PIC Math: A Modified SEIR Model for the Spread of Ebola in Western Africa

Chelsea Sandridge, Eric Jones, Kelsey Kalmbach, and Paul Diaz *

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Abstract

Ebola virus disease is a dangerous infectious pathogen that poses clear and present danger to the human population. The most recent outbreak of Ebola has taken root in Western Africa and has brought destruction to the countries of Guinea, Liberia, and Sierra Leone. The international community has provided some relief response to the epidemic, but this has not yet been enough to halt the epidemic (see Appendix A Figure 6). Our primary goal for this project was to determine the optimal location to place a hospital in Western Africa. To gain insight into the spread of the disease, we developed a modified SEIR model and fit parameters to data pertaining to the current epidemic. After running our simulations, we developed different metrics to determine the best location to place a hospital. Through our analysis, we established that the country of Liberia was most susceptible to changes in the rate of infectious individuals entering hospitals. This indicated a marginal benefit in adding a hospital to Liberia when compared to Sierra Leone, and a significant benefit compared to Guinea when measuring total change in cumulative infection numbers.

*Colorado School of Mines. Math 484: Capstone. Advisor: Stephen Pankavich

1 Background

The most recent Ebola outbreak began in December 2013 and has resulted in a devastating loss of life in the countries of Guinea, Liberia, and Sierra Leone. This outbreak has been the deadliest in the history of the disease claiming more than 10,000 lives thus far [1]. The severity of this epidemic has prompted an international response in an effort to halt the further spread of the disease.

The Ebola virus is infamous for its approximately 10 day incubation period, during which it is not infectious. However, once a patient turns symptomatic, they are able to pass on the virus to others. The virus can be transferred through direct contact with bodily fluids such as blood and vomit and is not known to be transmitted through the air. Another prevailing characteristic of the virus is that people who have died due to the disease are still able to transmit the virus. This is particularly a problem in areas where burial rituals require the handling of the deceased. Furthermore, the lack of adequate health care between Guinea, Liberia, and Sierra Leone has only perpetuated the disease. While several outside governments have contributed aid in the form of health care workers and treatment beds, this has not yet been enough to curb the rate of infection.

2 Goals of Project

Our first objective was to develop a model for the spread of Ebola in Western Africa, taking into account the characteristics of the disease. We then determined realistic parameters that fit the given data for Guinea, Liberia, and Sierra Leone. However, our main goal was to study and determine the optimal location to place a treatment facility amidst the countries of Guinea, Liberia, and Sierra Leone: we determined this location by looking at fitted parameter values, the sensitivity of our determined steady state, and by minimizing the infectious population with parameter variation.

3 Assumptions

While SEIR models are often implemented when studying the spread of infectious disease, for Ebola we needed to augment the original SEIR model with additional compartments, based on the assumptions which follow.

- Infected individuals can move to three different compartments: removed and infectious (i.e. not buried), removed and buried, removed and recovered
- Individuals who have died from the disease but who have not yet been buried can still infect susceptible individuals
- Individuals who recover from the disease are no longer susceptible
- Once hospitalized, infected individuals can still infect others. However, those who die in the hospital are buried immediately, and thus cannot infect others once dead
- Hospitals have unlimited space, but there is some delay in hospitalization
- Hospitalized individuals have a 10 percent greater chance of survival than non-hospitalized individuals
- There exist governmental, societal, or public health pressures encouraging the movement of symptomatic individuals to seek getting tested or entering Ebola treatment facilities

4 Mathematical Model

We used a modified SEIR model (Equation 1) to account for the assumptions related to the spread of Ebola.

$$\begin{cases} \frac{dS}{dt} = \alpha S - \beta_1 SI - \beta_2 SR_I - \beta_3 SH \\ \frac{dE}{dt} = \beta_1 SI + \beta_2 SR_I + \beta_3 SH - \delta E \\ \frac{dI}{dt} = \delta E - \gamma_1 I - \psi I \\ \frac{dH}{dt} = \psi I - \gamma_2 H \\ \frac{dR_I}{dt} = \rho_1 \gamma_1 I - \omega R_I \\ \frac{dR_B}{dt} = \omega R_I + \rho_2 \gamma_2 H \\ \frac{dR_R}{dt} = (1-\rho_1)\gamma_1 I + (1-\rho_2)\gamma_2 H \end{cases} \tag{1}$$

Here S is the susceptible population, E is the exposed population (i.e. those who have the disease but are not yet symptomatic), I is the infectious population, H is the hospitalized population, R_I is the removed and infectious population (i.e. those who have died but not yet been buried), R_B is the removed and buried population, and R_R is the removed and recovered population.

α = population growth constant [2]
β_1 = transmission rate between infected and susceptible
β_2 = transmission rate between removed and still infectious and susceptible
β_3 = transmission rate between hospitalized and susceptible
δ = rate at which people move from exposed to infected [3]
γ_1 = (average time with disease for unhospitalized individuals)$^{-1}$[5]
γ_2 = (average time with disease for hospitalized individuals)$^{-1}$
ψ = (average time for people to become hospitalized)$^{-1}$
$\rho_1 = 1.1 \times \rho_2$ = the proportion of people who die of the disease who are not hospitalized [4]
ρ_2 = the proportion of people who die of the disease who are hospitalized [4]
ω = (time until one is buried)$^{-1}$

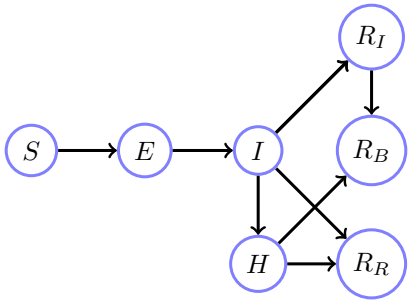


Figure 1: Illustration of mathematical model

5 Methods & Simulation

We ran simulations using data from a country time-series `GitHub` repository managed and maintained by Caitlin Rivers through the Network Dynamics and Simulation Science Laboratory at Virginia Tech [9]. The time-series contains the best data set we were able to find, with cumulative values of infections and deaths separate for each of the three countries. We parsed the data by removing outliers and areas containing a large number of days with no data. The resulting number of data points for Liberia, Guinea, and Sierra Leone were 30, 90, and 46 respectively.

