Department of Applied Mathematics and Statistics
COLORADO SCHOOL OF MINES
MATH 500: Linear Vector Spaces

Review Problems for Midterm Exam

1. Assume A is a real $p \times q$ matrix, $u \in \mathbb{R}^p$ satisfies $A^T u = 0$, and $v \in R(A)$. Show that u and v are orthogonal with respect to the standard inner product on $\mathbb{R}^p$.

2. Let V be a vector space over K and $\|\cdot\|$ be a norm on V. Show that for all $u, v \in V$,

$$\Big|\|u\| - \|v\|\Big| \leq \|u - v\|.$$

3. Let V be a vector space over K with subspaces U and W. Define

$$V_0 := \{u - w : u \in U, w \in W\}$$

and show that V_0 is a subspace of V.

4. Let $p, q \in \mathbb{N}$ with $p > q$ be given and let A be a $p \times q$ matrix with $\dim(R(A)) = q$. Show that $A^T A$ is nonsingular.

5. Find a normalized QR Factorization of $A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 2 & 4 \\ 4 & 0 & 4 \end{bmatrix}$.