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ELASTO-PLASTIC ANALYSES OF DEEP FOUNDATIONS IN COHESIVE SOIL

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SUMMARY

The bearing capacity of deep foundations has been analysed using finite elements in conjunction with elasto-viscoplasticity. The influence of elastic parameters, mesh size and strain conditions on computed collapse loads was observed. Axisymmetric problems in particular were of interest, because it was found that good numerical solutions could be obtained through to collapse provided reduced integration was used.

INTRODUCTION

The total resistance of a pile consists of base resistance and skin friction. Estimation of the base resistance components requires the use of bearing capacity factors which vary widely from one investigator to another in the case of frictional soils. Terzaghi's[1] approach, which is still popular, ignores the shear strength of the surcharging soil above foundation level but becomes increasingly conservative with depth. In any event, for soils with high friction angles, the bearing capacity of deep foundations is rarely of interest as settlements or even structural failure of the pile itself are more likely to govern the design.

The design of foundations in saturated clays is of greater interest to engineers. In this case, loading is either so rapid, or the soil permeability so low, that any changes in confining pressure are supported solely by the pore fluid with no resulting changes in effective stresses or strength. The strength of such a material is defined by its undrained cohesion C_u, which is a function of the effective stresses and strength parameters existing prior to undrained loading.

For 'frictionless' undrained clay materials, the range of N_c values due to various investigations is quite narrow. For a surface footing the exact Prandtl solution gives $N_c = 5{\cdot}14$. Increasing the footing depth causes N_c to rise, but not indefinitely. A certain depth is reached at which N_c reaches a maximum and no further base resistance can be mobilized. For a deep strip foundation, Skempton[2] and Meyerhof[3] give maximum N_c values of 7·5 and 8·3 respectively, but for circular foundations they were in closer agreement with N_c values of around 9. Meyerhof indicated that the maximum value was achieved at around two footing widths (diameters) below ground level although the more conservative factor of five footing widths is frequently used in practice.

The present paper gives some computed finite element predictions of the bearing capacity of deep foundations. The aim was to compare the finite element results with the established solutions mentioned above, and reach some general conclusions regarding the ability of finite elements to predict collapse loads. As finite element methods make no assumptions '*a priori*' as to the eventual mechanism of failure they represent a natural approach to bearing predictions.

Results from a series of meshes with varying D/B ratios are presented and both plane and axisymmetric strain conditions have been considered. In all cases, Tresca's failure criterion

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was used to simulate the strength of an undrained clay and the material was assumed weightless as self-weight stresses do not affect the strength of a cohesive soil.

The axisymmetric case was particularly interesting in view of recent work[4] suggesting that extremely complicated elements are necessary to predict collapse of nearly incompressible axisymmetric problems using exact integration schemes.

METHOD

All results presented in this paper were obtained using 8-node quadrilateral elements with the stiffness matrix computed using 'reduced' (2×2) integration. Non-linearity introduced by the plasticity was accounted for using the elasto-viscoplastic method.[5,6] This technique iterates using repeated elastic solutions until both equilibrium and yield are satisfied to some predetermined tolerances. The method falls into the 'initial strain' family of iterative solution approaches, and has been shown to be an efficient and versatile way of solving a wide class of boundary value problems.[7,8] It has also been shown to be more efficient than initial stress methods[5,9] in requiring approximately half as many iterations per load increment.

Although the name 'viscoplastic' implies a time dependence, in the absence of meaningful viscous properties of soil, time is treated as a dummy variable. A disadvantage of the method is that numerical stability is dependent on the choice of timestep size for each iteration. Cormeau[10] derived critical timesteps for some well-known yield functions with associated flow rules, and these almost invariably give stable solutions, even for non-associated flow rules with strain-hardening or softening.

Some problems with the critical timestep were experienced by the author as Poisson's ratio approached one half. It was found that the use of the critical timestep with a high Poisson's ratio tended to underestimate the true bearing capacity. This problem was overcome by arbitrarily halving the critical timestep for cases where Poisson's ratio exceeded 0·45.

BOUNDARY CONDITIONS

The basic meshes used for the deep foundation analyses are given in Figure 1. Most work was performed using Mesh B, but Mesh A was included so that the influence of boundary proximity could be observed. The footing was taken to be 2 m wide, but only half the problem was considered owing to symmetry. The computer program was arranged such that D could be altered by changing one piece of data enabling a range of D/B ratios to be considered. The effect of mesh size, both below and to the side of the footing was also taken into account by changing L and H.