The data set we worked with tracked the cumulative number of infections and deaths on any given day, while our model represents the number of active infections on a particular day, and so we had to modify the data set. To do this, we took the difference between the cumulative cases with the cumulative deaths, which allowed us to approximate the number of active cases on a particular day. These modified data sets were then applied in our parameter fitting simulations described below. In order to obtain our parameter values, we used Matlab's built in function `fminsearch`, which finds the minimum of unconstrained multivariable functions using a derivative-free method. The error, $\boldsymbol{\epsilon}$, that we sought to minimize was taken to be the sum of squares between the removed population in the data and our simulated removed infectious and removed buried populations as well as the difference between the number of infected in the data and our simulated infectious and hospitalized populations, (see Equation 2).

$$\boldsymbol{\epsilon} := \sum \sqrt{(R_{data} - (R_I + R_B))^2 + (I_{data} - (I + H))^2} \tag{2}$$

Due to the presence of several non-linear transmission terms, our model required a stiff ODE solver— we used Matlab's built in ODE solver `ode15s` to find the solution of our model once we obtained all parameter values. We were able to pass the Jacobian $\boldsymbol{J}$ of our system to obtain a better approximation of our solution (since we were passing in analytically determined derivatives rather than having Matlab using a finite difference approximation).

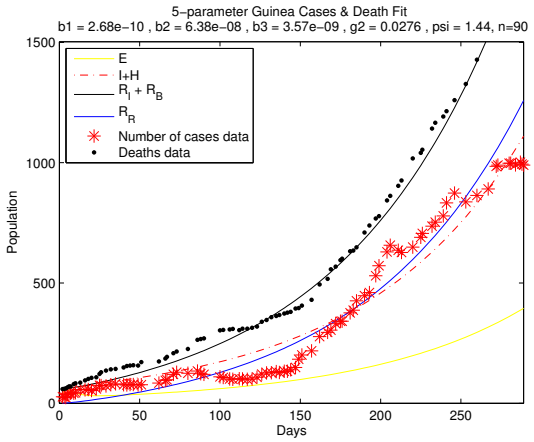
$$\boldsymbol{J} = \begin{pmatrix} \alpha - \beta_1 I - \beta_2 R_I - \beta_3 H & 0 & -\beta_1 S & -\beta_3 S & -\beta_2 S & 0 & 0 \\ \beta_1 I + \beta_2 R_I + \beta_3 H & -\delta & \beta_1 S & \beta_3 S & \beta_2 S & 0 & 0 \\ 0 & \delta & -\gamma_1 - \psi & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi & -\gamma_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_2\gamma_2 & \omega & 0 & 0 \\ 0 & 0 & 1 - \rho_1\gamma_1 & 1 - \rho_2\gamma_2 & 0 & 0 & 0 \end{pmatrix}$$

6 Parameter Values

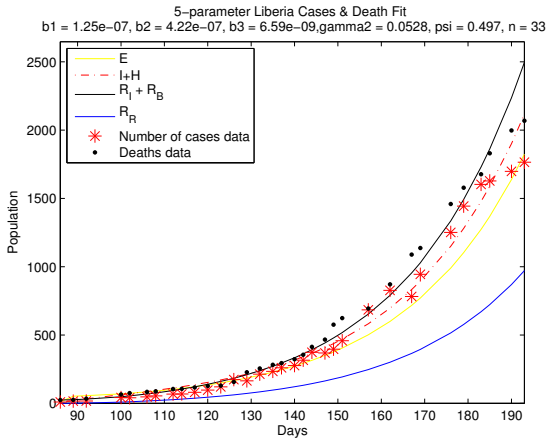
In Table 1 we compile the parameter values used in the model for each independent country. Six of these values were taken from literature, while five of the values were fitted to data (specifically, the values of β_1, β_2, β_3, γ_2, and ψ were fitted). These fitted parameters are in bold in Table 1. In fitting these parameters, we followed the procedure outlined in Section 5. Finally, we were able to run our model with the corresponding parameter values for Guinea, Liberia, and Sierra Leone— in Figure 2, we show the country plots alongside the data points.

	Guinea	Liberia	Sierra Leone
α	$\frac{.025}{365}$	$\frac{.024}{365}$	$\frac{.019}{365}$
$\boldsymbol{\beta_1}$	2.68×10^{-10}	1.25×10^{-7}	4.26×10^{-7}
$\boldsymbol{\beta_2}$	6.38×10^{-8}	4.22×10^{-7}	7.5×10^{-7}
$\boldsymbol{\beta_3}$	3.57×10^{-9}	6.59×10^{-9}	2.66×10^{-11}
δ	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
γ_1	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\boldsymbol{\gamma_2}$	0.0276	0.0528	0.0307
$\boldsymbol{\psi}$	1.44	0.497	2.21
ρ_1	0.649	0.77	0.451
ρ_2	0.59	0.7	0.41
ω	1	1	1

