Power generation with pressure retarded osmosis: An experimental and theoretical investigation

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1. Introduction

Pressure retarded osmosis (PRO) could be a viable source of renewable energy [1,2]. In a PRO system, water from a low salinity feed solution permeates through a membrane into a pressurized, high salinity brine/draw solution (e.g., seawater); power is obtained by depressurizing the permeate through a hydroturbine. A PRO model was developed to predict water flux and power density under specific experimental conditions. The model relies on experimental determination of the membrane water permeability coefficient (A), the membrane salt permeability coefficient (B), and the solute resistivity (K). A and B were determined under reverse osmosis conditions, while K was determined under forward osmosis (FO) conditions. The model was tested using experimental results from a bench-scale PRO system. Previous investigations of PRO were unable to verify model predictions due to the lack of suitable membranes and membrane modules. In this investigation, the use of a custom-made laboratory-scale membrane module enabled the collection of experimental PRO data. Results obtained with a flat-sheet cellulose triacetate (CTA) FO membrane and NaCl feed and draw solutions closely matched model predictions. Maximum power densities of 2.7 and 5.1 W/m² were observed for 35 and 60 g/L NaCl draw solutions, respectively, at 970 kPa of hydraulic pressure. Power density was substantially reduced due to internal concentration polarization in the asymmetric CTA membranes and, to a lesser degree, to salt passage. External concentration polarization was found to exhibit a relatively small effect on reducing the osmotic pressure driving force. Using the predictive PRO model, optimal membrane characteristics and module configuration can be determined in order to design a system specifically tailored for PRO processes.

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of this investigation was to evaluate the contributions of membrane characteristics and operating conditions to water flux and subsequently, to power density. Concentration polarization may severely reduce water flux and power density in PRO; however, the causes and effects of concentration polarization have not been investigated nearly as much for FO and PRO processes [15,18] as for RO processes [19,20]. In the current investigation, a predictive PRO model that includes the influence of draw solution, feed solution, concentration polarization, and hydraulic pressure on water flux and subsequent power output was developed. The model was tested using experimental results. Experiments were conducted using current-generation FO membranes and a well-established draw solution (NaCl). Water flux and power density were evaluated under different operating conditions.

With the availability of a PRO model, power density can be predicted for variable membrane characteristics, membrane structure, and module configuration. The optimization of these parameters would contribute to the design of membranes and membrane modules specifically tailored for PRO processes.

2. Theory

2.1. Osmotic processes

Osmosis is the transport of water across a selectively permeable membrane from a solution of higher water chemical potential (lower osmotic pressure) to a solution of lower water chemical potential (higher osmotic pressure). It is driven by a difference in solute molar concentrations across a membrane that allows passage of water, but rejects most solute molecules and ions. The osmotic pressure differential ($\Delta \pi$) is the pressure which, if applied as a hydraulic pressure ($\Delta P$) to the more concentrated solution, would prevent net transport of water across the membrane. FO uses the $\Delta \pi$ across the membrane, rather than a $\Delta P$ (as in RO), as the driving force for transport of water through the membrane. The FO process results in concentration of a feed stream and dilution of a highly concentrated stream (referred to as the draw solution). Flux in FO is in the opposite direction of RO. PRO can be viewed as an intermediate process between FO and RO, hydraulic pressure is applied to the draw solution (similar to RO) but the net water flux is still in the direction of the concentrated draw solution (similar to FO). The general equation describing water transport in FO, RO, and PRO is

$$J_w = A(\Delta \pi - \Delta P)$$

where $J_w$ is the water flux and $A$ is the water permeability coefficient of the membrane. For FO, $\Delta P$ is zero; for RO, $\Delta P > \Delta \pi$; and for PRO, $\Delta P < \Delta \pi$. The flux directions of the permeating water in FO, PRO, and RO are illustrated in Fig. 2. Also in Fig. 2, the orientation of an asymmetric membrane is indicated; in FO, the dense layer of the membrane faces the feed solution and in RO and PRO, the dense layer faces the draw solution.