Ground level at the top of the mesh was left open, the centreline was allowed to move vertically only, and the sides remote from the footing below and to the side were rigidly fixed. Two alternative boundary conditions were considered on face XY. The first allowed vertical movement only and the second placed no restriction on movement. These two cases are referred to as the 'closed' and 'open hole' analyses respectively. The reason that these alternatives were available was that in the finite element analyses, no actual foundation was placed in the hole. Stresses were applied to the soil in the form of prescribed vertical displacements at foundation level. Vertical stresses induced by these displacements were sampled at the first row of Gauss points beneath the displaced nodes, and averaged.

The closed hole analysis appeared more realistic in that it simulated the presence of a rigid, smooth-sided foundation placed in the hole. In this case, plastic flow could only propagate to ground level. The open hole case was much less confined however, and permitted plastic flow to occur back towards XY. The type of mechanism encouraged by the open hole analysis also seemed more in line with deep foundation theories in which the assumed mechanism is quite localized.

RESULTS

It is well known that the choice of elastic properties governing soil behaviour inside the failure surface has no influence on collapse loads. However, the elastic properties do influence the stress/deformation curve prior to failure. The curves of Figure 2 can easily be normalized with respect to Young's modulus which is directly proportional to the stresses generated by a given increment of vertical displacement, but the Poisson's ratio effect is more complex.