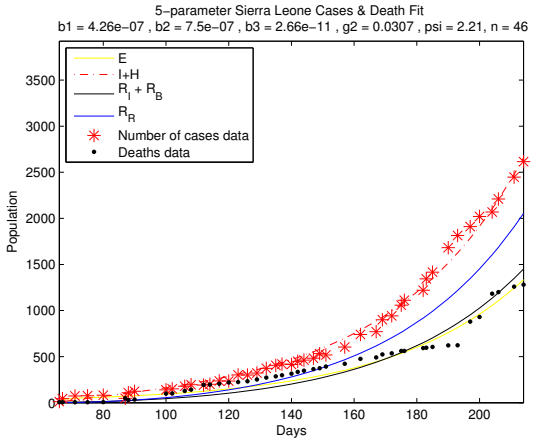
Table 1: Parameter Values



(a) Guinea



(b) Liberia



(c) Sierra Leone

Figure 2: Independent Country Parameter Fits

7 Steady States

To determine any non-zero steady state solutions, we set the rates of change for the S, E, I, H, and R_I populations to zero. We omit the rates of change for the removed-buried and removed-recovered populations since they have no feedback within the model (i.e. no rate of change depends on the variables R_B or R_R), and because these populations serve as sinks for our model. Following this analysis, we find

$$\begin{cases} \dfrac{dS}{dt} = \alpha S - \beta_1 SI - \beta_2 SR_I - \beta_3 SH = 0 \\ \dfrac{dE}{dt} = \beta_1 SI + \beta_2 SR_I + \beta_3 SH - \delta E = 0 \end{cases} \tag{3}$$

and

$$\begin{cases} \dfrac{dI}{dt} = \delta E - \gamma_1 I - \psi I = 0 \Leftrightarrow E = \dfrac{\gamma_1 + \psi}{\delta} I \\ \dfrac{dH}{dt} = \psi I - \gamma_2 H = 0 \Leftrightarrow H = \dfrac{\psi}{\gamma_2} I \\ \dfrac{dR_I}{dt} = \rho_1 \gamma_1 I - \omega R_I = 0 \Leftrightarrow R_I = \dfrac{\rho_1 \gamma_1}{\omega} I, \end{cases} \tag{4}$$

so that substitution of (4) into (3) yields

$$\begin{aligned} \frac{dS}{dt} &= S\left[\alpha - I\left(\beta_1 + \frac{\beta_2 \rho_1 \gamma}{\omega} + \frac{\beta_3 \psi}{\gamma_2}\right)\right] = 0 \\ \frac{dE}{dt} &= I\left[-(\gamma_1 + \psi) + S\left(\beta_1 + \frac{\beta_2 \rho_1 \gamma_1}{\omega} + \frac{\beta_3 \psi}{\gamma_2}\right)\right] = 0. \end{aligned} \tag{5}$$

Then, solving (5) explicitly for the steady state solution $\{\bar{S}, \bar{E}(\bar{I}), \bar{I}, \bar{H}(\bar{I}), \bar{R}_I(\bar{I})\}$ we find

$$\begin{aligned} \bar{S} &= \frac{\gamma_1 + \psi}{\beta_1 + \frac{\beta_2 \rho_1 \gamma_1}{\omega} + \frac{\beta_3 \psi}{\gamma_2}} \\ \bar{E}(\bar{I}) &= \frac{\gamma_1 + \psi}{\delta} \bar{I} \\ \bar{I} &= \frac{\alpha}{\beta_1 + \frac{\beta_2 \rho_1 \gamma_1}{\omega} + \frac{\beta_3 \psi}{\gamma_2}} \\ \bar{H}(\bar{I}) &= \frac{\psi}{\gamma_2} \bar{I} \\ \bar{R}_I(\bar{I}) &= \frac{\rho_1 \gamma_1}{\omega} \bar{I}. \end{aligned} \tag{6}$$

We have therefore solved for an endemic steady state solution namely, Equation 6. Notice that this steady state is completely defined by $\bar{S}$ and $\bar{I}$, and in turn, $\bar{S}$ and $\bar{I}$ are defined by the parameters of the model. We will use this explicitly defined steady state later, to investigate the model's sensitivity with respect to the system parameters.

8 Coupled System

Next, we added transportation coefficients to model transportation between countries. We assumed that since the susceptible population is so large and the movement between countries is relatively small, the movement of susceptible individuals was negligible. Additionally, we assumed that there are barriers to movement for infectious individuals and hospitalized individuals would be clearly infectious (and hence be prevented from crossing any borders) or must stay in the hospital, and therefore cannot move. Thus, we concentrated our system coupling on the movement of the exposed population between countries. Our

updated model, accounting for transportation

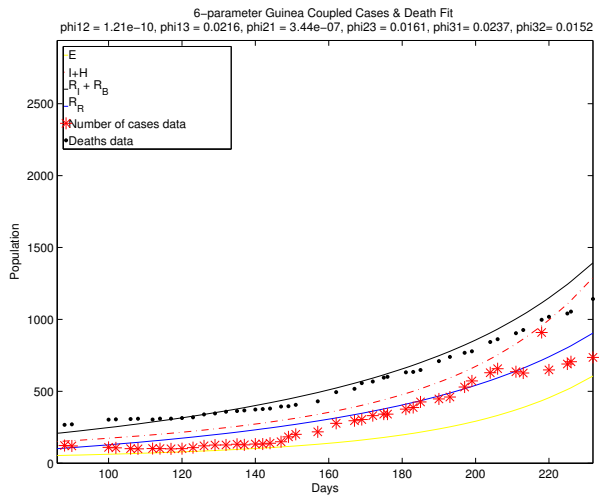
$$\begin{cases} \dfrac{dS_i}{dt} = \alpha S - \beta_1 SI - \beta_2 SR_I - \beta_3 SH \\ \dfrac{dE_i}{dt} = \beta_1 SI + \beta_2 SR_I + \beta_3 SH - \delta E - \phi_{ij} E_i + \phi_{ji} E_j \\ \dfrac{dI_i}{dt} = \delta E - \gamma_1 I - \psi I \\ \dfrac{dH_i}{dt} = \psi I - \gamma_2 H \\ \dfrac{dR_{I,i}}{dt} = \rho_1 \gamma_1 I - \omega R_I \\ \dfrac{dR_{B,i}}{dt} = \omega R_I + \rho_2 \gamma_2 H \\ \dfrac{dR_{R,i}}{dt} = (1-\rho_1)\gamma_1 I + (1-\rho_2)\gamma_2 H, \end{cases} \tag{7}$$

