Coda wave interferometry

An interferometer is an instrument that is sensitive to the interference of two or more waves (optical or acoustic). For example, an optical interferometer uses two interfering light beams to measure small length changes.

Coda wave interferometry is a technique for monitoring changes in media over time using acoustic or elastic waves. Sound waves that travel through a medium are scattered multiple times by heterogeneities in the medium and generate slowly decaying (late-arriving) wave trains, called coda waves. Despite their noisy and chaotic appearance, coda waves are highly repeatable such that if no change occurs in the medium over time, the waveforms are identical. If a change occurs, such as a crack in the medium, the change in the multiple scattered waves will result in an observable change in the coda waves. Coda wave interferometry uses this sensitivity to monitor temporal changes in strongly scattering media.

Coda wave interferometry can be used in two different modes. In the warning mode, the technique is used to detect a change in the medium, but this change is not quantified. This mode of operation is used to prompt further action, such as more elaborate diagnostics. In the diagnostic mode, coda wave interferometry is used to quantify the change in the medium.

Volcano monitoring. The use of this technique to detect changes in a medium can be illustrated with seismological data that have been recorded on the Merapi volcano in Java by U. Wegler and coworkers. As seismic (elastic) waves pass through a volcano, they are scattered by heterogeneities (scatters) such as voids, cracks, magma bodies, and faults.

In the experiment, an air gun placed in a small water basin dug in the side of the volcano was used to generate seismic waves. (An air gun is a device that emits a bubble of compressed air in water as a source of seismic waves.) The seismic waves generated by the same source recorded at a fixed receiver at two moments in time (a year apart) are shown in Fig. 1.

In the Fig. 1b and c, the two waveforms are shown superimposed in more detail. For the interval early in the seismogram, these waveforms are similar (Fig. 1b). For the later interval (coda), the waveforms are distinctly different (Fig. 1c), where one of the waveforms appears to be a time-shifted version of the other, indicating that the interior of the volcano had changed over time.

Note that in coda wave interferometry one needs only a single source and a single receiver, although in practice one may use more receivers to increase the reliability. This means that the hardware requirements of this technique are modest.

Theory. Suppose that a strongly scattering medium is excited by a repeatable source, and that the medium changes with time. Before the change in the medium, the unperturbed wave field \( u^{(0)}(t) \) can be written in Eq. (1) as a sum of the waves that propagate along the multiple scattering trajectories \( T \) in the medium, where \( t \) denotes time and \( A_T(t) \) is the wave that has propagated along trajectory \( T \).

\[
u^{(0)}(t) = \sum_T A_T(t) \quad (1)
\]

The change in the waveforms as shown in Fig. 1c

![Fig. 1. Waveforms recorded at the Merapi volcano for the same source and receiver on July 1997 (black) and July 1998 (color). (a) Complete waveforms and definition of the early (E) and late (L) time interval. (b) The recorded waves in the early time interval (E). (c) The recorded waves in the later interval (L). (Data courtesy of Ulrich Wegler)](image-url)
can be quantified by computing in Eq. (3) the time-
shifted cross-correlation over a time window with
center time \( t \) and width \( 2\tau_w \), where \( \tau_w \) is the
time shift of the perturbed waveform relative to the un-
perturbed waveform. Suppose that the waves are not
perturbed. In that case, \( u^p(t) = u^\omega(t) \) and the
time-shifted cross-correlation is equal to unity for a zero
lag time \( R(t_w = 0) = 1 \). When the perturbed wave
is a time-shifted version of the original wave, then
\( u^p(t) = u^\omega(t - \tau) \) and \( R(t_w) \) attain its maximum
for \( t = \tau \).

In general, the time-shifted cross-correlation \( R(t_w) \)
attains its maximum at a time \( t = t_{\text{max}} \) [Eq. (4)] when
\[
R(t_w) = \frac{1}{2} \omega^2 \sigma_t^2
\]

\[
t_{\text{max}} = (\tau)
\]

the shift time is given by the average perturbation of
the travel time of the waves that arrive in the em-
ployed time window, and the value \( R_{\text{max}} \) at its maxi-
mum [Eq. (5)] is related to the variance \( \sigma_t \) of the
travel-time perturbation of the waves that arrive in
the time window, where \( \omega \) is the angular frequency
of the waves. Given the recorded waveforms before
and after the perturbation, one can readily compute
the time-shifted cross-correlation and use Eqs. (4)
and (5) to obtain the mean and the variance of the
travel-time perturbation in the medium.

