Department of Applied Mathematics and Statistics
COLORADO SCHOOL OF MINES
MATH484: Capstone - Mathematical and Computational Modeling

Assignment #2
Due Thursday, February 6, 2014

For problems which require computational simulation, please print and submit both your code and results (e.g., pictures). In all problems, use parameter values given in the handout.

1. HIV Handout, p. 350, Problem **2**.

2. One class of antiretroviral drug that is currently used to combat HIV is called a *reverse transcriptase inhibitor* or *RTI*. Such drugs stop virions from copying their RNA within an infected cell by stifling a key viral enzyme that is required for reverse transcription. The overall effect is a decrease in latently, and hence actively, infected T-cells. We will incorporate this into the HIV model. Define $\epsilon_{RTI} \in [0,1]$ to be the efficacy of an RTI, where $\epsilon_{RTI} = 0$ when the drug is not present in the body and $\epsilon_{RTI} = 1$ when the drug is 100% effective. Assume that the person under study is being treated with an RTI and write down the new model. If $\epsilon_{RTI} = 1$ what happens to the condition that ensures the stability of the non-infective state? What efficacy is needed in order to guarantee that the non-infective state is stable? Provide an expression for this efficacy in terms of system parameters, and then use numerical parameter values (p. 350) to compute the ϵ_{RTI} that guarantees stability. Finally, simulate this system for two different values of ϵ_{RTI} to demonstrate that your value is accurate.

3. Assume $n = 1$ and write down the mutation model. One important class of equilibrium states are triples of the form

$$(v, x, z) = \left(v^*, \frac{g}{k}, \frac{h}{k}\right)$$

where $v^* > 0$ is arbitrary. Show that these equilibria are stable if and only if $N_{div} > 1$. This can be expanded for arbitrary $n \in \mathbb{N}$ to prove Observation 1 (on p. 353).

4. Assume that the individual in the mutation model (with $n \in \mathbb{N}$ arbitrary) is treated with an RTI of efficacy $\epsilon_{RTI} \in [0,1]$. How does this alter the model? Determine the efficacy needed to ensure that the infection is controlled - derive both an expression and a number for this by using given parameter values (p. 354) as in Problem 2. Finally, simulate this system with $n = 6$ for two different values of ϵ_{RTI} to demonstrate that your value is accurate.