where $i = 1, 2, 3$ represent Liberia, Guinea, and Sierra Leone respectively. The newly added term ϕ_{ij} represents the movement of exposed individuals from country i to country j, $j = 1, 2, 3, j \neq i$. To fit the transportation coefficients between countries, we used the previously fitted and compiled parameter values for each country (as discussed earlier), and then fit the transportation coefficients using the same error function as for the previous fits.

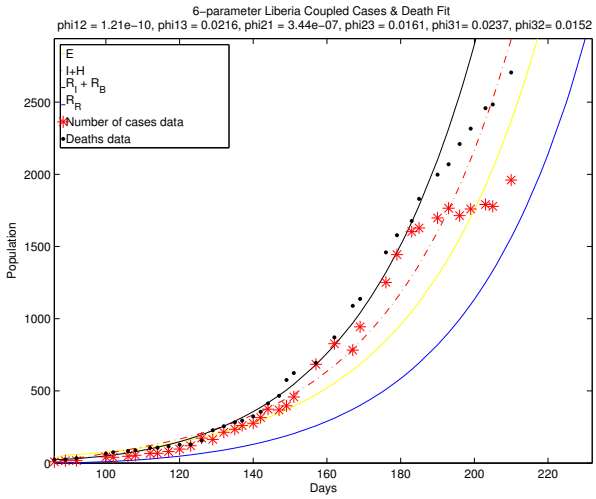
In coupling these countries, we needed to splice together some numerical results to compensate for the different initial conditions associated with each country. If we call the day of the first data point for each of the three countries t_1, t_2, and t_3 (let's assume $t_1 < t_2 < t_3$), then to splice together the coupled model we simulated t_1 through t_3 to arrive at country 1's "initial condition" (which is the state of country 1 at t_3), and similarly simulated t_2 through t_3 to find country 2's state at t_3. After this process, we had a legitimate initial condition at the latest starting point of the data t_3, and we were able to solve the ODEs using a numerical solver (`ode15s` in our case). We should note that in evaluating the error associated with the difference between the model and the data, we assumed each country's error was weighted equally, and we only calculated the error after the simulation time $t > t_3$. The fitted ϕ_{ij} values can be seen in Table 2, and the numerical simulations of our coupled model are in Figure 3.

ϕ_{12}	ϕ_{13}	ϕ_{21}	ϕ_{23}	ϕ_{31}	ϕ_{32}
1.21E-10	0.0216	3.44E-7	0.0161	0.0237	0.0152

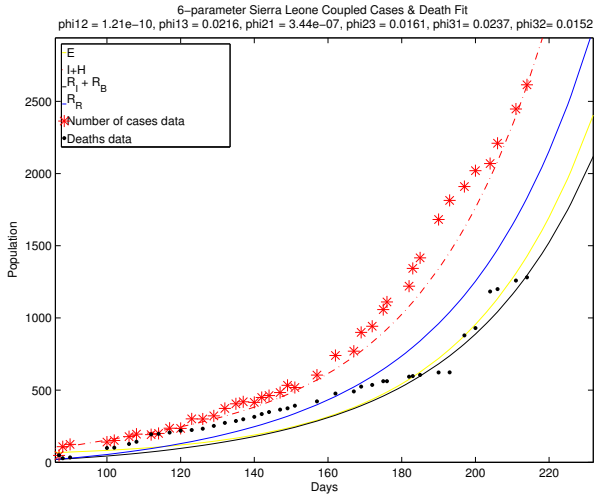
Table 2: Transportation coefficients for country coupling (countries 1, 2, 3, are Liberia, Guinea, Sierra Leone respectively)



(a) Guinea



(b) Liberia



(c) Sierra Leone

Figure 3: Independent Country Parameter Fits

9 Sensitivity Analysis

If the change in the steady state value with respect to a certain parameter value is large, we can determine that that parameter has a significant impact on the model. Accordingly, we will take the derivative of the steady state values in Equation 6 with respect to each of the parameters. Since we are primarily concerned with the I population to understand the disease dynamics and the effects of hospitalization, we have only taken the derivatives of I with respect to each of the parameters.

We see that some of the most sensitive parameters are β_1, β_2 and ψ, all values that we fit, while the values of γ_1 and ρ_1 (which we took from literature) are not as sensitive. The values that were more sensitive were all values that we fit. This indicates that a small change in one of these parameters can have a large impact on the disease dynamics; our values were fit to minimize error in the model compared to the data set, and so are as accurate as we could make them.