Measuring velocity change. Figure 2a shows an ex-
periment in which ultrasound waves were propa-
gated through a granite cylinder and recorded. The
waves are complex due to the reverberations within
this cylinder. With a heating coil, the temperature
of the cylinder was raised 5 °C (9 °F). The perturbed
waveforms have the same character as the unper-
turbed waves shown in Fig. 2a. The tail of the wave
trains was divided in 15 nonoverlapping time inter-
vals. For each time interval, the time shift between
the perturbed and unperturbed waves was deter-
mined by computing the time-shifted cross-
correlation of Eq. (3) and by picking the time for
which it attains its maximum \( t_{\text{max}} \). The relative
velocity change for each time interval is given by
\[
\delta v/v = -\frac{\Delta t_{\text{max}}}{t}
\]

This quantity is shown in Fig. 2b as a function of the center time of the employed time
windows.

Since the employed time windows are nonover-
lapping, the measurements of the velocity change in the
different time windows are independent. The scatter
in the different estimates of the velocity change is
small; this provides a consistency check of coda wave
interferometry. The variability in the measurements
can be used to estimate the error in the velocity
change. Note that the relative velocity change in
this example is only about 0.16% with an error of
about 0.03%. This extreme sensitivity to changes in
the medium is due to the sensitivity of the multiply
scattered waves to changes in the granite.

In an elastic medium such as granite, there is no
single wave velocity. Compressional \( (P) \) waves and
shear \( (S) \) waves propagate with different velocities
\( v_P \) and \( v_S \), respectively. The change in the velocity
inferred from coda wave interferometry [Eq. (6)] is a
\[
\frac{\delta v}{v} = \frac{\delta v^3}{2v_P^3 + v_S^3} + \frac{2\delta v^3}{2v_P^3 + v_S^3}
\]

weighted average of the change in the velocities for
the two wave types. For a Poisson medium, an elas-
tic medium, where \( v_P = \sqrt{3} v_S \), the relative velocity
change is given by \( \delta v/v \approx 0.098 \delta v_P/v_P + 0.91 \delta v_S/v_S \).

This means that in practice coda wave interfero-
metry provides a constraint on the change in the shear
velocity \( v_S \).

Other applications. Some applications may involve
a change in the location of scatterers, or a change
in the source position. This can be used to moni-
tor the properties of a turbulent fluid. One can seed
the fluid with neutrally buoyant particles that scatter
waves. Acoustic waves that propagate through the
fluid are scattered by these particles. Over time, the
particles are swept along by the turbulent motion.
When acoustic waves are sent into the fluid once
more from the same source, the waves are scattered
by particles that have been displaced by the turbu-
 lent flow. The resulting change in the recorded waves
can be used to characterize the properties of the tur-
bulent flow.

Coda wave interferometry, to a certain extent,
can be used to distinguish between different
perturbations. When the velocity changes, the mean travel-time perturbation is nonzero and is proportional to the total travel time. Since the waves that propagate along all paths experience the same travel-time change, the variance of the travel-time perturbation vanishes. When the location of the scatterers is perturbed randomly, the mean travel-time perturbation vanishes, because some paths are longer, while others are shorter. However, the variance of the travel-time perturbation is nonzero and can be shown to grow linearly with time. When the source is displaced, the mean travel-time perturbation vanishes, and the variance of the travel time is constant with time. These results are summarized in the table. The mean and the variance of the travel-time change can be computed from the unperturbed and the perturbed waveforms. Since different types of perturbations leave a different imprint on these quantities, coda wave interferometry can help determine the mechanism of the change over time.

For background information see ACOUSTIC EMISSION; ACOUSTICS; EARTHQUAKE; INTERFEROMETRY; SEISMOGRAPHIC INSTRUMENTATION; SEISMOLOGY; SOUND; VOLCANO; VOLCANOLOGY in the McGraw-Hill Encyclopedia of Science & Technology.

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