

Department of Applied Mathematics and Statistics
COLORADO SCHOOL OF MINES
MATH 500: Linear Vector Spaces

Assignment #8: Normal Operators, Spectral Theorem, and SVD
Due Tuesday, December 7, 2021

1. (12 points) Define $A \in \mathbb{C}^{2 \times 2}$ by

$$A = \begin{bmatrix} 2 & -2i \\ 2i & 2 \end{bmatrix}.$$

Show that A is normal and construct a unitary diagonalization of A ; that is, find a diagonal matrix D and unitary matrix P such that $A = PDP^H$.

2. (10 points) Let $u \in \mathbb{R}^p$ and $v \in \mathbb{R}^q$ be unit vectors. Show that $A \in \mathbb{R}^{p \times q}$ defined by $A = u \otimes v = uv^T$ satisfies

$$\|A\|_2 = 1.$$

3. (18 points) Construct the matrices U, Σ, V in the singular value decomposition $A = U\Sigma V^T$ of

$$A = \begin{bmatrix} 4 & -2 \\ 2 & -1 \\ 0 & 0 \end{bmatrix}.$$