In looking at our sensitivity analysis, we consider the absolute sensitivity $\dfrac{\partial \bar{I}}{\partial q}$ as well as the relative sensitivity $\dfrac{\partial \bar{I}}{\partial q}\Delta q \sim \dfrac{\partial \bar{I}}{\partial q} q$ for some parameter in our model q; we make the assumption that $\Delta q \sim q$, or that a change in some parameter is proportional to the size of the parameter. With this, we can consider, roughly, $\Delta \bar{I}$ for variation in each of the parameters (a measure which yields more productive values than the absolute

	Guinea (absolute)	Guinea (relative)	Liberia (absolute)	Liberia (relative)	Sierra Leone (absolute)	Sierra Leone (relative)
$\frac{\partial \bar{I}}{\partial \alpha}$	5.245×10^{6}	344.856	4.555×10^{6}	299.526	2.167×10^{6}	112.736
$\frac{\partial \bar{I}}{\partial \beta_1}$	-1.809×10^{9}	-0.485	-1.364×10^{9}	-170.554	-2.442×10^{8}	-104.010
$\frac{\partial \bar{I}}{\partial \beta_2}$	-1.174×10^{8}	-7.489	-1.051×10^{8}	-44.3358	-1.101×10^{7}	-8.259
$\frac{\partial \bar{I}}{\partial \beta_3}$	-11.539	-7.489	-57.579	-44.336	-18.312	-8.259
$\frac{\partial \bar{I}}{\partial \gamma_1}$	-74.8896	-7.48896	-443.358	-44.3358	-82.586	-8.259
$\frac{\partial \bar{I}}{\partial \gamma_2}$	7.489	7.489	44.336	44.336	8.259	8.259
$\frac{\partial \bar{I}}{\partial \psi}$	-9.436×10^{10}	-336.882	-1.284×10^{10}	-84.637	-1.758×10^{10}	-0.468
$\frac{\partial \bar{I}}{\partial \rho_1}$	-233.946	-336.882	-170.295	-84.637	-0.212	-0.468
$\frac{\partial \bar{I}}{\partial \omega}$	12205.900	336.882	1602.970	84.637	15.2287	0.468

Table 3: Sensitivity Analysis Values

sensitivity). We have supplied all of the sensitivities for our model for each of the three countries in Table 3.

10 Hospital Placement

We considered three different metrics to determine the optimal location to place a hospital. First we looked at some of the specific parameter values (see Table 4)— we found that Liberia has the smallest ψ value, corresponding to the longest wait time for hospitalization. Liberia also had the largest fatality rates, ρ_1 and ρ_2, indicating that the disease is the most fatal in Liberia. Thus, based on comparing ψ, ρ_1 and ρ_2 for each country, Liberia is the country most in need of additional hospital capacity.

	Guinea	Liberia	Sierra Leone
(hospitalization rate)$^{-1}$ $\boldsymbol{\psi}$	1.44	0.497	2.21
death rate (not hospitalized) ρ_1	0.649	0.77	0.451
death rate (hospitalized) ρ_2	0.59	0.7	0.41

Table 4: Parameter Values

Next, we considered the hospital placement's impact on our nonzero steady state populations by investigating how the infectious steady state changes given a small relative change in the hospitalization parameter ψ. The important results are highlighted in the Table 5. Since this value is large for Guinea and Liberia, we can conclude that a small relative change in the hospitalization parameter would lead to a drastic change in the infectious steady state. This might suggest that these two countries would be good places to add additional hospital aid. If the wait time for hospitalization decreases slightly in either of these countries, the infectious steady state decreases significantly. Thus, hospital aid in either of these countries could largely affect the model dynamics. However, these conclusions can only be made for the steady state, which our simulations have not yet reached by the end of the simulation. Thus,the rest of the model dynamics may

react differently to a small change in ψ. However, this metric is still a good indicator relating the effectiveness of a hospital in each country respectively.

$\frac{\partial I}{\partial \psi}\Delta\psi$	
Guinea	-336.882
Liberia	-84.6367
Sierra Leone	-0.467521

Table 5: Steady State Values

Finally, we looked at the change in the infected population based on changing ψ. In Figure 4 we demonstrate the relative change in the infectious population at the end of the time span given a relative change in ψ (for this, we use the coupled model). For each country, ψ was varied from the initial fitted value up to twice that that amount, corresponding to a hospital wait time decreased by a factor of 2.

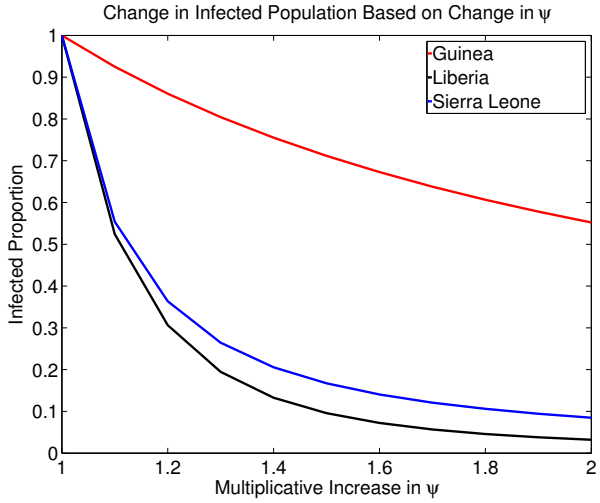


Figure 4: Sierra Leone

We see that a decrease in hospitalization time has the least impact on Guinea. It has similar impacts on Sierra Leone and Liberia, but the largest impact on Liberia. An increase in ψ by 10 percent leads to a nearly 50 percent decrease in infectious individuals in Liberia and Sierra Leone by the end of the simulation.

With these three metrics, we have concluded that the optimal placement for a hospital in West Africa is Liberia. Based on our parameter values, Liberia is most in need of a hospital, and based on our simulations, a hospital would be effective at decreasing the end infectious population in Liberia. Our sensitivity analysis also suggests that additional hospital capacity would help curb the infection in Liberia.

