Department of Applied Mathematics and Statistics
COLORADO SCHOOL OF MINES
MATH484: Capstone - Mathematical and Computational Modeling

Assignment #5
Due Thursday, March 19, 2015

For problems which require computational simulation, please print and submit both your code and results (e.g., pictures).

1. Handout (Lin & Segel) p.31, Problem **2b**
Let $\chi, \mu, D > 0$. Assume that $a(t,x)$ and $\rho(t,x)$ satisfy the nonlinear system of PDEs

$$\frac{\partial a}{\partial t} = \frac{\partial}{\partial x}\left(\mu\frac{\partial a}{\partial x} - \chi a\frac{\partial \rho}{\partial x}\right)$$
$$\frac{\partial \rho}{\partial t} = fa - k\rho + D\frac{\partial^2 \rho}{\partial x^2}$$

Perform a stability analysis of the uniform state $a = a_0 \in \mathbb{R}$ and $\rho = \rho_0 \in \mathbb{R}$ assuming $k = k(\rho)$ and $f = f(\rho)$.

2. Handout (Lin & Segel) p.31, Problem **3**
Consider two-dimensional variations so that a and ρ satisfy

$$\frac{\partial a}{\partial t} = \nabla \cdot (\mu\nabla a - \chi a\nabla\rho)$$
$$\frac{\partial \rho}{\partial t} = fa - k\rho + D\Delta\rho.$$

Perform a stability analysis of the uniform state $a = a_0 \in \mathbb{R}$ and $\rho = \rho_0 \in \mathbb{R}$ assuming all parameters are positive constants. Do this by assuming perturbations of the form $\sin(q_1 x + q_2 y + \theta)e^{\sigma t}$ where $q_1, q_2, \theta \in \mathbb{R}$. More specifically, show that if $q^2 = q_1^2 + q_2^2$, then the instability condition remains the same as the one-dimensional case.

3. Assume $x \in \mathbb{R}$ and use the FTBS method to approximate solutions to the system of PDEs given by (10) and (11) in the handout. In particular, use the spatial interval $[-5, 5]$; constant parameter values $\mu = 1, D = 0.2, f = 0, k = 1, \chi = 10$; and initial conditions

$$a_0(x) = \begin{cases} 2 & \text{if } |x| < \frac{1}{10} \\ 0 & \text{else} \end{cases}$$

$$\rho_0(x) = \begin{cases} 1 & \text{if } x \in [-1.1, -0.9] \bigcup [-3.1, -2.9] \bigcup [-4.1, -3.9] \\ 0 & \text{else} \end{cases}$$

For step sizes use $dx = 0.1$, $dt = 1 \times 10^{-4}$. Create three different figures - a 1×2 matrix of plots with $a_0(x)$ in the left column and $\rho_0(x)$ in the right column, a 1×2 matrix of plots with $a(0.5, x)$ in the left column and $\rho(0.5, x)$ in the right column, and a 1×2 matrix of plots with $a(1, x)$ in the left column and $\rho(1, x)$ in the right column.

4. Friedman & Littman, p.53, Problem **3.3.3**
For the equation

$$\partial_t u = \partial_{xx} u$$

use the von Neumann criterion to discuss the stability of the Forward Euler scheme

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta x)^2}.$$

5. Friedman & Littman, p.54, Problem **3.5.1**
Use a letter of your choice (preferably with a corner or two, and not "L") to create 2 plots - one for the dose $D(x)$ and another containing the backscattered exposure $E(x)$. To create the plot of E, use the parameters $\alpha = \frac{1}{4}$, $\beta = \frac{1}{2}$, and $\eta = \frac{1}{2}$, and implement the Forward Euler method (from Problem 4) to solve the diffusion equation with $dx = dy = 0.05$ and $dt = 2.5 \times 10^{-4}$ on the two-dimensional grid $[-2, 2] \times [-2, 2]$.