

IN SITU MEASUREMENTS OF SHEAR-WAVE VELOCITY IN SEDIMENTS WITH HIGHER-MODE RAYLEIGH WAVES*

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ABSTRACT

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A seismic survey was carried out on a tidal flat in the SW-Netherlands in order to determine shear-wave velocities in sediments by means of higher-mode Rayleigh waves. The dispersion properties of these Rayleigh waves were measured in the 2-D amplitude spectrum—or f,k -spectrum—and resulted in phase velocities for six different modes as a function of frequency (5–30 Hz). These observed phase velocities were inverted for a nine-layer model for the shear-wave velocity to a depth of 50 m.

INTRODUCTION

Whenever seismic surveys are carried out on land, surface waves dominate the motion. In seismic prospecting for oil this ‘ground roll’ tends to obscure valuable reflections. In these surveys the surface waves are regarded as source-generated noise, and correspondingly one attempts to remove them. However, surface waves do contain independent information on properties of the medium through which the waves propagate, notably on the S-wave velocity as a function of depth. This information can be retrieved, though only with more complicated techniques of processing and interpretation. Inversion techniques have been developed in the context of global seismology to retrieve information from measured surface wave dispersion (e.g., Takeuchi and Saito 1972, Nolet 1981).

In 1985 a seismic survey was carried out on a tidal flat in the SW-Netherlands to test if higher-mode Rayleigh waves can be used to determine shear-wave velocities at shallow depths. Rayleigh waves are surface waves which are characterized by an elliptical ground motion in the vertical plane through source and receiver, an amplitude decrease of $r^{-1/2}$ from the source and an exponential (evanescent) character

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below a certain, frequency-dependent, depth. Phase velocities of Rayleigh waves can be related to the propagation characteristics of the layers near the surface. The object of this survey was to determine, in situ, the shear wave velocity in the uppermost layers of marine sediments.

The applications of surface wave analysis in exploration geophysics are relatively few. Al-Husseini, Glover and Barley (1981) analyzed dispersion patterns of 'ground roll' in Saudi Arabia in the f,k -spectrum in order to determine the frequency and wavenumber filters for a field acquisition system in a reflection survey. McMechan and Yedlin (1981) used a method for surface wave analysis earlier developed in global seismology (Nolet and Panza 1976). Their analysis of real (marine) data was limited to the fundamental mode surface wave, which has a limited penetration depth at frequencies employed in exploration seismics. Essen et al. (1981) investigated surface waves up to mode 2 by using a multiple filter technique, and by determination of the ratio of horizontal to vertical amplitude. They fitted their data with a simple three-layer model for shear-wave velocity to a depth of about 20 m.

However, for a precise and unambiguous investigation of the dispersion of higher modes, an analysis in the frequency-wavenumber (or $f-k$) domain is mandatory (Nolet and Panza 1976). Contrary to the situation in global seismology, where stations are sparse and unevenly distributed (e.g., Nolet 1977), there is little danger in a controlled experiment of spatial aliasing at the frequencies of interest, so a 2-D Fourier transform can be used for analysis. We show that with the frequencies and geophone spacing used in this survey the shear-wave velocity can be determined to a depth of about 50 m (the maximum depth depends on the frequency, recording aperture and the medium), using the phase velocities of six Rayleigh modes.

Knowledge of the shear-wave velocity with depth in sediments is important, since together with information on the density, one can determine the shear modulus. This is critically important in studies of foundation vibrations, effects of earthquakes and slope stabilities. If the compressional wave-velocity variation with depth is known one can determine the Poisson's ratio (Hamilton 1979). Finally, the velocity of shear waves is also needed to compute static corrections for shear wave profiling (Mari 1984).

DATA ACQUISITION

The experiment was performed on the *Plaat van Oude Tonge*, a tidal flat between the islands of *Schouwen-Duiveland* and *Goeree-Overflakkee* in the Province of *Zeeland*, SW-Netherlands (fig. 1). A simple weight drop source of 30 kg was used. The receivers were vertical component geophones with a natural frequency of 10 Hz (for a complete description of the field equipment, see Doornenbal and Helbig 1983).

