Optimized Aperiodic Concentric Ring Arrays

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Abstract: This paper uses a genetic algorithm to optimize the element spacings in a concentric ring array. Each ring has equally spaced elements. The number of elements in a ring and/or the spacing between rings is optimized in order to minimize the maximum sidelobe level.

Keywords: concentric ring array, thinned array, aperiodic array, genetic algorithm.

1. Introduction

A ring array has elements lying on a circle. Two or more ring arrays with different radii and a common center form a concentric ring array. A concentric ring array with ring \( n \) having \( N_n \) elements and a radius of \( r_n \) is shown in Figure 1. The distance between elements on ring \( n \) is given by \( d_n \).

![Figure 1. A concentric ring array.](image)

The usual approach to obtaining low sidelobes in the array factor of a concentric ring array is amplitude weighting the elements [1], [2]. Another approach is to use aperiodic spacing. A circular Taylor amplitude taper projected onto a line in the azimuth plane relates to the element density in the concentric ring array [3]. The radii of the rings and the element densities in the rings contribute to give the desired low sidelobe taper. High sidelobes appear in the azimuth pattern of a Taylor weighted ring array when the spacing between element in a ring exceeds \( 0.4\lambda \) [4]. The steepest descent method can find the weights and ring spacing that results in desirable sidelobes and main lobe characteristics [5]. An investigation was also done to find the ring spacing of small concentric ring arrays that results in either the minimum...
maximum sidelobe level or the maximum directivity [6]. Ring spacing determines the sampling of a low sidelobe Taylor taper for concentric ring arrays. Adjusting the ring spacing can significantly reduce the maximum sidelobe level of a concentric ring array when the main beam is steered from boresight [7]. Milligan provides an excellent overview of generating a space taper for concentric ring arrays. He demonstrates calculating the spacing to obtain approximate low sidelobe Taylor tapers [8].

This paper presents results from optimizing the ring spacing and/or the number of elements in a ring in order to produce a low sidelobe array factor as a function of θ.

2. Uniform Concentric Ring Arrays

The array factor for the concentric ring array with a single element at the center (Figure 1) is given by

$$ AF = 1 + \sum_{n=1}^{N_e} w_n \sum_{m=1}^{N_n} e^{jkr_n \cos\phi_m \sin\theta \cos\phi + jkr_n \sin\phi_m \sin\theta \sin\phi} $$

where

- \( N_n \) = number of elements in ring \( n \)
- \( N_e \) = number of rings
- \( k = 2\pi / \lambda \)
- \( \lambda \) = wavelength
- \( w_n \) = element weights for ring \( n \)
- \( r_n \) = radius of ring \( n \)
- \( (x_n, y_n) \) = location of element \( n \)
- \( x_n = r_n \cos\phi_m \)
- \( y_n = r_n \sin\phi_m \)
- \( u = \sin\theta \cos\phi \)
- \( v = \sin\theta \sin\phi \)
- \( \phi_m = 2\pi (m-1) / N_n \)

In this formula, all the elements in the same ring have the same weight. A uniform array has equal element spacing and weighting. For a uniform concentric ring array, the ring spacing, \( r_n \), is a constant times the ring number and the spacing between elements within a ring, \( d_n \), is approximately constant for all rings. Figure 2 is a diagram of a 279 element concentric ring array. There are nine rings with having the characteristics listed in Table 1.