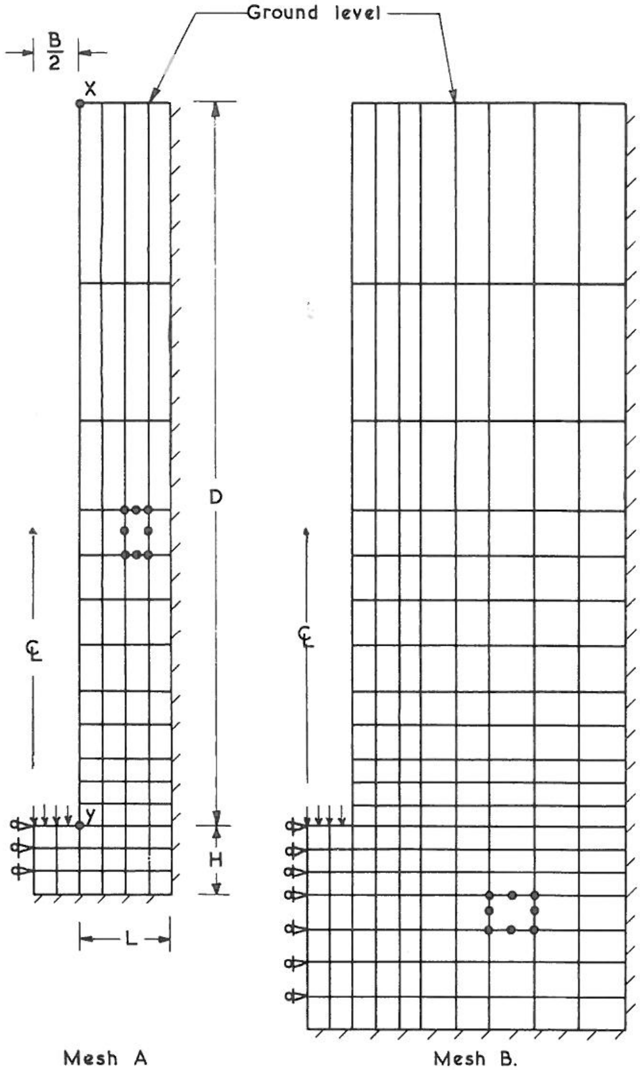

Figure 1. Deep foundation meshes

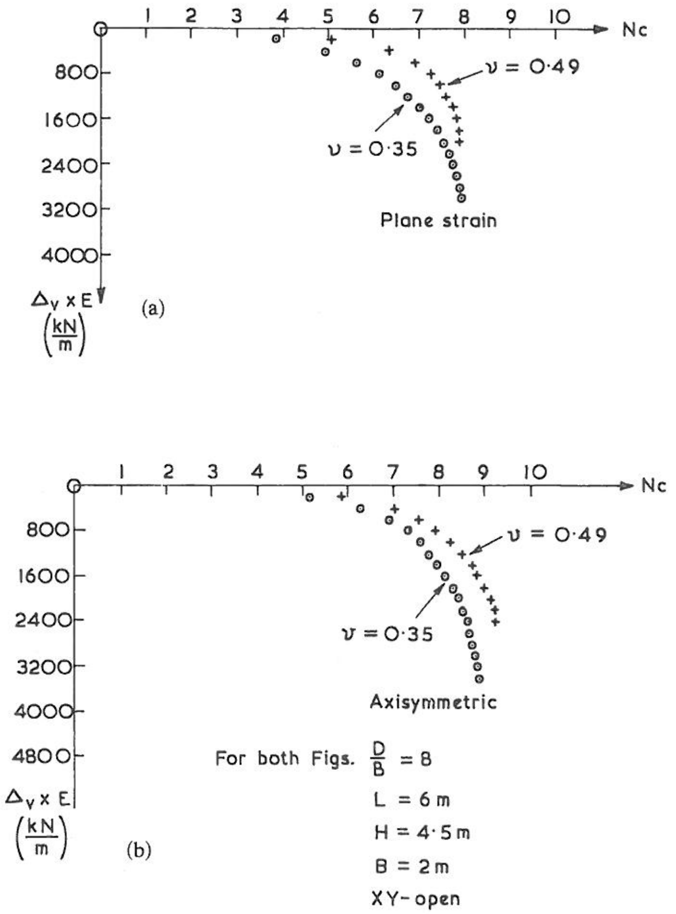

Figure 2. Effect of Poisson's ratio

From Figure 2 it is seen that although Poisson's ratio does not influence collapse, it tends to stiffen the system as it approaches 0·5. This of course could also be demonstrated in a simple elastic analysis.

It is interesting to note how well conditioned the stress/deformation curves are, especially for the axisymmetric case. The curves smoothly approach failure even with $\nu = 0{\cdot}49$, and is a good illustration of 'reduced' integration giving solutions to problems which 'exact' schemes could not cope with using such simple elements; other investigators[4] have concluded that the simplest element that could tackle such an incompressible axisymmetric problem using exact integration is a fifteen-noded triangle.

Before embarking on the full analyses, a brief study was made of the influence of mesh size on computed collapse loads by repeating calculations using Mesh A (Figure 1). It was not expected that collapse predictions would be very sensitive to mesh size, because any increase in confining pressure imposed by an adjacent boundary has no effect on material strength on a $\phi_u = 0$ material. Figure 3 confirms this, with Meshes A and B yielding results that were barely distinguishable to plotting accuracy. It should be noted that although the collapse

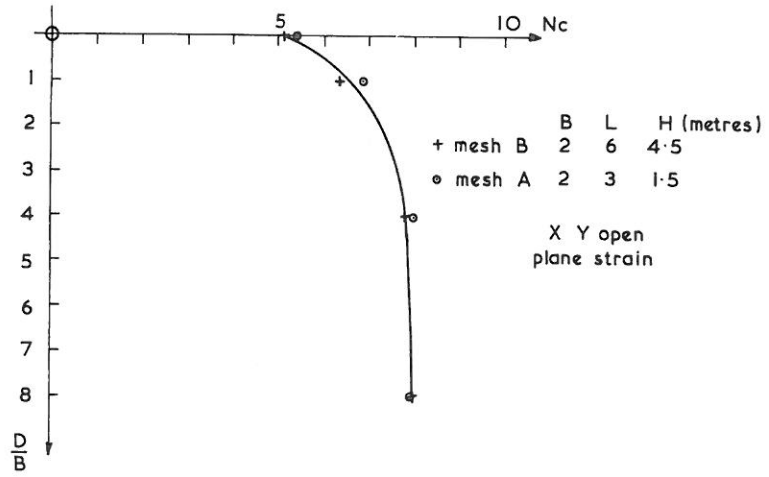

Figure 3. Effect of mesh size

stresses were not changed by mesh size, the displacements to reach failure were higher in Mesh B.

Continuing with Mesh B, a number of analyses were carried out for various D/B ratios. For each ratio, both plane and axisymmetric cases were considered, and both 'open' and 'closed hole' analyses performed.