Two considerations set the dimensions of the layout of the geophones and shotpoints. Firstly, the spacing of the geophones has to be small enough to avoid, or at least minimize, spatial aliasing of the wavetypes under study. Secondly, the length of the geophone array should be large enough to separate higher modes with only a small difference in wavenumber.

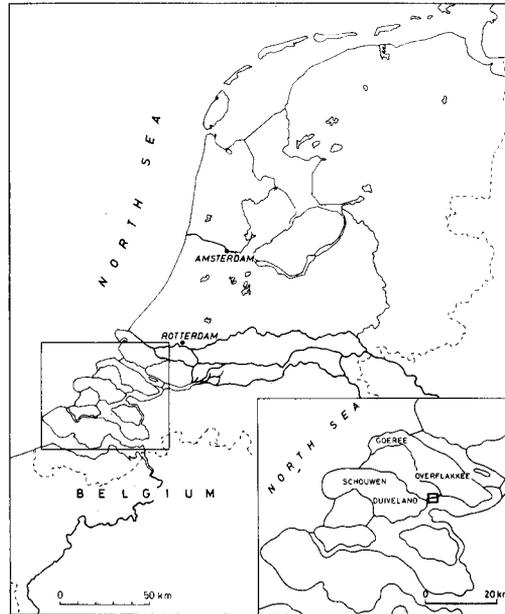


Fig. 1. Topographic map showing the location of the survey area in the province of Zeeland, SW-Netherlands (inset). The mark in the inset indicates the Plaat van Oude Tonge.

Twenty-four geophones were placed in a straight line with a spacing of 1 m. The shortest distance between a shotpoint and geophone was 40 m, the distance between the consecutive shotpoints was 24 m. The sets of 24 traces of 12 shotpoints were combined, thereby simulating a time-offset record due to a single source at the end of an extended line of 288 receivers with offsets 40, 41, ..., 327 m. This approach can introduce source-related discontinuities in waveform and statics, which were minimized by selecting a site that had negligible topographic variation.

The time-offset record is shown in fig. 2. Surface waves clearly dominate the record. The higher modes arrive first as a diffuse band of seismic energy. After this, the fundamental mode arrives as a distinct wave packet. In the fundamental mode, the wave crests can be seen to move through the wave packet as a function of offset, indicating differences between the phase- and group-velocity of this mode.

INTERPRETATION OF DISPERSIVE RAYLEIGH WAVES

There are several methods available for analysis of Rayleigh-wave dispersion. The objective of these methods is to measure the dispersion, either the phase velocity $c = 2\pi f/k$ or the group velocity $U = 2\pi df/dk$. Either or both can be used to find the S-velocity profile in sediments. Two examples are the 'multiple filter technique' and the 2-D Fourier method. The multiple filters measure group velocities as a function of frequency. In the 2-D Fourier method one transforms data from the x,t -domain