In PRO, the power that can be generated per unit membrane area (i.e., the power density) is equal to the product of the water flux and the hydraulic pressure differential across the membrane:

$$W = J_w \Delta P = A(\Delta \pi - \Delta P)\Delta P$$

By differentiating Eq. (2) with respect to $\Delta P$, it can be shown that $W$ reaches a maximum when $\Delta P = \Delta \pi/2$ (Fig. 3). Substituting this value for $\Delta P$ in Eq. (2) yields:

$$W_{\text{max}} = A\frac{\Delta \pi^2}{4}$$

$J_w$ as a function of $\Delta P$ is illustrated in Fig. 3 for both real and ideal conditions. The FO point (at $\Delta P = 0$), the PRO zone (where $\Delta P < \Delta \pi$), and the RO zone (where $\Delta P > \Delta \pi$) are indicated. The flux reversal point occurs where $\Delta P = \Delta \pi$. Fig. 3 also illustrates $W$ and $W_{\text{max}}$ in the PRO zone.

2.2. Salt permeability

Membranes for osmotically-driven processes are susceptible to reverse salt diffusion, where a small amount of salt permeates the membrane from the draw solution to the feed solution due to the concentration gradient across the membrane [21]. Reverse salt diffusion reduces the effective osmotic pressure difference across the membrane. The salt permeability coefficient of a semi-permeable membrane can be obtained from RO experiments [17] and is given by:

$$B = \frac{A(1 - R)(\Delta P - \Delta \pi)}{R}$$
where $B$ is the salt permeability coefficient and $R$ is salt rejection. $R$ is defined as:

$$R = 1 - \frac{C_P}{C_F}$$  \hspace{1cm} (5)

where $C_P$ is the salt concentration in the permeate solution and $C_F$ is the salt concentration in the feed solution.

### 2.3. Concentration polarization

Another, more severe phenomenon, which also reduces the effective osmotic pressure difference across the membrane, is concentration polarization [15]. Concentration polarization is the accumulation or depletion of solutes near an interface. As a result of water crossing the membrane, the solute is concentrated on the feed side of the membrane surface and diluted on the permeate side of the membrane surface.

Because the membranes used for osmotic processes are typically asymmetric [22] (comprised of a thin dense layer on top of a porous support layer), concentration polarization occurs externally on the dense layer side and internally in the support layer side. In PRO applications, the dense layer of the membrane faces the draw solution and the porous support layer faces the feed solution (Fig. 2). This configuration is necessary to ensure that the membrane can sustain the hydraulic pressure induced on the draw solution side [12,17,23]. In this configuration, both dilutive external concentration polarization and concentrative internal concentration polarization occur (Fig. 4).

#### 2.4. External concentration polarization in PRO

Dilutive external concentration polarization is concentration polarization that results in the solute being diluted on the draw solution side of the membrane. Dilutive external concentration polarization can be calculated from film theory [19,24]. The external concentration polarization modulus ($\pi_{D,m}/\pi_{D,b}$) is calculated using:

$$\frac{\pi_{D,m}}{\pi_{D,b}} = \exp \left( -\frac{J_{W}}{k} \right)$$  \hspace{1cm} (6)

where $\pi_{D,m}$ is the osmotic pressure at the membrane surface and $\pi_{D,b}$ is the bulk osmotic pressure of the draw solution. $J_{W}$ is negative in this equation because the water flux is in the direction of the more concentrated solution and the concentration polarization effect is dilutive ($\pi_{D,m} < \pi_{D,b}$). The mass transfer coefficient ($k$) in the draw solution is calculated using:

$$k = \frac{Sh D}{d_h}$$  \hspace{1cm} (7)

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**Fig. 2.** Representation of solvent flow in FO, PRO, and RO. Membrane orientation is indicated in each system by the thick black line representing the membrane dense layer.

**Fig. 3.** Magnitude and direction of $J_W$ for FO, PRO, and RO and magnitude of $W$ for PRO is shown. Figure adapted from Ref. [17].

**Fig. 4.** Illustration of osmotic driving force profiles for an asymmetric membrane with the dense layer facing the draw solution (PRO mode). Internal and external concentration polarization are also shown.
where $D$ is the diffusion coefficient of the solute in the draw solution and $d_h$ is the hydraulic diameter of the flow channel. It has been shown that in spacer-filled flow channels the flow becomes turbulent at relatively low Reynolds numbers ($Re$) (e.g., $Re < 50$) [25–27]. Under these conditions, the Sherwood number ($Sh$) is determined using the correlation [27]:

$$Sh = 0.2 Re^{0.57}Sc^{-0.40} \quad (8)$$

where $Sc$ is the Schmidt number.

