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ARTICLE

Spatial correlation length of normalized cone data in sand: case study in the north of Denmark

S. Firouzianbandpey, D.V. Griffiths, L.B. Ibsen, and L.V. Andersen

Abstract: The main topic of this study is to assess the anisotropic spatial correlation lengths of a sand layer deposit based on cone penetration testing with pore pressure measurement (CPTu) data. Spatial correlation length can be an important factor in reliability analysis of geotechnical systems, yet it is rarely estimated during routine site investigations. Results from two different sites in the north of Denmark are reported in this paper, indicating quite strong anisotropy due to the depositional process, with significantly shorter spatial correlation lengths in the vertical direction. It is observed that the normalized cone resistance is a better estimator of spatial trends than the normalized friction ratio.

Key words: spatial correlation length, cone penetration testing with pore pressure measurement (CPTu), soil inhomogeneity, normalized cone resistance, normalized friction ratio.

Résumé : Le sujet principal de cette étude est l'évaluation des longueurs de corrélation spatiale anisotrope d'un dépôt sous forme de couches de sable, à partir de résultats d'essais de pénétration du cône avec la mesure des pressions interstitielle (CPTu. La longueur de corrélation spatiale est considérée comme un facteur important lors de la réalisation d'analyses de fiabilité de systèmes géotechniques, cependant elle est rarement estimée durant les investigations de routine sur le terrain. Les résultats provenant de deux sites différents dans le nord du Danemark sont présentés dans cet article. Ces résultats indiquent une anisotropie relativement forte associée au processus de déposition, en raison de longueurs de corrélation spatiale significativement plus faibles dans la direction verticale. Il est observé que la résistance normalisée du cône est un meilleur paramètre pour l'estimation des tendances spatiales plutôt que le ratio de friction normalisé. [Traduit par la Rédaction]

Mots-clés : longueur de corrélation spatiale, essai de pénétration du cône avec la mesure des pressions interstitielle (CPTu), inhomogénéité du sol, résistance normalisée du cône, ratio de friction normalisé.

Introduction

An understanding of the variability of soil properties for the purpose of foundation design and analysis, and as a basis for the calibration of new design codes, is of considerable importance in geotechnical engineering. Soil data used in foundation design is usually accompanied with significant uncertainty because of limited site data and natural variability.

In addition to the mean and standard deviation of soil parameters, the spatial correlation has been increasingly recognized as an influential property in determining probabilistic outcomes. This paper seeks to estimate anisotropic spatial correlation length at two sand deposits in northern Denmark, by analysis of the measured spatial variability of sampled soil properties using statistical trends and correlations, and to interpolate soil properties at unsampled locations. When the available information is very limited, geotechnical design is inevitably more conservative (e.g., Baecher 1986). In this study, we study the results of stress normalized cone penetration testing with pore pressure (CPTu) data to estimate spatial correlation length. Numerous studies, such as Alonso and Krizek (1975), Tang (1979), Nadim (1986), Campanella et al. (1987), Wu et al. (1987), Reyna and Chameau (1991), Kulhawy et al. (1992), Fenton (1999), Elkateb et al. (2003*a*, 2003*b*), and Phoon et al. (2003), have reported assessments of inherent soil variability using cone penetration tests (CPT). However, results using stress-normalized CPT data are more limited (Uzielli et al. 2005).

There are several different methods for modeling the inherent variability of a soil property represented as a random field. For example, the local average subdivision method, as proposed by Fenton and Vanmarcke (1990) was subsequently combined with finite element methods to analyze geotechnical problems of practical interest (see Griffiths and Fenton 1993; Fenton and Griffiths 2008). The local average subdivision method requires probability density functions (pdf) of key input parameters described by the mean, μ, and standard deviation, σ, of the property at each point in space, and a spatial correlation length δ. These parameters can be estimated from field data obtained at discrete locations across a site.