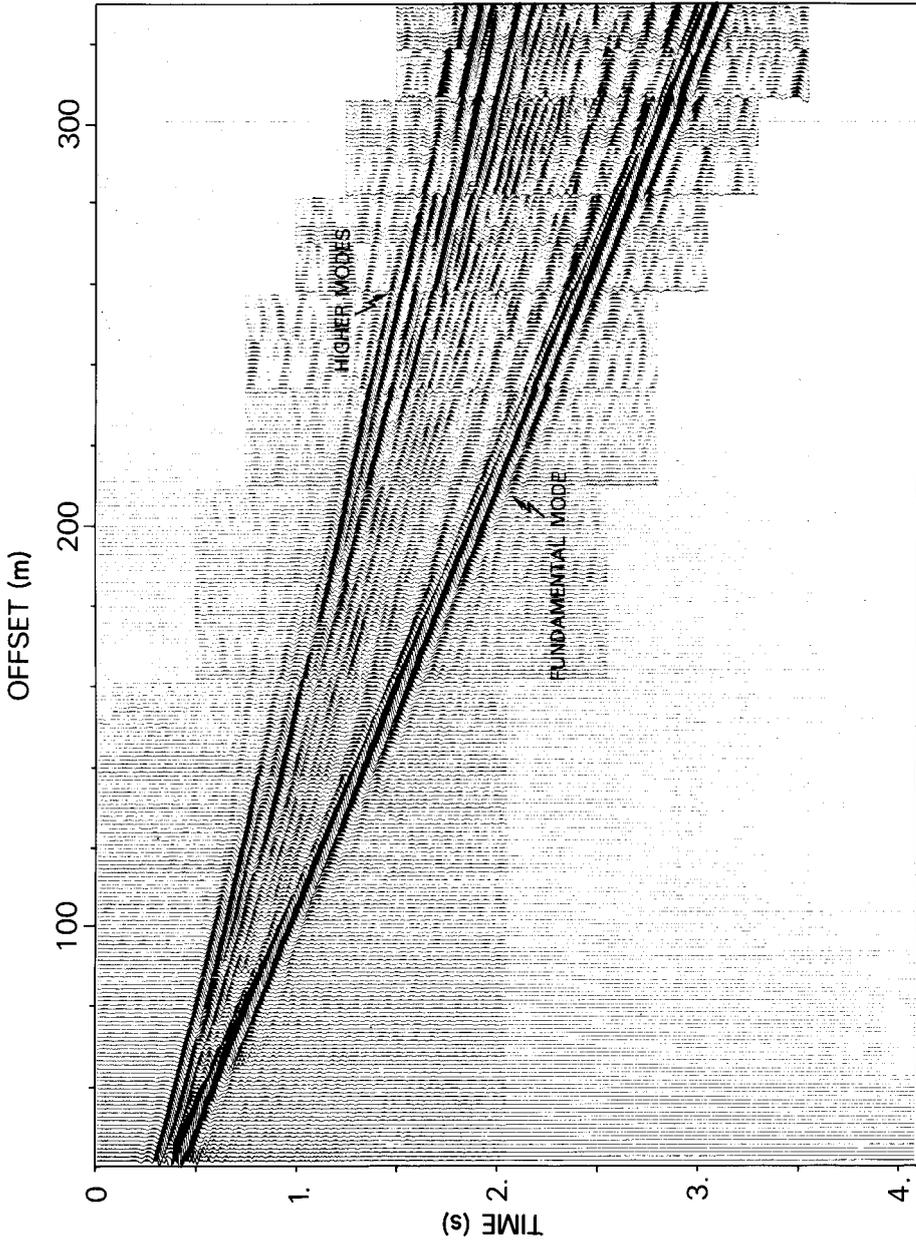


Fig. 2. Time-offset record for 288 traces from vertical geophones spaced 1 m apart (offset: 40–327 m). The traces are corrected for geometrical spreading (amplitude decrease $\sim r^{-1/2}$) and field amplification.

to the f,k -domain. Phase velocities as a function of frequency can be determined from energy maxima in the $f-k$ domain.

In the multiple filter technique (Dziwowski and Hales 1972) an individual trace is filtered by narrow bandpass filters at a sequence of center frequencies. The filtered signal is then retrieved by an inverse Fourier transform; the maximum of the instantaneous amplitudes of the filtered signal propagate approximately with the group velocity. The multiple filter analysis, therefore, results in the group velocity as a function of frequency.

This technique did not give useful results in our case and the results are not shown in this paper. The maxima in the multiple filter diagrams were not sharp enough to derive a reliable group-velocity curve as a function of frequency, except for part of the fundamental mode. Moreover, the multiple filter diagrams of several traces showed significant differences. One reason for the disappointing results for the higher modes is their overlap in time, which makes it difficult to identify the waves unambiguously. At low frequencies, the relatively small aperture also hampered accuracy.