### 2.5. Internal concentration polarization in PRO

Concentrative internal concentration polarization is concentration polarization that results in the solute being concentrated inside the support layer of the membrane. Lee et al. [17] derived an expression to model the effect of internal concentration polarization in PRO applications. $J_w$ is determined by:

$$J_w = A \left[ \pi_{D,m} \frac{1 - \frac{C_{D,b}}{C_{D,m}} \exp(J_wK)}{1 + \frac{C_{D,b}}{C_{D,m}} \exp(J_wK) - 1} - \Delta P \right] \quad (9)$$

where $C_{D,b}$ is the salt concentration of the bulk feed solution and $C_{D,m}$ is the salt concentration of the draw solution at the membrane surface. The solute resistivity for diffusion through the porous support layer ($K$) is defined by:

$$K = \frac{\tau \tau}{D} \quad (10)$$

where $t$, $\tau$, and $\varepsilon$ are the thickness, tortuosity, and porosity of the support layer, respectively. Similar to $K$ for external concentration polarization, $K$ can be used to determine the influence of internal concentration polarization on water flux.

### 2.6. Water flux in PRO

In order to consider the effects of both internal and external concentration polarization on water flux in PRO, Eq. (9) can be modified by assuming that $C_{D,b}/C_{D,m} = \pi_{F,b}/\pi_{D,m}$ [23], and substituting $\pi_{D,m} = \pi_{D,b} \exp(-J_w/k)$ (from Eq. (6)). The resulting equation for water flux in the presence of both internal and external concentration polarization is:

$$J_w = A \left[ \pi_{D,b} \exp \left( \frac{-J_w}{k} \right) \frac{1 - \pi_{D,b} \exp(J_wK) \exp \left( \frac{J_w}{k} \right)}{1 + \frac{C_{D,b}}{C_{D,m}} \exp(J_wK) - 1} - \Delta P \right] \quad (11)$$

where $J_w$ is a function of the membrane characteristics ($A$ and $B$), mass transfer coefficient ($k$), solute resistivity ($K$), bulk osmotic pressures ($\pi_{F,b}$ and $\pi_{D,b}$), and applied hydraulic pressure ($\Delta P$).

The parameters necessary to calculate water flux in PRO (Eq. (11)) are obtained from RO and FO experiments and from calculation. $k$ is calculated using Eqs. (6)–(8). $A$ and $B$ are determined under RO conditions; $A$ is calculated using Eq. (1) and measuring $J_w$ and $\Delta P$ when $\Delta \pi = 0$ and $B$ is calculated using Eqs. (4) and (5). $K$ is obtained from FO experiments ($\Delta P = 0$) with deionized (DI) water as the feed and using Eq. (9). In this case Eq. (9) is rearranged to:

$$K = \frac{1}{J_w A} \ln \left( \frac{A \pi_{D,b} \exp \left( \frac{-J_w}{k} \right) - J_w}{B} \right) + 1 \quad (12)$$

### 2.7. Power density in PRO

The power density in PRO is equal to the product of the water flux through the membrane and the hydraulic pressure differential across the membrane (Eq. (2)). In the case of current generation membranes (experiencing reverse salt diffusion and concentration polarization), the equation governing the process is:

$$W = J_w \Delta P$$

$$= A \left[ \pi_{D,b} \exp \left( -\frac{J_w}{k} \right) \frac{1 - \frac{\pi_{D,b}}{\pi_{D,m}} \exp(J_wK) \exp \left( \frac{J_w}{k} \right)}{1 + \frac{C_{D,b}}{C_{D,m}} \exp(J_wK) - 1} - \Delta P \right] \quad (13)$$

After Eq. (11) is solved numerically to determine $J_w$, Eq. (13) can be solved algebraically to determine $W$ as a function of $\Delta P$.

Qualitatively, $J_w$ and $W$ as functions of $\Delta P$ are illustrated in Fig. 3. Under ideal conditions, as hydraulic pressure increases, water flux decreases until reaching zero at $\Delta P = \Delta \pi$ (the flux reversal point). Simultaneously, power density increases until reaching a maximum when $\Delta P = \Delta \pi/2$ and then decreases to zero at the flux reversal point. As expected, under real conditions, reverse salt diffusion (represented by $B$) and concentration polarization (represented by $k$ and $K$) cause the effective osmotic pressure differential ($\Delta \pi$) across the membrane to be lower than the bulk osmotic pressure differential. This phenomenon reduces the attainable $J_w$ and $W$ compared to the ideal membrane case.