A nine ring concentric ring array has the array factor shown in Figure 3. The array has a directivity of 29.35 dB and a peak relative sidelobe level of -17.4 dB.

| Table 1. Ring radius and number of elements per ring for a 9-ring uniform concentric ring array. |
|---|---|---|---|---|---|---|---|---|
| \( n \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| \( r_n (\lambda) \) | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 |
| \( N_n \) | 6 | 12 | 18 | 25 | 31 | 37 | 43 | 50 | 56 |
3. Nonuniformly Spaced Concentric Ring Arrays

The first approach keeps $d_n = \lambda/2$ while varying $r_n$. In effect, the element density in the radial direction relates to a low sidelobe amplitude taper.

$$r_n = r_{n-1} + \Delta_n$$

where $0 \leq \Delta_n \leq \lambda$ and the minimum spacing between rings is $\lambda/2$. A hybrid genetic algorithm that combines a continuous genetic algorithm with a Nelder Mead algorithm is used to find the $r_n$ that result in the minimum maximum sidelobe level in the array factor. Table 2 has the optimized ring spacing and the number of elements in each ring. This new array has a slightly larger radius than the nine ring uniform concentric array but has only 201 elements or 72% of the elements of the uniform array. Its directivity is 27.38 dB with a maximum relative sidelobe level of -22.94 dB. Figure 4 is a diagram of the array. Figure 5 shows the corresponding array factor. The optimized array has a higher directivity and lower sidelobe level than the thinned array. All the sidelobes have nearly the same height.

Table 2. Ring radius and number of elements per ring for a nine ring uniform concentric ring array with optimized $r_n$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_n (\lambda)$</td>
<td>1.001</td>
<td>1.587</td>
<td>2.137</td>
<td>2.875</td>
<td>3.659</td>
<td>4.977</td>
</tr>
<tr>
<td>$N_n$</td>
<td>12</td>
<td>19</td>
<td>26</td>
<td>36</td>
<td>45</td>
<td>62</td>
</tr>
</tbody>
</table>
Figure 4. Optimized $r_n$ for an array with six concentric rings.

Figure 5. Array factor associated with optimized $r_n$ for array with six concentric rings.

A second approach to minimizing the maximum sidelobe level lets $r_n = n\lambda/2$ and optimizes the $N_n$. A genetic algorithm is used to find the optimum $N_n$ listed in Table 3. The first four rings have the same number of elements as the uniform array in Table 1. The resulting array has 183 elements (65.6% of the elements of the uniform array) arranged as shown in Figure 6. Its directivity is 28.5 dB with a maximum relative sidelobe level of -25.58 dB. The array factor shown in Figure 7 does not have the constant sidelobe level of Figure 5.

Figure 6. Optimized $N_n$ in nine concentric rings with equal ring spacing.
Figure 7. Array factor associated with optimized $N_n$ in nine equally spaced concentric rings.

Table 3. Ring radius and number of elements per ring for a nine ring uniform concentric ring array with optimized $N_n$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_n$ ($\lambda$)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.5</td>
</tr>
<tr>
<td>$N_n$</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>25</td>
<td>17</td>
<td>23</td>
<td>22</td>
<td>27</td>
<td>32</td>
</tr>
</tbody>
</table>

A final approach simultaneously optimizes $r_n$ and $N_n$. Table 4 has the resulting optimum $r_n$ and $N_n$. This array has 142 elements or 50.9% of the 9 ring uniform concentric array. Its directivity is 28.89 dB with a maximum relative sidelobe level of -27.82 dB. The optimized array appears in Figure 8 with the corresponding array factor in Figure 9. The directivity and the sidelobe level exceed the results obtained through the three other approaches.

Table 4. Ring radius and number of elements per ring for a nine ring uniform concentric ring array with optimized $r_n$ and $N_n$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_n$ ($\lambda$)</td>
<td>0.758</td>
<td>1.355</td>
<td>2.090</td>
<td>2.989</td>
<td>3.783</td>
<td>4.702</td>
</tr>
<tr>
<td>$N_n$</td>
<td>9</td>
<td>17</td>
<td>25</td>
<td>31</td>
<td>26</td>
<td>33</td>
</tr>
</tbody>
</table>

Figure 8. Optimized $r_n$ and $N_n$ in each ring for array with six concentric rings.
4. Conclusions

A genetic algorithm can be used to minimized the maximum sidelobe level of a concentric ring array by optimizing the ring spacing and/or number of elements in each ring.

References