The second technique, aimed at measuring phase velocities rather than group velocities, was much more successful. The 2-D amplitude spectrum was calculated by 2-D Fourier transformation of the data (after a $r^{1/2}$ correction for geometrical spreading). The sample interval in the data was 8 ms, so the Nyquist frequency is 62.5 Hz, which is well above the recorded range of frequencies (5–30 Hz). The Nyquist wavenumber equals 0.5 m^{-1} . Use of 256 traces gave a good resolution for the higher modes. This number of traces appears to be necessary: with only 128 traces, i.e., half the aperture but unchanged geophone spacing, we obtained a blurred image in the f,k -domain, and the higher modes could not be separated unambiguously.

The result of the 2-D Fourier transformation of 256 traces (offsets 72–327 m) is the 2-D amplitude spectrum shown in fig. 3. In this figure one can clearly see the fundamental Rayleigh mode and the higher modes. Energy is presented on the 2-D amplitude spectrum from light to dark in 5 dB steps (range 80–100 dB). Note the slight downward curvature for each mode for the higher frequencies, which shows that higher frequencies have smaller phase velocities. In a second analysis we muted the fundamental mode in the x,t -domain and calculated the f,k -spectrum for the higher modes only in order to minimize the disturbing influence from the high-amplitude fundamental. We were thus able to determine phase velocities for five higher modes (see fig. 4) by drawing curves in the f,k -domain, and determining the phase velocities $c = \pi f/k$.

INVERSION OF PHASE VELOCITIES

The observed phase velocities can be used to determine the structure of the sediments by comparison with synthetic phase velocities in a model. The phase velocity depends primarily on the shear wave velocity and is insensitive to realistic variations in density and compressional wave velocity with depth. This can be seen by compar-

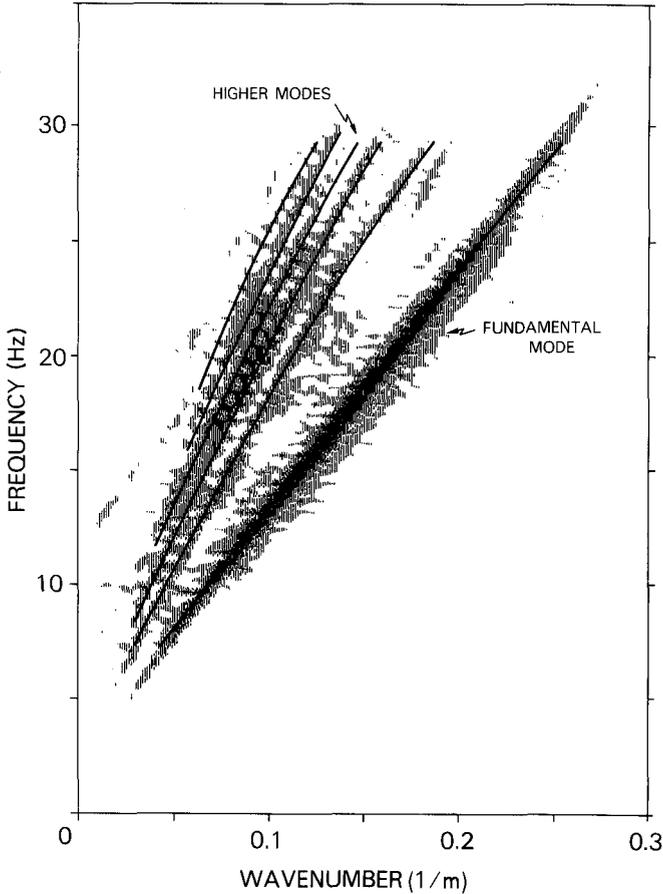


Fig. 3. 2-D amplitude spectrum for 256 traces with offsets 72–327 m. The energy is contoured at 5 dB intervals in the range 80 dB–100 dB. The superimposed curves are calculated for the final model extracted from these data.

ing the partial derivatives of the phase velocity (Nolet 1981) with respect to S-wave velocity, P-wave velocity and density. For the two latter parameters, a realistic profile was adopted, and only the shear wave velocity (V_s) of the model was allowed to vary. We assigned larger errors to higher mode phase velocities, and within one mode, large absolute errors were assigned to the lower frequencies.