### 3. Experimental

#### 3.1. Membranes

A flat-sheet cellulose triacetate (CTA) FO membrane (Hydration Technology Innovations, Albany, OR) was used in the experiments. The physical characteristics of this membrane are unique compared to other commercially available semi-permeable membranes and it has been determined to be the best available membrane for most FO applications [22,28].

#### 3.2. Solution chemistries

Certified ACS-grade NaCl (Fisher Scientific, Pittsburg, PA) was used to prepare both feed and draw solutions. Osmotic pressures, viscosities, and densities of all solutions (Table 1) were calculated using software from OLI Systems Inc. (Morris Plains, NJ). Diffusion coefficients were calculated according to Stokes [29] and Mills [30] and are also shown in Table 1.

#### 3.3. RO experiments

#### 3.3.1. Bench-scale system

A flat-sheet, bench-scale RO test unit was used to determine the membrane coefficients $A$ and $B$. The test unit had a channel on the feed side of the membrane to allow the feed solution to flow tangential to the membrane. The channel was 75 mm long, 25 mm wide, and 2.5 mm deep; it had an effective membrane area of 18.75 cm$^2$. Mesh spacers placed in the feed channel supported the membrane and enhanced the turbulence in the feed stream. The feed solution was contained in a 2-L reservoir. A high-pressure positive displacement pump (Wanner Engineering Inc., Minneapolis, MN) was used as the feed pump; the pump rate was controlled with a digital display.

<table>
<thead>
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<th>$C$ (g/L NaCl)</th>
<th>$\pi$ (kPa)</th>
<th>$\mu$ (mPa·s)</th>
<th>$\rho$ (g/L)</th>
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<td>0.976</td>
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</table>

Table 1 NaCl aqueous solution characteristics.
to recirculate the feed solution at 0.5 L/min. The permeate was collected in a 1-L graduated cylinder placed on an analytical balance linked to a computer. Flux through the membrane was calculated based on the change of weight of water in the graduated cylinder. The conductivity of the permeate was monitored (Accumet Basic, Fisher Scientific, Hampton, NH) and recorded.

3.4. Determination of membrane coefficients

RO tests were conducted with the dense layer of the membrane facing the feed solution. The first set of experiments was performed to determine $A$ following Eq. (1) and using DI water as the feed solution. Pressure was increased at 345-kPa (50-psi) increments from 345 to 1035 kPa (50–150 psi) for a duration of 3 h each. A second experiment was performed to determine $B$ following Eqs. (4) and (5). These experiments were performed for 12 h with 2 g/L NaCl as the feed solution and a pressure of 1035 kPa. The conductivity of the permeate was recorded at the end of the experiment. The temperature was held constant at 24 $\pm$ 1 $^\circ$C.

3.5. PRO experiments

3.5.1. Bench-scale system

A schematic drawing of the flat-sheet bench-scale PRO system is shown in Fig. 5. The custom-made membrane module had symmetric channels on both sides of the membrane. This allowed for the feed and draw solutions to flow tangential to the membrane. The membrane area and the dimensions of the feed and draw solution channels were the same as those in the bench-scale RO module and mesh spacers were placed in both channels. A variable-speed gear pump (Cole-Palmer, Vernon Hills, IL) was used to recirculate the feed solution at 0.5 L/min. A high-pressure positive displacement pump (Wanner Engineering Inc., Minneapolis, MN) was used to recirculate the draw solution at 0.5 L/min. The feed solution was contained in a 5.4-L constant-level reservoir; the volume was kept constant by a float valve and DI water continuously replenished the water that crossed the membrane. The DI water reservoir was placed on an analytical balance linked to a computer that collected data continuously. Flux through the membrane was calculated based on the change of weight of the DI water reservoir. Hydraulic pressures on the feed and draw solutions were monitored and recorded. The hydraulic pressure of the draw solution loop was controlled with a needle valve. The draw solution concentration was held constant by continuous reconcentration with a pilot-scale RO subsystem. The RO subsystem (different from the bench-scale RO system used for conducting membrane characteristics tests) is described in a previous publication [31].