The present study focuses on the spatial variability of cone data normalized with respect to vertical stress. The data were collected at two different sites in the north of Denmark, where the ambient soil type is classified as either sand or silty sand. By calculating the statistical parameters of the cone data, the spatial correlation length of the field was estimated in the vertical and horizontal directions.

Vanmarcke (1977) estimated spatial correlation functions and spatial correlation lengths using common functions such as exponential, exponential oscillatory, quadratic exponential oscillatory, and bilinear. In the present research, the function describing spatial correlation with respect to relative distance is an exponential model. Furthermore, the parameter of the model, D, is calculated separately in the horizontal and vertical directions. At both

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S. Firouzianbandpey, L.B. Ibsen, and L.V. Andersen. Department of Civil Engineering, Aalborg University, 9000 Aalborg, Denmark.
D.V. Griffiths. Department of Civil and Environmental Engineering, Colorado School of Mines, 80401 Golden, CO, USA, and Australian Research Council Centre of Excellence for Geotechnical Science and Engineering, University of Newcastle, Callaghan NSW 2308, Australia.
Corresponding author: S. Firouzianbandpey (e-mail: sf@civil.aau.dk).

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Fig. 1. Terminology for cone penetrometers.

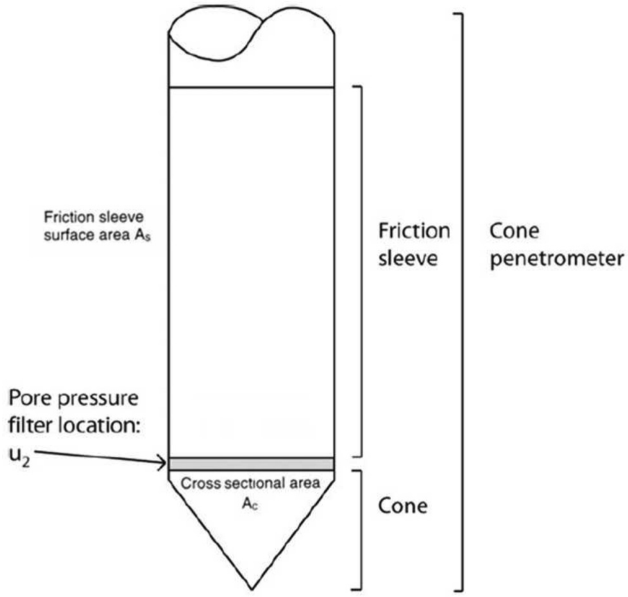


Fig. 2. Map of field test area.

sites, the cone data show that the vertical and horizontal correlation structures in soil properties are strongly anisotropic, with shorter correlation lengths in the vertical direction. Also in the vertical direction, it was observed that cone resistance normalized by vertical stress is more spatially correlated than normalized values of the friction ratio, with spatial correlation lengths estimated in the range of 0.5 and 0.2 m, respectively.

Cone penetration test

The cone penetration test (CPT) and its enhanced versions (i.e., piezocone-CPTu and seismic-CPT) have various applications in a wide range of soils. During testing, a cone mounted on a series of rods is pushed into the ground at a constant rate, while continuous measurements are made of the resistance to the penetration of the cone and of a surface sleeve surrounding the rods. The CPTu is preferred among other in situ testing methods because of the advantages of fast and continuous profiling, repeatable and reliable data (not operator-dependent), being economical and productive to conduct, and having a strong theoretical basis for interpretation.

Figure 1 illustrates the main parts of a cone penetrometer. The total force acting on the cone, Q_c, divided by the projected area of the cone, A_c, produces the cone resistance, q_c. The total force acting on the friction sleeve, F_s, divided by the surface area of the friction sleeve, A_s, is the sleeve friction, f_s. A piezocone also measures pore water pressure, typically just behind the cone in the location u_2, as shown in Fig. 1.