A linearization around a reference model (consisting of a finite number of layers) makes it possible to find a reliable model that fits the data, and facilitates the computation of the model error (Nolet 1981). The linear equations, thus obtained, are overdetermined, and we defined the solution as the model that minimizes

$$\|C^{-1/2}A\beta - C^{-1/2}c\|^2,$$

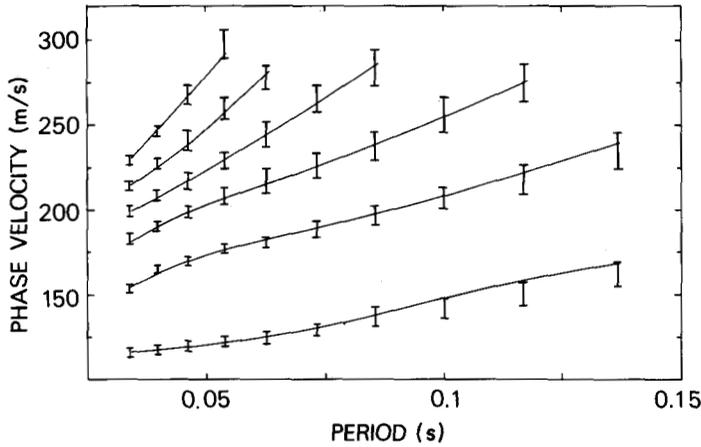


Fig. 4. Phase velocities for different modes as a function of the period. The curves are calculated for the model. Vertical bars give the observed values with their absolute error.

where \mathbf{A} is a matrix of partial derivatives of phase velocities

$$A_{ij} = \frac{\partial c_j}{\partial \beta_i}$$

(see Nolet 1981). \mathbf{C} is the covariance matrix of the data, $\boldsymbol{\beta}$ the vector of shear-wave perturbations in each layer, and \mathbf{c} is the vector of differences in calculated and measured phase velocities for each data point.

As shown in Nolet (1981), the a posteriori uncertainty in the shear-wave velocity (β_i) can be estimated for uncorrelated data with

$$(\Delta\beta_i)^2 = \sum_j A_{ji}^{-1} A_{ij}^{-1} \sigma_j^2,$$

Table 1. *The shear wave velocity with depth.*

Depth(m)	V_s (m/s)
0-1	101 ± 9
1-2	126 ± 5
2-4	127 ± 3
4-6	146 ± 4
6-8	172 ± 5
8-12	184 ± 5
12-20	200 ± 5
20-30	232 ± 6
30-50	307 ± 40