3.6. Determination of mass transport coefficient

Three experiments were performed at $\Delta P = 0$ (i.e., in FO mode) and Eq. (12) was used to determine $K$. These experiments were performed for 12 h with DI water as the feed solution and 20, 35, and 60 g/L NaCl as the draw solution. The temperature of the system was held constant at 24 $\pm$ 1 $^\circ$C.

3.7. PRO performance tests

PRO performance tests were conducted with the dense layer of the membrane facing the draw solution. A matrix of experiments was performed (Table 2). Three feed solution concentrations (0,

<table>
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<th>C_{DS} (g/L NaCl)</th>
<th>$\Delta P$ (kPa)</th>
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Temperature of the system was held constant at 25 °C. Water flux was constant at an average of 6.78 m/s throughout the experiment. The NaCl rejection of the membrane was calculated to be 94% and B was calculated to be 1.14 × 10^{-6} m/s. These results are comparable to values reported in a previous study using similar membranes [32].

4.2. Calculations of mass transport coefficient (k)

k values (Table 3) were calculated using Eq. (7). For the three draw solution concentrations, Eq. (8) was used to calculate the Sh number. The hydraulic diameter (d_h) of the flow channel was calculated as proposed by [25]. k values increased slightly with draw solution concentration due to increases in density, viscosity, and diffusion coefficient as the solute concentration increases.

4.3. Determination of solute resistivity (K)

A set of FO experiments was performed to evaluate K using Eq. (12). The term t/r/s was then calculated for each experimental condition using Eq. (10) with a D of 1.51 × 10^{-9} m^2/s, adopted from Mills [30]. The same D was used for all three experiments because the same solution (DI water) was used as the feed stream. The term t/r/s represents the structural characteristic of the membrane support layer, and therefore, should be constant for a given membrane. Results (Table 4) reveal that the term t/r/s remains fairly constant under the three different experimental conditions. The resulting average t/r/s was used to calculate K coefficients for different feed solution concentrations. K is expected to change slightly since D is a function of the solute concentration. The calculated K coefficients for the different feed solution concentrations used during the PRO performance tests are summarized in Table 5.

4.4. PRO model results

Theoretical PRO water flux and power density curves as a function of hydraulic pressure for three feed solution concentrations and two draw solution concentrations are illustrated in Fig. 8. The two draw solution concentrations evaluated were 35 g/L NaCl to simulate sea water and 60 g/L NaCl to simulate concentrate from RO desalination facilities [33]. Water flux values are reported on the primary y-axis while power density values are reported on the primary x-axis.

### Table 3

<table>
<thead>
<tr>
<th>Draw Solution (g/L)</th>
<th>D (10^{-9} m^2/s)</th>
<th>Average t/r/s (10^{-4} m)</th>
<th>k (10^{-5} m/s)</th>
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### Table 4

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### Table 5

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<th>Average t/r/s (10^{-4} m)</th>
<th>K (10^{-5} s/m)</th>
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secondary y-axis. In both cases, as hydraulic pressure increases, water flux decreases until reaching zero (the flux reversal point). Power density reaches a maximum when the hydraulic pressure is approximately half of the hydraulic pressure of the flux reversal point. For example, in the case of the 35 g/L NaCl draw solution (Fig. 8a) and 2.5 g/L NaCl feed solution, the flux reversal point occurs at 2450 kPa and the maximum in power density occurs at 1280 kPa. As the feed concentration increases, water flux and power density decrease because of the reduced Δπ and also likely due to increased concentration polarization and increased salt passage. When comparing Fig. 8a and b, it can be seen that when the higher draw solution concentration is used (Fig. 8b), the water fluxes and power densities are much higher. This is simply due to the higher driving force. From the model predictions in Fig. 8, it is estimated that the maximum power density can be between 1.6 and 2.9 W/m² for the 35 g/L NaCl draw solution and between 5.1 and 8.1 W/m² for the 60 g/L NaCl draw solution. These ranges are close to (in the case of the 35 g/L NaCl draw solution), or in exceedence of (in the case of 60 g/L NaCl draw solution) the 4 W/m² target that has been set for successful development of osmotic power technology [4,16].

