Department of Applied Mathematics and Statistics
COLORADO SCHOOL OF MINES
MATH484: Capstone - Mathematical and Computational Modeling

Assignment #4
Due Thursday, February 26, 2015

For problems which require computational simulation, please print and submit both your code and results (e.g., pictures).

1. Friedman & Littman, p.39, Problem **2.7.2**
Let $U > 0$ be given. For the equation

$$\partial_t c + U \partial_x c = 0,$$

investigate the **stability** of the FTFS (forward time, forward space) scheme

$$\frac{c_j^{n+1} - c_j^n}{\Delta t} + U \frac{c_{j+1}^n - c_j^n}{\Delta x} = 0.$$

2. Friedman & Littman, p.40, Problem **2.7.5**
For the same equation, investigate the **stability** of the Lax-Wendroff scheme

$$c_j^{n+1} = c_j^n - \frac{U}{2} \frac{\Delta t}{\Delta x} \left(c_{j+1}^n - c_{j-1}^n \right) + \frac{U^2}{2} \left(\frac{\Delta t}{\Delta x} \right)^2 \left(c_{j+1}^n - 2c_j^n + c_{j-1}^n \right).$$

3. Friedman & Littman, p.40, Problem **2.7.6**
Use the Lax-Wendroff scheme for Problem 2.4.1 and compare your results with those of the FTBS scheme we previously used. As in the directions for the first problem of HW#3, use $dt = 0.001$ and $dx = 0.1$. For the spatial interval use $[-2, 200]$, fix the y axis in your plots to $[0, 5]$, and create a 5×1 matrix of plots on the same figure.

4. Draw the stencil for the Lax-Wendroff scheme above. Then, show that this numerical method is **consistent** with the advection equation

$$\partial_t c + U \partial_x c = 0$$

and use this to demonstrate that it is first-order accurate in time and second-order accurate in space, i.e.

$$|E_j^n| \leq O\left(\Delta t + (\Delta x)^2\right).$$

5. Friedman & Littman, p.44, Problem **2.8.3**
We will do this problem together in class. Please make sure to submit the results with your solutions.