Figure 4 gives solutions for the 'closed' hole analyses with face XY supported on vertical rollers. For both plane and axisymmetric strain, it was found that the computed N_c values were unaffected by the presence of rollers for D/B less than 2 to 3. This was because there was little tendency for soil to move past face XY at relatively shallow depths, with failure mechanisms preferring to outcrop at ground level. For greater D/B ratios, the 'open hole' values of N_c levelled out whereas the 'closed hole' values continued to rise. N_c values for the axisymmetric case appeared to rise more swiftly than in plane strain owing to the extra

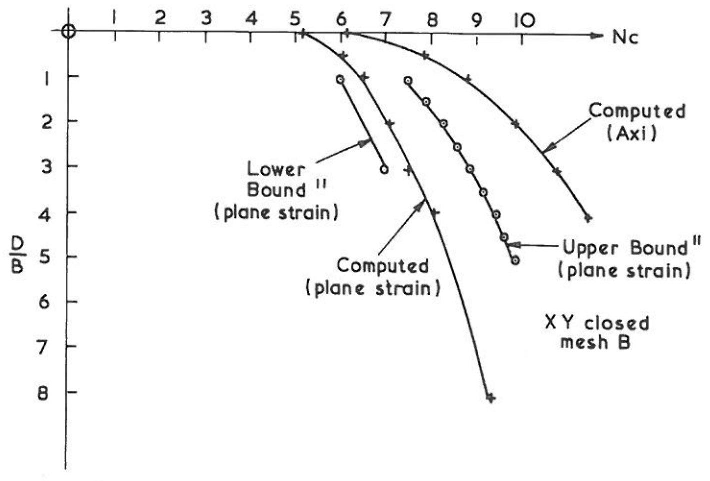

Figure 4. Comparison of F.E. solution with limit analyses

confinement inherent in the problem. Increasing N_c values observed with the 'closed hole' is what would also be predicted by any limit analysis in which the presence of a smooth, rigid pile compelled the failure mechanism to outcrop at ground level. No kinematically admissible mechanism of the Meyerhof-type would be possible for the 'closed hole' case. The deeper the foundation therefore, the greater the amount of soil that must be sheared, and the higher the bearing capacity. Such a limit analysis[11] for the plane strain case is given in Figure 4, and the computed solutions fall nicely between the bounds.

More realistic N_c values were obtained by the 'open hole' analyses. The results in Figure 5 compare well with classical predictions of Skempton and Meyerhof in both plane strain and axisymmetry where N_c reached a maximum of 7·8 and 9 respectively at a depth to width (diameter) ratio of about 3.

Although the stress/deformation plots in plane strain and axisymmetry (Figure 2) appeared quite similar, it is interesting to note from Figure 6 that the nodal displacements at failure in

