Learning Objective:
Interpret the informal definition of limit. Namely, \( \lim_{x \to a} f(x) = L \) if \( f(x) \) is arbitrarily close to \( L \) for all \( x \) sufficiently close to \( a \).

Example 1

\[
y = \frac{x^2 - 1}{x+1} \quad \text{Domain: all reals except } x = -1.
\]

If \( x \neq -1 \) then

\[
y = \frac{x^2 - 1}{x+1} = \frac{(x+1)(x-1)}{(x+1)} = x - 1
\]

\[
\lim_{x \to 1} \frac{x^2 - 1}{x+1} = 0
\]
**Interpretation:**

\[
\frac{x^2 - 1}{x + 1}
\]

can be as close as we want to 0 (arbitrarily close) by moving \( x \) as close as we need to 1 (sufficiently close).

**Example 2**

\[
f(x) = \frac{\sin(x)}{x}
\]

Domain is all reals except \( x = 0 \).

\[
f(x) \text{ gets closer and closer (arbitrarily close) to } 1 \text{ as } x \text{ gets closer and closer (sufficiently close) to } 0.
\]

Symbolically,

\[
\lim_{{x \to 0}} f(x) = 1
\]
Example 3

(The Heaviside function)

\[
h(x) = \begin{cases} 
  0 & \text{if } x < 0 \\
  1 & \text{if } x \geq 0.
\end{cases}
\]

There is no \( L \) such that \( h(x) \) gets arbitrarily close to \( L \) for all \( x \) sufficiently close to \( 0 \).

We say \( \lim_{x \to 0} h(x) \) does not exist.

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<th>( h(x) )</th>
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