11 Future Work

In studying the Ebola epidemic, we were (as is common) restricted by our access to current and correct data. If we had data with more resolution (e.g. legitimate data at the province level), we could fit parameters to individual towns and cities instead of fitting parameters to each country as a whole— this would better capture the density of the infection and thus lead to a more accurate placement of the treatment facility.

We would also like to look at more current data. Our hospital placement is valid up through the validity of our data, which is December 2014. Accordingly, given more current data we could make a more current decision about the optimal hospital placement. Since December 2014, China has sent aid to Liberia which have proved highly effective, which lends credence to our hospital placement decision.

We would also like to add a logistic growth factor to our current model to account for the carrying capacity of hospitals. However, this modification would only be realistic if we were able to achieve a higher resolution of the data, with more detailed information about the current hospital situation in Western Africa,

data needed due to the discrepancies in hospitalization rates that occur in the different regions of Guinea, Liberia, and Sierra Leone. While conducting our research, we found that in some rural areas, hospital beds were available but not being used, while in larger cities people were often waiting to get into hospitals. Thus, when we ran the simulations for each country as a whole, it was hard to capture the disparities between hospital capacity regions within each country. With the addition of logistic growth our model would become

$$\begin{cases} \frac{dS}{dt} = \alpha S - \beta_1 SI - \beta_2 SR_I - \beta_3 SH \\ \frac{dE}{dt} = \beta_1 SI + \beta_2 SR_I + \beta_3 SH - \delta E \\ \frac{dI}{dt} = \delta E - I(\gamma_1 + \psi)\left(1 - \frac{H}{\kappa}\right) \\ \frac{dH}{dt} = I(\gamma_1 + \psi)\left(1 - \frac{H}{\kappa}\right) - \gamma_2 H \\ \frac{dR_I}{dt} = \rho_1 \gamma_1 I - \omega R_I \\ \frac{dR_B}{dt} = \omega R_I + \rho_2 \gamma_2 H \\ \frac{dR_R}{dt} = (1-\rho_1)\gamma_1 I + (1-\rho_2)\gamma_2 H. \end{cases} \tag{8}$$

With this modificatoin, our hospitalized population H now grows to a carrying capacity κ and no further. Such a change would warrant new analysis and simulations, but most importantly would require accurate data as to the historical values of κ in each of the countries of interest (i.e. the number of beds available to Ebola patients in each country). For this extended model, we would change κ as opposed to ψ to determine how additional hospital beds would effect the overall disease dynamics.

We would also like to investigate additional metrics for optimal hospital placement. In particular, we investigated how additional hospitalization would effect the model, given that it starts at the beginning of the model. More realistically, aid is given at some point further in to the epidemic. It would be interesting to see if the overall dynamics changed with additional hospitalization introduced on some given day in the model as opposed to at the beginning.

Finally, we would like to model the spread of Ebola staring with the initial infection, 'patient zero'. Then, using our coupled system, accurately capture the spread of the disease over all three countries. This would give a better indication about the disease dynamics for the entire outbreak. We would hope to see similar parameter values as the ones we have fit previously. This would add validity to our model and suggest that our results are valid for a much longer time frame than just for the data we used. Even further, introducing time-varying coupling parameters (presumably the ease of moving between countries has grown more difficult as the epidemic has progressed) would allow for a more cohesive understanding of our model.

12 Conclusion

For this project, we developed a modified SEIR model to address the characteristics of the Ebola epidemic in Western Africa. After creating this model, we were able to use Matlab to fit values to our unknown parameters and run simulations against known data for Guinea, Liberia, and Sierra Leone. We then coupled our individual models to account for transportation of exposed individuals between the three countries. With this coupled model in place, we were able to use different metrics in determining the optimal location to place a hospital. First, we looked at parameter values and their variation between Guinea, Liberia, and Sierra Leone. Second, we looked at the sensitivity of our steady state with respect to the parameters. Finally, we looked at the effect the hospitalization rate parameter ψ had on reducing the infectious population in each country.

Our results indicate that Liberia is the best location to place a hospital. This conclusion is valid up until the end of our data set which ends in December 2014. These results are partially verified by the state of Liberia in April 2015. As of April 29, there have been no new cases in the state of Liberia for 34 days (see Appendix A Figure 6) and many of the Ebola treatment centers of Liberia have closed, are scheduled to scale-down, or are closing (see Appendix A Figure 7). The reason for this improvement might be related to the placement of a treatment facility by China in the state of Liberia in December 2014 [11]. Further,

the fact that the Chinese government was planning to open another Ebola treatment facility in Liberia as of December 2014 lends credence to our findings, as we were able to come to the same conclusion using data specifically from March-December 2014. Based on our model and assumptions, we have determined the optimal placement for a hospital in December 2014 (at least based upon our various metrics for hospital placement). Given a more current and more detailed data set, this same model and process for parameter fitting could be used to determine the current optimal placement for a hospital.

All of the code used in for this project is compiled at `github.com/erijones/ebola_modeling_484`, and may be used by any parties wishing to further our research.

