Section 3.3 Rules of Differentiation

Question: Do we really need to use the definition of $f'(x)$ to find the derivative?

Objectives

1. Compute derivatives of polynomials
2. Recognize standard notation for derivative
3. Recall the derivative of the exponential function.
   \[ \frac{d}{dx}(e^x) = e^x \]
4. Recognize that derivative rules come from the definition of the derivative.

The constant rule.

\[ \frac{d}{dx} (c) = 0 \]

why?
\[ \frac{d}{dx} (c) = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0 = 0 \]

The power rule

\[ \frac{d}{dx} (x^n) = nx^{n-1} \quad (n \text{ is a nonzero integer}) \]

why?
\[ \frac{d}{dx} (x^n) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \to 0} \frac{x^n + nx^{n-1}h + \text{terms with } h^2 \text{ at least } h^2 - x^n}{h} \]
\[ = \lim_{h \to 0} nx^{n-1} + \text{ terms with } h \text{ at least } h \]
\[ = nx^{n-1} + 0 \]
Constant Multiple Rule

\[
\frac{d}{dx} (cf(x)) = c \cdot \frac{d}{dx} f(x)
\]

why?

\[
\frac{d}{dx} (cf(x)) = \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h}
\]

\[= \lim_{h \to 0} \frac{c(f(x+h) - f(x))}{h}\]

\[= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\]

\[= c \cdot \frac{df}{dx}\]

Sum/Difference Rule

\[
\frac{d}{dx} (f(x) \pm g(x)) = \frac{df}{dx} \pm \frac{dg}{dx}
\]

why?

\[
\frac{d}{dx} (f(x) \pm g(x)) = \lim_{h \to 0} \frac{(f(x+h) \pm g(x+h)) - (f(x) \pm g(x))}{h}
\]

\[= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \pm \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}\]

\[= \frac{df}{dx} \pm \frac{dg}{dx}\]
Example

Find $f'(x)$ for

(a) $f(x) = x^2 + 2x + 3$

(b) $f(x) = x^9 - x^3 + 1$

(c) $f(x) = (x+1)(x-1)$

(a) $f'(x) = 2x + 2$

(b) $f'(x) = 99x^9 - 30x^2 9$

(c) $f'(x) = x^2 - 1$

$f'(x) = 2x$

The derivative of $f(x) = b^x$

Assumption

At $x=0$, $f'(0) = 1$.

That is, $1 = \lim_{h \to 0} \frac{b^h - b^0}{h}$

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Then

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{b^{x+h} - b^x}{h}$

$= \lim_{h \to 0} \frac{b^x b^h - b^x}{h}$

$= b^x \lim_{h \to 0} \frac{b^h - 1}{h}$

So $\frac{d}{dx}(e^x) = e^x \cdot 1 = e^x$
Example

5. If \( f(x) = 2e^x + x^2 - 3 \) then
\[
f'(x) = 2e^x + 2x.
\]

6. If \( f(x) = 5e^x - 1 \) find the equation of the tangent line at \( x = 2 \).

\[
f'(x) = 5e^x
\]

\[
f'(2) = 5e^2 \quad \text{slope of tangent line}
\]

\[
f(2) = 5e^2 - 1 \quad \text{y-value of point}
\]

\[
y - f(2) = f'(2)(x - 2)
\]

\[
y - 5e^2 + 1 = 5e^2(x - 2)
\]