Description of the sites

As illustrated in Fig. 2, the analysis concerns two different sites located in Jutland, in the north of Denmark, a few metres from the sea. Statistical characteristics of cone tip resistance in the Aalborg site were estimated using piezocone penetration test (CPTu) data of nine soundings obtained from the industrial part of the city. A wind turbine blade storage will be constructed at this location. The site is close to the Limfjord river, meaning it is a basin deposit area. The soil sediment is 1–4 m of clay/gyttia marine deposits, while the lower layers are mostly silty sand. The CPTu tests reached a depth of approximately 8 m. The nine soundings are arranged in a cross-shaped pattern, with 10 m separation between the holes. The cross is framed by four bore holes (Fig. 3*a*). In both sites and all CPT soundings, the interval distance between the samples in the vertical direction is 2 cm. Figure 4*a* shows a representative CPT profile performed in the field. Statistical analysis of the CPT data was performed using MATLAB, after programming the soil behavior classification system suggested by Robertson (1990) as shown in Fig. 5.

The second site is located in the northeast of Denmark, at the harbor of Frederikshavn. The soil layers are mostly sand, with some thinly interbedded stiff clays. Twelve soundings were performed at this site to depths of 7–8 m, in a cross-shaped pattern as illustrated in Fig. 3*b*. Figure 4*b* shows a representative CPT profile from this site. The soil types inferred from the representative cone profile, based on the Robertson classification chart, are plotted in Fig. 6. The ambient layers are various types of sands, gravelly sands, and sand mixtures.

Normalized cone data

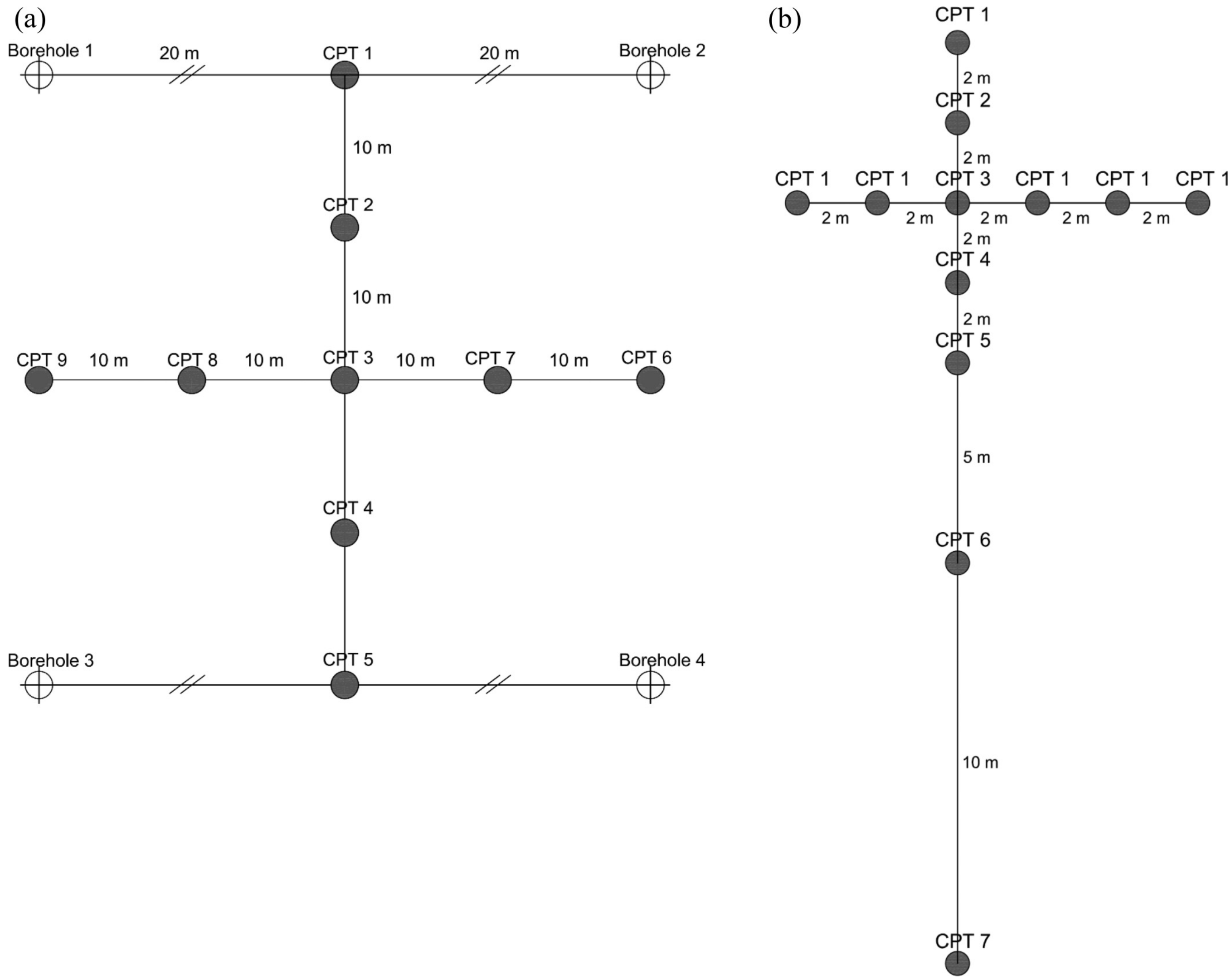
Effective overburden stress can have a significant influence on CPT measurements and lead to an incorrect assessment of soil strength parameters. Low overburden stresses result in a reduced sleeve and tip resistance, whereas at greater soil depths, a logarithmically pronounced increase occurs in measured tip and sleeve resistance (Moss et al. 2006).

