MATH 332 Chapter 6 Objectives
By the end of Chapter 6 students should be able to:

6.1 Inner Product, Length, and Orthogonality
- Apply terminology of inner (dot) product, length, unit vector, distance between vectors, orthogonal vectors, orthogonal compliment.
- Use the inner product to determine the length of a vector, the distance between two vectors, or when two vectors are orthogonal.
- Apply the Pythagorean Theorem to orthogonal vectors.
- Determine when a vector is in $W^\perp$.
- Recognize $W^\perp$ as a subspace.
- Apply theorem 3 to find bases for $(\text{Row} A)^\perp$ and $(\text{Col} A)^\perp$

6.2 Orthogonal Sets
- Determine if a collection of vectors is an orthogonal/orthnormal set/basis.
- Given an orthogonal basis express a vector $y$ as a linear combination of the basis vectors. (Determined by theorem 5.)
- Apply the fact that an orthogonal set of nonzero vectors is linearly independent.
- Determine the projection of one vector onto another vector.
- Apply theorems 6 and 7 (properties of matrices with orthonormal columns).

6.3 Orthogonal Projection
- Use the Orthogonal Decomposition Theorem to decompose a vector into two vectors - one in a subspace $W$ and the other in the orthogonal complement $W^\perp$.
- Recognize that the orthogonal projection of $y$ onto $W$ is the best approximation of $y$ in the subspace $W$. (i.e., the closest point in $W$ to $y$.
- Recognize that the projection of $y$ onto $W$ can be written as $U U^T y$ where the columns of $U$ form an orthonormal basis for $W$.

6.4 The Gram-Schmidt Process
- Use Gram-Schmidt to find an orthonormal basis for a subspace (from a given basis).
- Find the $QR$ factorization of $A$ (when possible).

6.5 Least-Squares Problems
- Find the least squares solution to $Ax = b$ using the normal equations $A^T Ax = A^T b$.
- Calculate the least squares error $||b - Ax||$.  
• Recognize when there is a unique least squares solution to $Ax = b$ for each $b$. (By theorem 14, the columns of $A$ are linearly independent or $A^T A$ is invertible).

• Find the least squares solution to $Ax = b$ using the orthogonal projection of $b$ onto $\text{Col} A$ (when the columns of $A$ are orthogonal).

• Recognize a $QR$ factorization of $A$ find the (unique) least squares solution.

• Use the method of least squares to find a line of best fit. (Section 6.6)