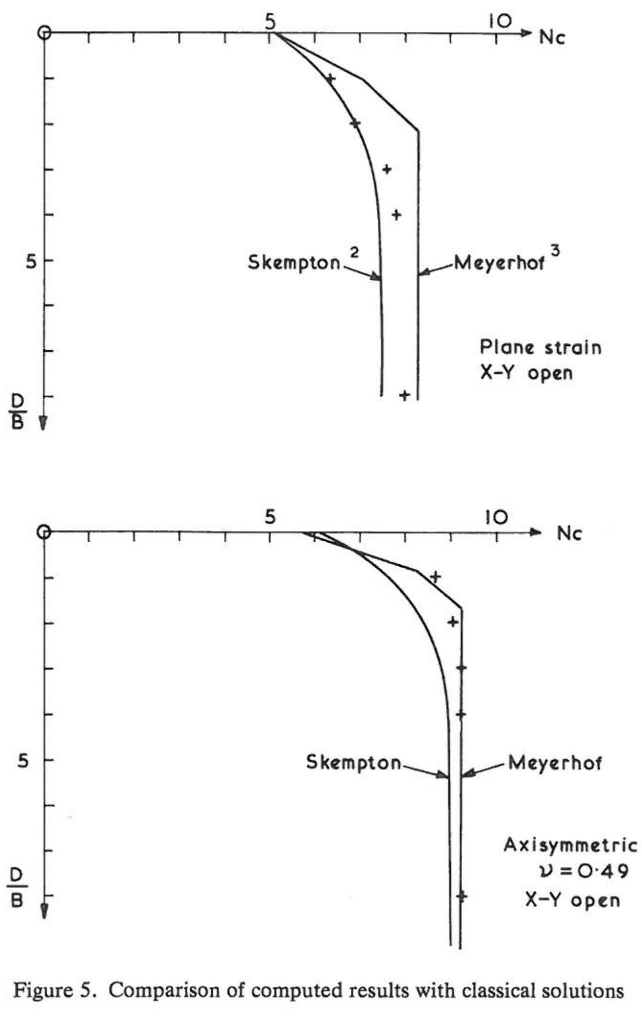

Figure 5. Comparison of computed results with classical solutions

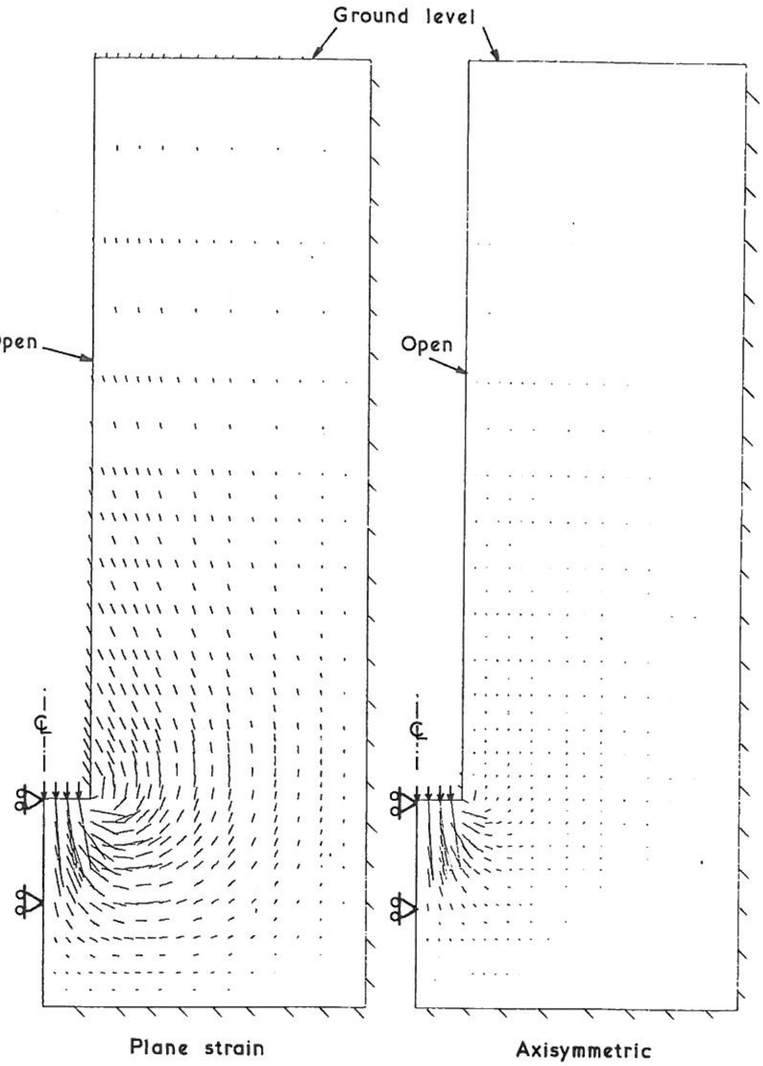

Figure 6. Comparison of displacement vectors at failure

the two cases were strikingly different. Both the diagrams in this figure were scaled to represent the nodal displacement pattern after 1·5 m of vertical footing displacement. The plane strain case readily formed a mechanism with some nodal displacements travelling vertically upwards and to the left in search of the open vertical face. No analogous mechanism exists for the axisymmetric problem in spite of the similar appearance of the meshes in cross-section. Nodal movements in axisymmetry were much more constrained with elements locking against each other as they attempted to maintain compatibility.* A very localized mechanism eventually appeared at the open face.

* Reduced integration has the effect of relaxing compatibility.

CONCLUSIONS

Finite element viscoplastic analyses of deep foundations have given collapse load predictions for undrained clays which were in close agreement with classical solutions provided the right boundary conditions were selected.

Even in nearly incompressible axisymmetric problems, accurate collapse predictions were made using 8-node quadrilateral elements with 'reduced' integration. In view of this, objections to the use of 'reduced' integration on the grounds of compatibility violations seems secondary provided only collapse loads are required.

The 'open' and 'closed hole' analyses gave similar results for D/B less than 2 to 3 when the mechanism tended to ground level, but for higher D/B rates, the closed hole values continued to rise while the open hole values levelled out.

The computed collapse loads were found to be insensitive to the mesh size below footing level when dealing with 'Tresca' materials where strength was not a function of mean confining stress.

Although N_c values computed in axisymmetry were only slightly higher than their plane strain counterparts, their nodal displacements at failure differed considerably. Movements were much more localized and constrained in the axisymmetric case, whereas a mechanism formed readily in plane strain.

Using displacement control and reduced integration with 8-node quadrilateral elements, it was necessary to reduce the critical timestep in order to maintain stable solutions as Poisson's ratio approached one half.

Using the same computer program, it is possible to examine the behaviour of foundations in frictional materials. Their strength becomes a function of (at least) confining pressure and strain, and the results will be reported in a second paper.

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