The effects of external concentration polarization, internal concentration polarization, and salt passage on water flux and power density are illustrated in Fig. 9. In the ideal scenario, no external concentration polarization, internal concentration polarization, or salt passage are considered. Theoretical water flux and power density are computed considering a membrane with an A of 1.87 \times 10^{-9} (m/s)/kPa. In the K scenario, the effects of external concentration polarization are introduced. In the K scenario, the effects of internal concentration polarization are introduced by considering a membrane with a support layer. In the K and B scenario, the effects

![Image](https://via.placeholder.com/150)

**Fig. 8.** Model results for water flux (Jw) and power density (W) as a function of applied hydraulic pressure (ΔP). Draw solution concentrations are (a) 35 g/L NaCl (2763 kPa) and (b) 60 g/L NaCl (4882 kPa).

![Image](https://via.placeholder.com/150)

**Fig. 9.** Model results for water flux (Jw) and power density (W) as a function of applied hydraulic pressure (ΔP) for different feed and draw solution concentrations.
of both internal concentration polarization and salt passage are considered (a B of 1.11 × 10⁻⁷ m/s was used). In the complete scenario, the combined effects of external concentration polarization, internal concentration polarization, and salt passage are considered. In Fig. 9a and d, a K scenario does not exist; when DI water is used as the feed solution and no salt passage is considered, there is no internal concentration polarization because there is no salt on the feed side of the membrane. It is also worth noting that the effect of salt passage alone (i.e., a B scenario) is not considered. Salt passage cannot be isolated from internal concentration polarization because salt passage affects process performance only if the membrane has a support layer, and if the membrane has a support layer, internal concentration polarization must be considered.

As expected, in all six figures, water flux and power density values decrease as the model gets more complex. Concentration polarization and salt passage reduce the driving force across the membrane; however, their effects vary depending on the feed and draw solution concentrations.

The effects of internal concentration polarization (K scenario) are greater than the effects of external concentration polarization (k scenario). Also, the effects of internal concentration polarization (K scenario) are enhanced by salt passage (K and B scenario). The magnitudes of internal concentration polarization (K scenario) and internal concentration polarization plus salt passage (K and B scenario) increase with increasing draw solution concentrations while the magnitude of external concentration polarization (k scenario) decreases with increasing feed solution concentrations and increases with increasing draw solution concentrations. Explanations for these trends are given below.

The external concentration polarization modulus is dependent on flux (Eq. (6)) and decreases as water flux decreases. Increasing feed solution concentration (0 to 2.5 to 5 g/L) decreases the driving force and subsequently, the attainable water flux. This causes a decrease in the effects of external concentration polarization. This can be seen in Fig. 9 by comparing the difference between the ideal and k scenarios either for Fig. 9a–c or d–f. Similarly, increasing draw solution concentration (35 to 60 g/L) increases the driving force and flux. This causes an increase in the effect of external concentration polarization, which can be seen by comparing the difference between the ideal and k scenarios either for Fig. 9a and d or b and e or c and f.

The analysis of internal concentration polarization is more complex. Its modulus, implicitly contained in Eq. (9), depends on both water flux and feed concentration; it decreases as water flux decreases and increases as feed concentration increases. Results show that internal concentration polarization becomes more severe as feed concentration increases (and water flux decreases) (0 to 2.5 to 5 g/L), thus, it appears that internal concentration polarization may be more dependent on feed concentration than on water flux. This can be seen by comparing the difference between the ideal and K scenarios for Fig. 9b and c or e and f. The effect of water flux alone on internal concentration polarization can be isolated by comparing the difference between the ideal and K scenarios between Fig. 9b and e or c and f. When the feed solution is constant, the increased draw solution concentration results in higher water fluxes and increased internal concentration polarization.

The analysis of internal concentration polarization in PRO depend on water flux; when water flux approaches zero (i.e., the hydraulic draw solution pressure approaches the osmotic pressure differential across the membrane), the contributions of internal and external concentration polarization to water flux reduction also approach zero and fluxes for the k and K scenarios approach that for the ideal scenario. However, the K and B and complete scenarios do not follow this behavior.