Different methods are used in the literature for normalizing CPT measurements for vertical stress. This study applies the technique proposed by Robertson and Wride (1998) to cone tip resistance measurements:

$$(1)\qquad q_{c1N} = \left(\frac{q_c}{P_a}\right)C_Q, \quad C_Q = \left(\frac{P_a}{\sigma'_{v0}}\right)^n$$

where q_{c1N} is the dimensionless cone resistance normalized by the weight of soil above the cone, q_c is the measured cone tip resistance, and C_Q is a correction for overburden stress. The exponent n takes the values 0.5, 1.0, and 0.7 for cohesionless, cohesive, and intermediate soils respectively, while σ'_{v0} is the effective vertical stress, and P_a is the reference pressure (atmospheric pressure) in the same units as σ'_{v0} and q_c, respectively. q_{c1N} is a dimensionless cone resistance normalized by the weight of the soil on top of the cone. In most studies, this value is used instead of the raw cone resistance in the correlations.

Fig. 3. Position of boreholes and CPTu: (*a*) Aalborg site; (*b*) Frederikshavn site.



Also, the normalized friction ratio is calculated using the equation proposed by Wroth (1984)

$$(2)\qquad F_R = 100\frac{f_s}{q_c - \sigma_{v0}}$$

where f_s is the measured sleeve friction and σ_{v0} is the total vertical stress. In this case, q_c, σ_{v0}, and f_s are all in the same units.

These procedures were applied to all soil types, to decrease the effect of overburden pressure on the results.

Estimation of soil behavior type from cone data

The two charts shown in Figs. 5*a* and 6*a* represent the classification system proposed by Robertson (1990) that incorporates two pieces of normalized cone data (tip resistance and pore water pressure). Using the chart, the soil type can be estimated using cone data after a normalization in which the overburden pressure and the initial water level surface is accounted for. Seven different zones can be identified in the chart by plotting the data for the normalized cone resistance and pore pressure ratio.

