Department of Applied Mathematics and Statistics
COLORADO SCHOOL OF MINES
MATH 500: Linear Vector Spaces

Review Problems for Midterm Exam

1. Let $u \in \mathbb{R}^p \setminus \{0\}$ and $v \in \mathbb{R}^q \setminus \{0\}$ be given and define $A = uv^T \in \mathbb{R}^{p\times q}$.

(a) Show that $\{v\}$ is a basis for $\mathrm{Col}\left(A^T\right)$.

(b) Show that $\mathrm{Nul}\,(A) = \mathrm{Col}(A^T)^{\perp}$.

2. Let $\mathcal{V}$ be a normed space over $\mathbb{K}$ with norm $\|\cdot\|$. Show that for all $u, v \in \mathcal{V}$,

$$\Big|\|u\| - \|v\|\Big| \leq \|u - v\|.$$

3. Let $\mathcal{V}$ be a vector space over $\mathbb{K}$ with $U, W \subseteq \mathcal{V}$ subspaces. Define

$$M = \{u - w : u \in U, w \in W\}$$

and show that M is a subspace of $\mathcal{V}$.

4. Let $p, q \in \mathbb{N}$ be given and let A be a real $p \times q$ matrix with $\dim(\mathrm{Col}(A)) = q$. Show that $A^T A$ is nonsingular.

5. Find an orthonormal basis for $M = \mathrm{span}\left\{\begin{bmatrix}3\\0\\4\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}, \begin{bmatrix}4\\0\\4\end{bmatrix}\right\}$.