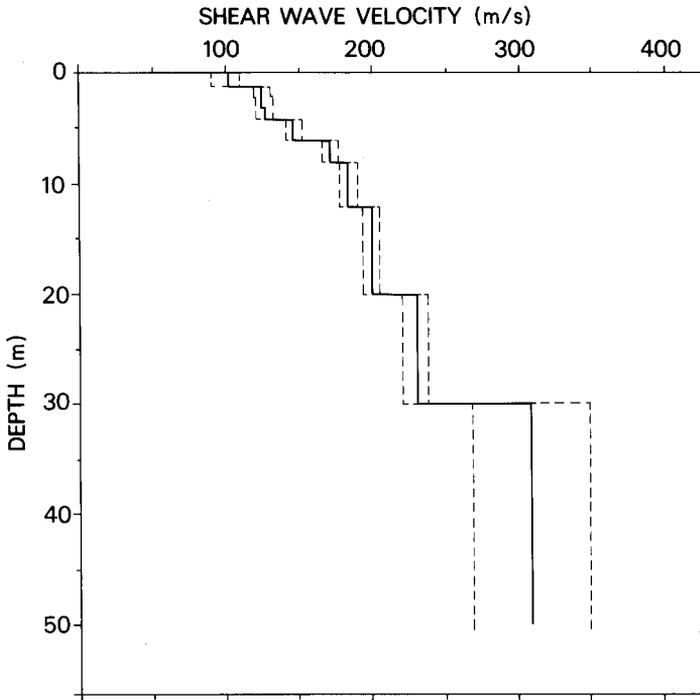


Fig. 5. The final model for the shear-wave velocity (solid line) with its standard deviation (dashed lines).

where \mathbf{A}^{-1} is the generalized inverse of \mathbf{A} , and σ_j is the error in c_j . In general, there is a trade-off between resolution (the number of layers in a certain depth range in the model) and variance, see Nolet (1981). The number of layers in our model was chosen in such a way that the variance of the final solution was acceptable. The final model we found, and its standard deviation, is presented in table 1 and in fig. 5. The fit of the phase velocities produced by this model is shown in fig. 4.

DISCUSSION

By means of a relatively simple seismic investigation, a multi-layered model for the shear-wave velocity can be determined accurately using higher-mode Rayleigh waves. Since we were able to identify six modes in the f, k -spectrum, a reliable model could be determined to a depth of 50 m. Comparison of the measured and synthetic phase velocities shows satisfactory agreement. This method can be applied in any region where lateral heterogeneity is small over distances of the order of 300 m. In other regions the method can still be applied at smaller offsets, but the resolution will then be more limited in depth.

The model obtained shows a reasonable agreement with shear-wave velocity

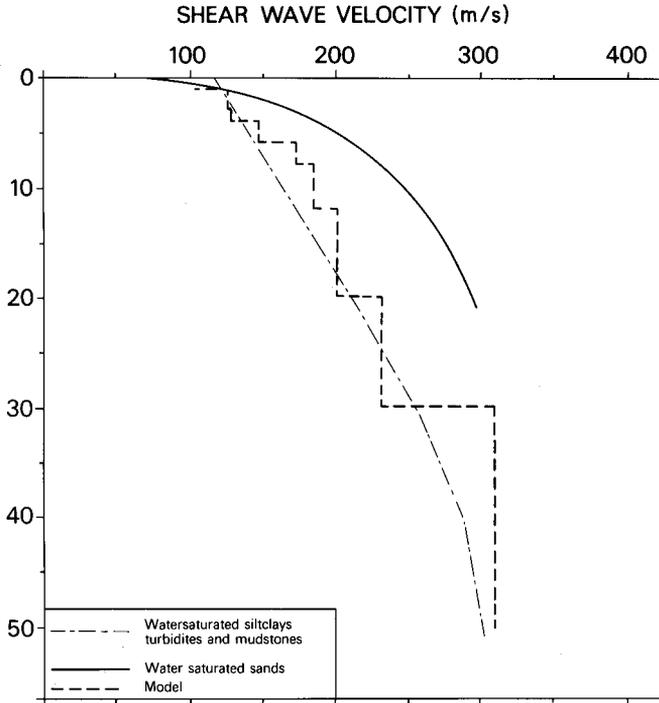


Fig. 6. Shear wave velocity with depth for different marine sediments (Hamilton 1979), and the final model.

profiles for water saturated silt-clays, turbidites and mudstones as compiled by Hamilton (1976, 1979), see fig. 6. From analysis of boreholes to a depth of 50 m in the neighbourhood of the seismic line it is known that there is a sand fraction in the sediments. The Westland formation at the surface of the tidal flat (to a depth of 25–30 m) is a sequence of coarse to fine-grained sands with intercallations of peat and clay (Doornenbal and Helbig 1983). These layers were not resolved in this inversion.

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