MATH 111 FALL 2016
Final Exam Review

- Somewhere between one third and one half of the points on this exam will come from Chapter 5 (the new material). The remaining points will come from Chapters 2-4.

- The in-class activities, written homework assignments, MyMathLab homework assignments, exams 1 and 2, and the attached review problems are nice tools to prepare for the final exam.

- For constrained optimization problems other than a cube or cylinder, geometric formulas for the surface area and volume will be provided. You should know how to compute the surface area and volume of a cube and cylinder.

- The chapters covered are listed below. The hints/tips below are listed to give you some help in organizing your studies. This list is not exhaustive so just because something is not listed here does not mean it will not be on the final. The course map on Blackboard gives an exhaustive list of possible final exam topics and is a highly recommended tool to use for studying.

- Chapter 1: The main focus of chapter 1 and the covered sections was pre-calculus review. There will not be any questions on the exam that focus solely on the chapter 1 material. However it is certainly possible to see this material incorporated into other questions.

- Chapter 2: The main focus of chapter 2 was the evaluate of limits and the notion of continuity.
  - Be able to interpret the informal definition of a limit.
  - Limit computations can be evaluated using L'Hôpital's Rule when appropriate.
  - Be able to interpret limits involving infinity.
  - Know the Intermediate Value Theorem and be able to apply it.
  - You will not be directly asked about the Squeeze Theorem.
  - You will not be asked about the epsilon-delta definition of a limit.

- Chapter 3: The main focus of chapter 3 was the concept of the derivative.
  - Be able to compute derivatives using any combinations of the rules (product, quotient, chain, etc..) or basic formulas.
  - You should know all trigonometric derivatives, derivatives for (general) logarithms/exponentials, and the derivative of inverse tangent.
  - Be able to interpret the meaning of the derivative geometrically from the definition.
  - Given a function with a physical meaning, be able to interpret the derivative as a rate of change with correct units
  - Be able to find the equation of a tangent line or other similar problem involving the slope of the tangent line.
  - Don't forget about implicit differentiation!

- Chapter 4: The main focus of chapter 4 is the use of calculus for optimization and approximation.
  - Recall that “linear approximation” is synonymous with “tangent line approximation”.
  - Know the definition of critical point/number.
Ex

Compute \( \lim_{x \to 3} \frac{2x^2 + 5x - 1}{x^2 + 3x - 1}\).

\[
\lim_{x \to \infty} \frac{2x^2 + 5x - 1}{x^2 + 3x - 1} = 2
\]

Ex

Compute \( \lim_{x \to 7^+} \frac{5}{(x - 7)^2} \).

\[
\lim_{x \to 7^+} \frac{5}{(x - 7)^2} = \frac{5}{(7 - 7)^2} = \frac{5}{0}
\]

\begin{itemize}
  \item If \( x > 7 \) then \( (x - 7)^2 > 0 \).
  \item Thus \( \frac{5}{(x - 7)^2} > 0 \) for all \( x > 7 \).
\end{itemize}

That is, \( \lim_{x \to 7^+} \frac{5}{(x - 7)^2} = \infty \).
Ex  \[ \lim_{x \to 0} x^{2x} \]

This is a \( 0^0 \) indeterminate form.

\[
L = \lim_{x \to 0} x^{2x}
\]

\[
\ln(L) = \lim_{x \to 0} 2x \ln(x)
\]

\[
\ln(L) = \lim_{x \to 0} \frac{\ln(x)}{\frac{1}{2x}} \quad \text{as } \quad \frac{\infty}{\infty} \quad \text{indeterminate form}
\]

\[
\ln(L) = \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{2x^2}}
\]

\[
\ln(L) = \lim_{x \to 0} \frac{1}{x} \left(-2x^2\right)
\]

\[
\ln(L) = \lim_{x \to 0} -2x = 0
\]

\[ L = e^0 = 1 \]

Thus, \( \lim_{x \to 0} x^{2x} = 1 \)
\[ \lim_{x \to 0^+} (\cos(x) - \frac{1}{x}) \to \infty \]

\[ \lim_{x \to 0^+} \frac{\cos(x) - \frac{1}{x}}{x} = \frac{0}{0} \text{ ind. form} \]

\[ \lim_{x \to 0^+} \frac{x \cos(x) - \sin(x)}{x \sin(x)} \to 0 \]

\[ \lim_{x \to 0^+} \frac{1 \cdot \cos(x) - x \sin(x) - \cos(x)}{1 \cdot \sin(x) + x \cos(x)} = \frac{0}{0} \text{ ind. form} \]

\[ \lim_{x \to 0^+} \frac{-x \sin(x)}{\sin(x) + x \cos(x)} \to 0 \]

\[ \lim_{x \to 0^+} \frac{1 \cdot \sin(x) + x \cos(x)}{\cos(x) + 1 \cdot \cos(x) - x \sin(x)} \]

\[ \lim_{x \to 0^+} \frac{\sin(x) + x \cos(x)}{-x \sin(x) + 2 \cos(x)} = \frac{\frac{\sin(0) + 0 \cdot \cos(0)}{0 \cdot \sin(0) + 2 \cos(0)}}{0 + 0.1} = \frac{0}{2} = \frac{0}{0} \]
Ex

Let \( y (x^2 + 4) = 8 \)

(a) Find \( \frac{dy}{dx} \)

(b) Find the equation of the tangent line when \((x, y) = (z, 1)\).

\[
\begin{align*}
\frac{d}{dx} \left( y (x^2 + 4) \right) &= \frac{d}{dx} (8) \\
\frac{dy}{dx} (x^2 + 4) + y (2x) &= 0 \\
\frac{dy}{dx} (x^2 + 4) &= -2x y \\
\frac{dy}{dx} &= \frac{-2x y}{x^2 + 4}
\end{align*}
\]

\[
\begin{align*}
(b) \quad m &= \left. \frac{dy}{dx} \right|_{(2, 1)} = \frac{-2 (2) (1)}{(2^2) + 4} = \frac{-4}{4 + 4} = \frac{-4}{8} = -\frac{1}{2}
\end{align*}
\]

\[
y - y_1 = m (x - x_1)
\]

\[
y - 1 = -\frac{1}{2} (x - 2)
\]
Ex

Let \( f(x) \) be the elevation in ft above sea level of a hiking trail at a distance of \( x \) miles from the trailhead.

(a) What are the units of \( f'(x) \)?

units are \( \text{ft per mile} \).

(b) Suppose \( f'(3) = 1000 \). What does this tell you about the trail?

After 3 miles, the elevation of the trail is increasing at a rate of 1000 ft/mi.