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A

Appendix A: Figures

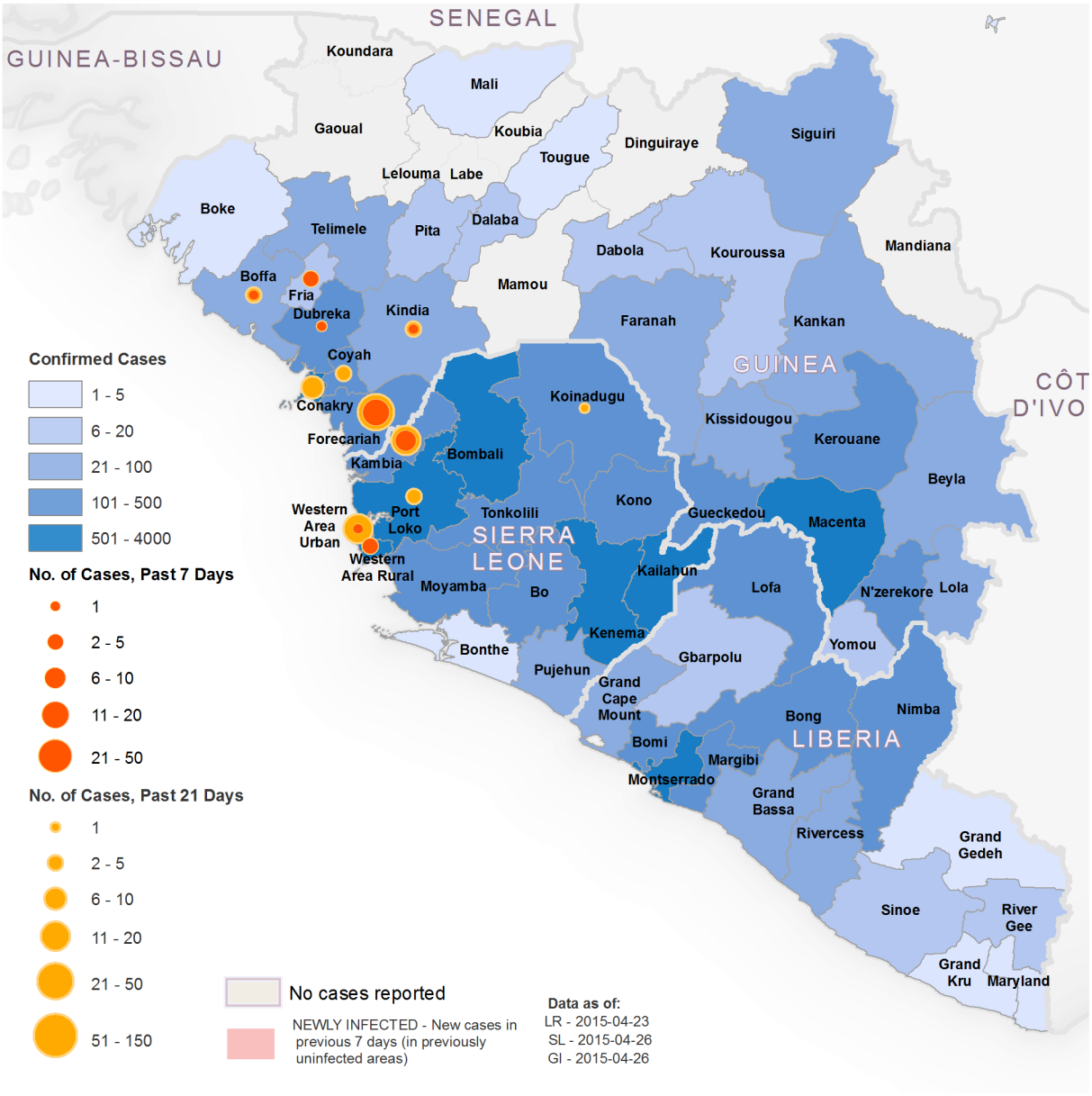

Figure 5: Case Counts - 29 April 2015 [10]