![Fig. 10. Experimental results for water flux (Jw) and power density (W) as a function of applied hydraulic pressure (∆P) (points) compared with model results (lines) under different feed and draw solution concentrations. Temperature was 25 °C.](image-url)
Table 6
Comparison between power density results from the literature and from the current study.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\Delta \pi$ (kPa)</th>
<th>$\Delta P$ (kPa)</th>
<th>$J_w$ ($10^{-6}$ m/s)</th>
<th>$W$ (W/m²)</th>
<th>$W([\Delta \pi - \Delta P])$ ($10^{-3}$ W/m² x kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[9]</td>
<td>2533</td>
<td>1216</td>
<td>0.29</td>
<td>0.35</td>
<td>0.27</td>
</tr>
<tr>
<td>[10]</td>
<td>10132</td>
<td>1925</td>
<td>0.81</td>
<td>1.56</td>
<td>0.19</td>
</tr>
<tr>
<td>[11]</td>
<td>8106</td>
<td>4053</td>
<td>0.81</td>
<td>3.27</td>
<td>0.81</td>
</tr>
<tr>
<td>[12]</td>
<td>7802</td>
<td>4053</td>
<td>0.77</td>
<td>3.12</td>
<td>0.83</td>
</tr>
<tr>
<td>This study</td>
<td>2763</td>
<td>972</td>
<td>2.81</td>
<td>2.73</td>
<td>1.52</td>
</tr>
<tr>
<td>This study</td>
<td>4882</td>
<td>972</td>
<td>5.21</td>
<td>5.06</td>
<td>1.30</td>
</tr>
</tbody>
</table>

The salt that crosses the membrane increases the salt concentration of the feed solution at the membrane surface ($C_{F,m}$), which reduces the osmotic pressure differential across the dense layer of the membrane, and therefore, reduces the pressure at which the flux reversal point occurs. Reverse salt diffusion is not flux-dependent and is solely responsible for the reduction of the flux reversal point pressure. If reverse salt diffusion could be eliminated, the flux reversal point, and hence $W_{max}$, would be greater.

4.5. PRO experimental results

Fig. 10 compares water flux and power density experimental results (symbols) with model results (lines). The maximum hydraulic pressure differential that could be achieved during the experiments was 970 kPa. Larger pressures caused perforation of the membrane because the spacers placed in the feed channels essentially tore the membrane. The experimental data closely follow the model results.

At a pressure of 970 kPa, the water fluxes achieved were between $1.8 \times 10^{-6}$ and $2.8 \times 10^{-6}$ m/s for the 35 g/L NaCl draw solution and between $3.6 \times 10^{-6}$ and $5.2 \times 10^{-6}$ m/s for the 60 g/L NaCl draw solution, depending on feed solution concentrations. These flux values are well above results previously reported in the literature (Table 6), likely due to the improved membrane and membrane module used in the current investigation. Loeb et al. [9], Loeb and Mehta [10], and Mehta and Loeb [11,12] used hydrophobic polymeric membranes and hollow-fiber membrane modules not particularly suited for PRO applications. The power densities achieved in the current investigation (Fig. 10) were up to 2.7 W/m² for a 35 g/L NaCl (2763 kPa) draw solution and up to 5.1 W/m² for a 60 g/L NaCl (4882 kPa) draw solution. In order to compare these values with those previously reported in the literature, a normalized power density value which takes into account the draw solution concentrations and hydraulic pressures used in previous investigations, was calculated ($W([\Delta \pi - \Delta P])$). From Table 6 (last column) it can be seen that the normalized power density values from the current investigation are well above results previously reported in the literature.

The proposed model provides guidance on how to increase power density in PRO operations. Changing operating conditions (e.g., increasing cross-flow on the membrane) can marginally improve power density by decreasing the effect of external concentration polarization. Greater improvement could be achieved by modifying the membrane structure. For example, a substantial improvement in the power density output could be made by optimization of the membrane support layer to reduce the internal concentration polarization that occurs. Specifically, the power density could be increased by up to 70% by reducing internal concentration polarization (seen by comparing the complete scenarios with the $k$ scenarios in Fig. 10). Thus, development of new membranes with thinner, less tortuous, and more porous support layers may be a key factor in successful osmotic power development.

