Department of Applied Mathematics and Statistics
COLORADO SCHOOL OF MINES
MATH484: Capstone - Mathematical and Computational Modeling

Assignment #6
Due Tuesday, April 7, 2015

For problems which require computational simulation, please print and submit both your code and results (e.g., pictures).

1. Let $\eta = \frac{1}{4}, \alpha = \frac{2}{10}$, and $\beta = 2$. Use Fourier Series with $N = 15$ to solve the inverse Electron Beam Lithography problem (i.e., find and plot $D_{15}(x)$) on the spatial interval $[-1, 1]$ with $dx = 10^{-3}$ and

$$E(x) = \begin{cases} 1 & \text{if } |x| < \frac{1}{2} \\ 0 & \text{else.} \end{cases}$$

Also compute and plot $E_{15}(x)$ - the approximation to $E(x)$ given by the first 15 terms of the Fourier Series expansion - in the same figure as $D_{15}(x)$.
Hint: To check your code for $D_{15}(x)$, try it with $N = 10$ and compare to Figure 3.9.

2. Do the same as for Problem 1, but use the Fejer sums (i.e. $\tilde{D}_{15}(x)$ and $\tilde{E}_{15}(x)$) instead. Compare the new dosing function approximation to the old one - how are they different and why is one better?

3. Friedman & Littman, p.102, Problem **5.7.3** - Implicit Method only
Define $L = 10$, $t_0 = 4$, $u_0 = 2$, as well as the functions

$$S_1(t) = 1 \qquad \text{for } t \in [0, 4]$$

and

$$S_2(t) = \begin{cases} \frac{1}{2} & \text{if } t \in [0, 2] \\ \frac{3}{2} & \text{if } t \in [2, 4] \end{cases}$$

where L is the length of the converter and impose the boundary condition $\frac{\partial T}{\partial x} = 0$ at the right endpoint. Using these values and the implicit method on p.98 with $dx = 0.05$ and $dt = 0.01$, solve (5.19)–(5.23) and determine which control function makes $J(S)$ smaller, S_1 or S_2. Recall that $J(S)$ is defined for the simplified problem by (5.25).