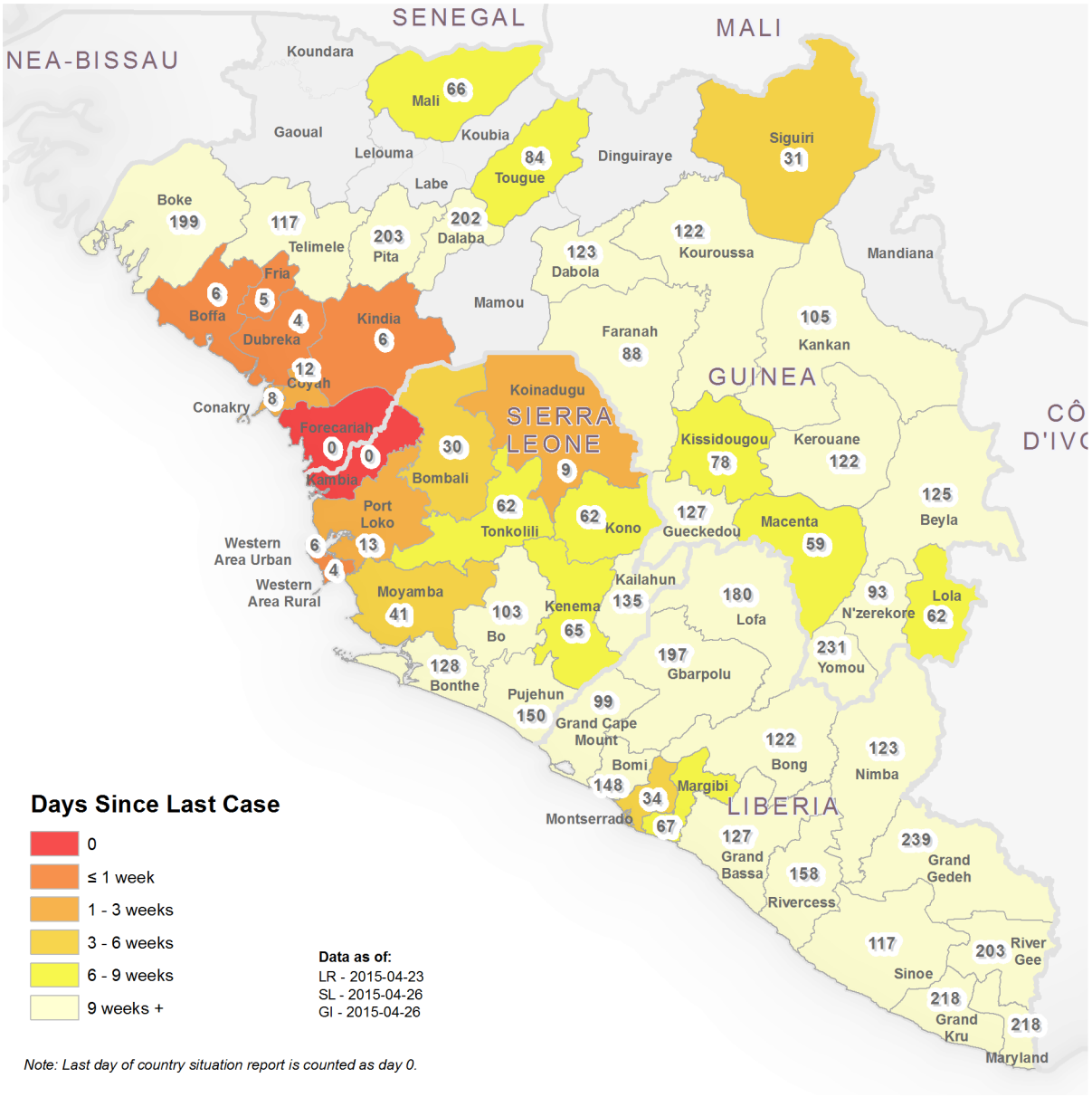

Figure 6: Days Since Last Case - 29 April 2015 [10]

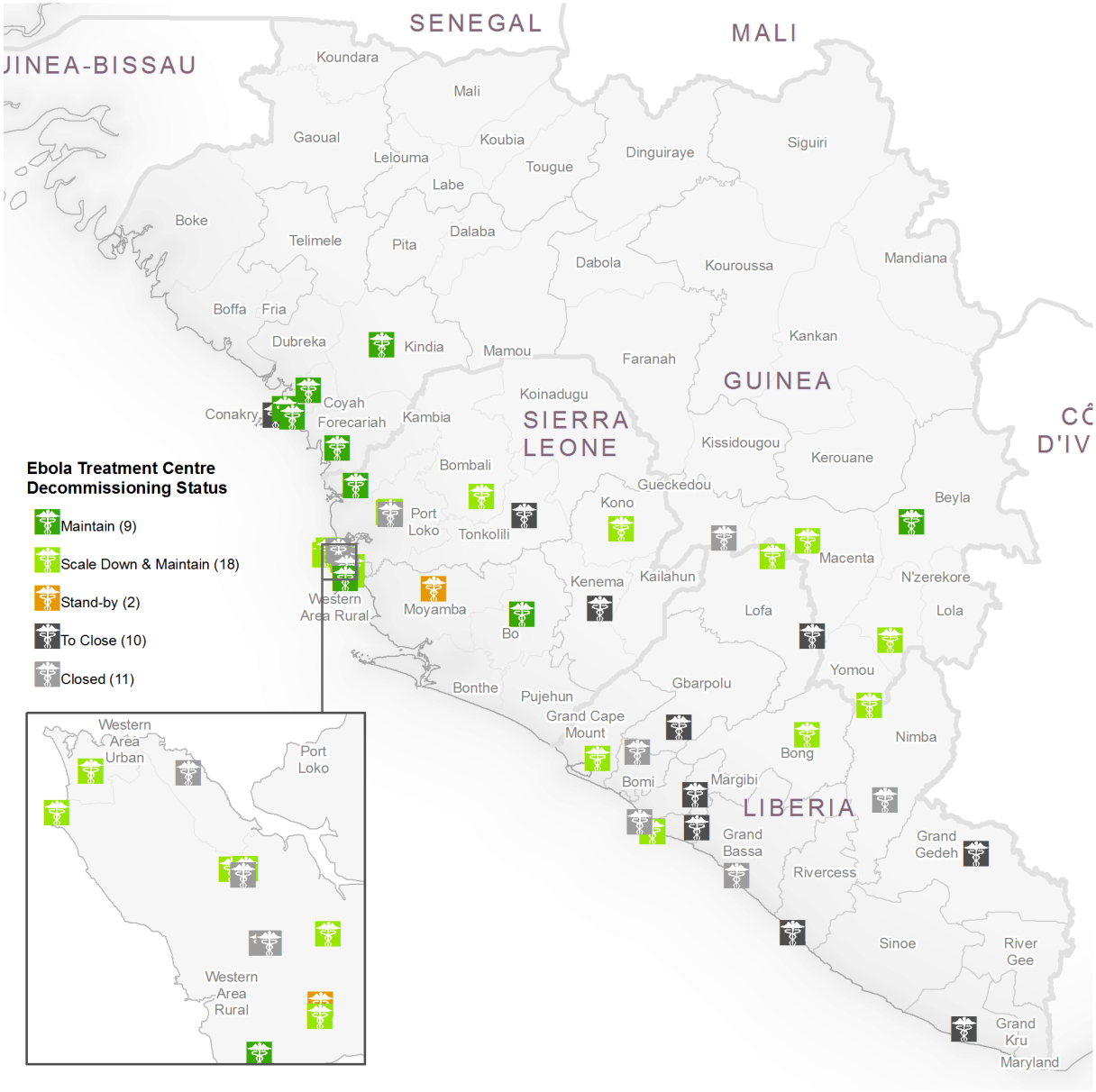

Figure 7: ETC Decommission Status - 29 April 2015 [10]