Furthermore, in the current study, the pressure drop along the membrane module was directly measured and found negligible due to the small size of the module itself. Practically, the pressure drop along the membrane module is a function of the flow hydraulics of the system, and when designing future PRO membrane modules pressure drop should be taken into consideration using appropriate correlations in order to minimize its magnitude.
Fig. 11 illustrates how the maximum power density would change by varying $A$, $k$, and $K$. This example was constructed using 35 g/L NaCl as the draw solution, DI water as the feed solution, and a $\Delta P$ of approximately 1300 kPa. In this example, $A$ was increased up to 10 times from the values determined earlier in this investigation to represent the typical permeability of a nanofiltration membrane. $K$ was increased up to 2.5 times compared to the values determined earlier to represent a mass transfer coefficient with $Re$ equal to 1000, which is the typical upper limit for spacer-filled flow channels [25,27]. $k$ was reduced up to a tenth of the value determined earlier to simulate a very thin membrane support layer with low tortuosity and high porosity. These values are extreme and not achievable with current membrane technologies but are used to demonstrate the relative contribution of the three parameters to power density. Results show that combining the improvements of all three parameters could lead to a power density of over 28 W/m$^2$; this is a ten-fold increase from the earlier experimental PRO results shown in this study. The results also show that $A$ is the parameter that influences $W_{\text{max}}$ the most, closely followed by $K$. The influence of both $K$ and $k$ on $W_{\text{max}}$ is greater when $A$ is high, due to their dependency on $J_w$, as explained in the model results section.

5. Conclusions

In this investigation, for the first time, experimental PRO results were compared with model predictions. The PRO model was developed to predict water flux and power density under specific experimental conditions. PRO experiments were performed using a bench-scale PRO system utilizing a custom-made cross-flow membrane module. Data collected from the PRO experiments closely matched model predictions. At 970 kPa hydraulic pressure on the draw solution side, the power densities achieved were up to 2.5 times compared to the values determined earlier in this investigation. The hydraulic pressure of the bulk draw solution ($\Delta P$) was increased up to 2.5 times from the values determined earlier in this investigation to represent the typical permeability of a nanofiltration membrane. $K$ was increased up to 2.5 times compared to the values determined earlier to represent a mass transfer coefficient with $Re$ equal to 1000, which is the typical upper limit for spacer-filled flow channels [25,27]. $k$ was reduced up to a tenth of the value determined earlier to simulate a very thin membrane support layer with low tortuosity and high porosity. These values are extreme and not achievable with current membrane technologies but are used to demonstrate the relative contribution of the three parameters to power density. Results show that combining the improvements of all three parameters could lead to a power density of over 28 W/m$^2$; this is a ten-fold increase from the earlier experimental PRO results shown in this study. The results also show that $A$ is the parameter that influences $W_{\text{max}}$ the most, closely followed by $K$. The influence of both $K$ and $k$ on $W_{\text{max}}$ is greater when $A$ is high, due to their dependency on $J_w$, as explained in the model results section.

Acknowledgments

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**Nomenclature**

$\Delta P$ hydraulic pressure differential (kPa)

$\Delta \pi$ osmotic pressure differential (kPa)

$\varepsilon$ porosity

$\pi_{D,b}$ osmotic pressure of the bulk draw solution (kPa)

$\pi_{D,m}$ osmotic pressure of the draw solution at the membrane surface (kPa)

$\pi_{F,b}$ osmotic pressure of the bulk feed solution (kPa)

$\pi_{F,m}$ osmotic pressure of the feed solution at the membrane surface (kPa)

$t$ tortuosity

$A$ water permeability coefficient ((m/s)/kPa)

$B$ salt permeability coefficient (m/s)

$C_{D,b}$ salt concentration of the bulk draw solution (g/L)

$C_{D,m}$ salt concentration of the draw solution at the membrane surface (g/L)

$C_{F,b}$ salt concentration of the bulk feed solution (g/L)

$C_{F,m}$ salt concentration of the feed solution at the membrane surface (g/L)

$C_r$ salt concentration in the feed solution (g/L)

$C_p$ salt concentration in the permeate solution (g/L)

$D$ salt diffusion coefficient (m$^2$/s)

$d_h$ hydraulic diameter (m)

$J_w$ water flux (m/s)

$k$ external concentration polarization mass transfer coefficient (m/s)

$L$ length of channel (m)

$R$ salt rejection

$Re$ Reynolds number

$Sc$ Schmidt number

$Sh$ Sherwood number

$t$ thickness (m)

$W$ power density (W/m$^2$)

**References**


