ transmission rate between removed and still infectious and susceptible

$\beta_3 =$ transmission rate between hospitalized and susceptible

$\delta =$ rate at which people move from exposed to infected [4]

$\gamma_1 =$ (average time with disease for unhospitalized individuals)$^{-1}$

$\gamma_2 =$ (average time with disease for hospitalized individuals)$^{-1}$

$\psi =$ (average time that people become hospitalized)$^{-1}$

$\rho_1 = 1.1 \times \rho_2 =$ the proportion of people who die of the disease who are not hospitalized [5]

$\rho_2 =$ the proportion of people who die of the disease who are hospitalized [5]

$\omega =$ (time until one is buried)$^{-1}$

References

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[3] "Population Growth (annual %)." The World Bank. Web. 26 Mar. 2015. http://data.worldbank.org/indicator/SP.POP.GROW.

[4] "Signs and Symptoms." Centers for Disease Control and Prevention. N.p., n.d. Web. 27 Mar. 2015. http://www.cdc.gov/vhf/ebola/symptoms/index.html.

[5] "Case Fatality Rate for Ebolavirus." Epidemic : Molecular Epidemiology and Evolution of Viral Pathogens. Web. 17 Apr. 2015. http://epidemic.bio.ed.ac.uk/ebolavirus-fatality-rate.

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[9] Rivers, Caitlin. "Consolidated, machine-readable ebola situation reports"*Virgina Tech Network Dynamics and Simulation Science Laboratory.* 31 Dec, 2014. https://github.com/muxspace/ebola.sitrep