The normalized tip resistance is calculated as

$$(3)\qquad Q_t = \frac{q_t - \sigma_{v0}}{\sigma'_{v0}}$$

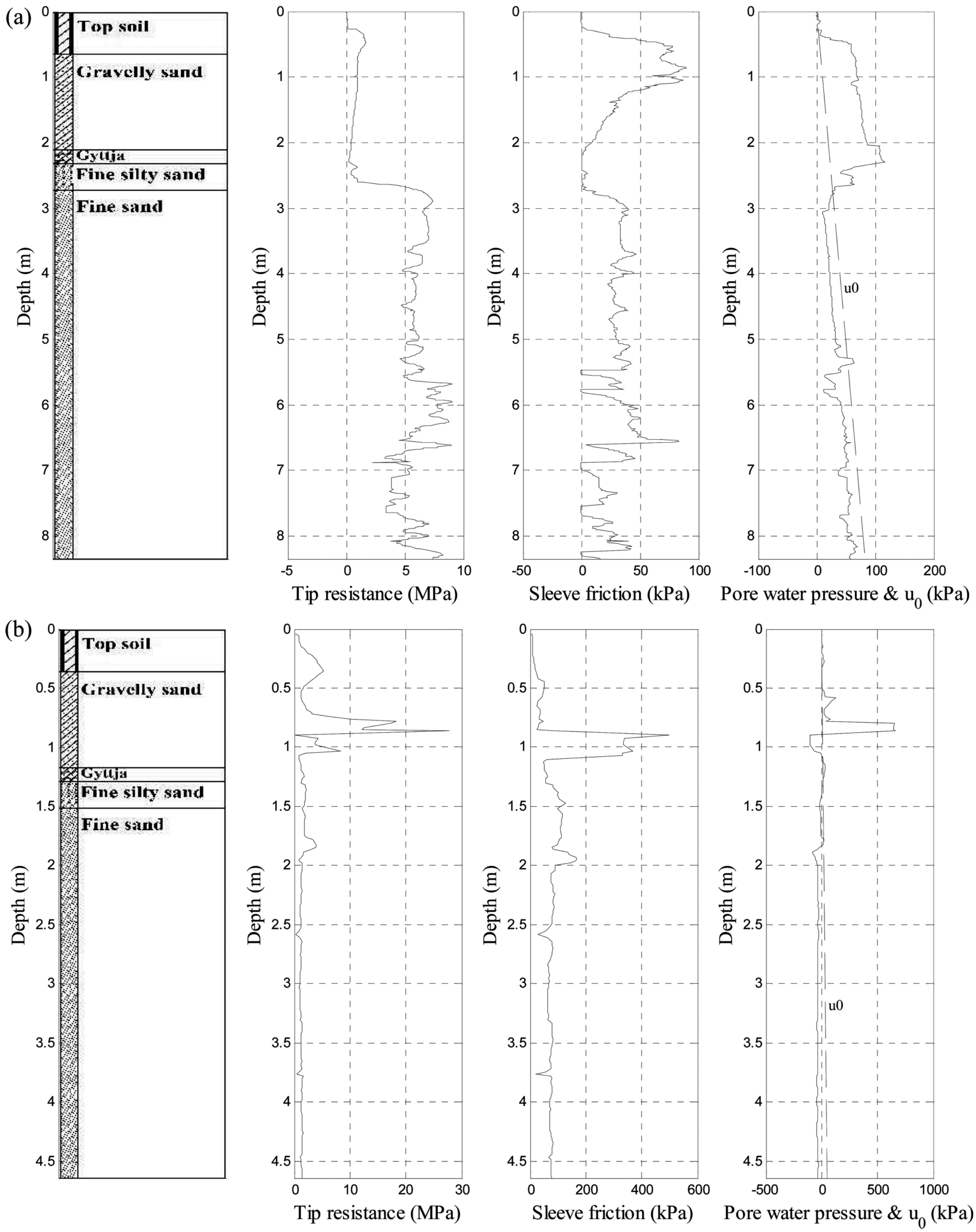
where q_t is the measured resistance, and σ_{v0} and σ'_{v0} are the total and effective overburden pressures estimated by the density of the soil and cone depth at the measured data, respectively. The normalized pore pressure (also known as the pore pressure ratio in the soil classification chart by Robertson 1990) is defined as follows:

$$(4)\qquad B_q = \frac{u_2 - u_0}{q_t - \sigma_{v0}}$$

where u_0 is the initial pore pressure and u_2 is the pore pressure measurement at the back of the cone penetrometer.

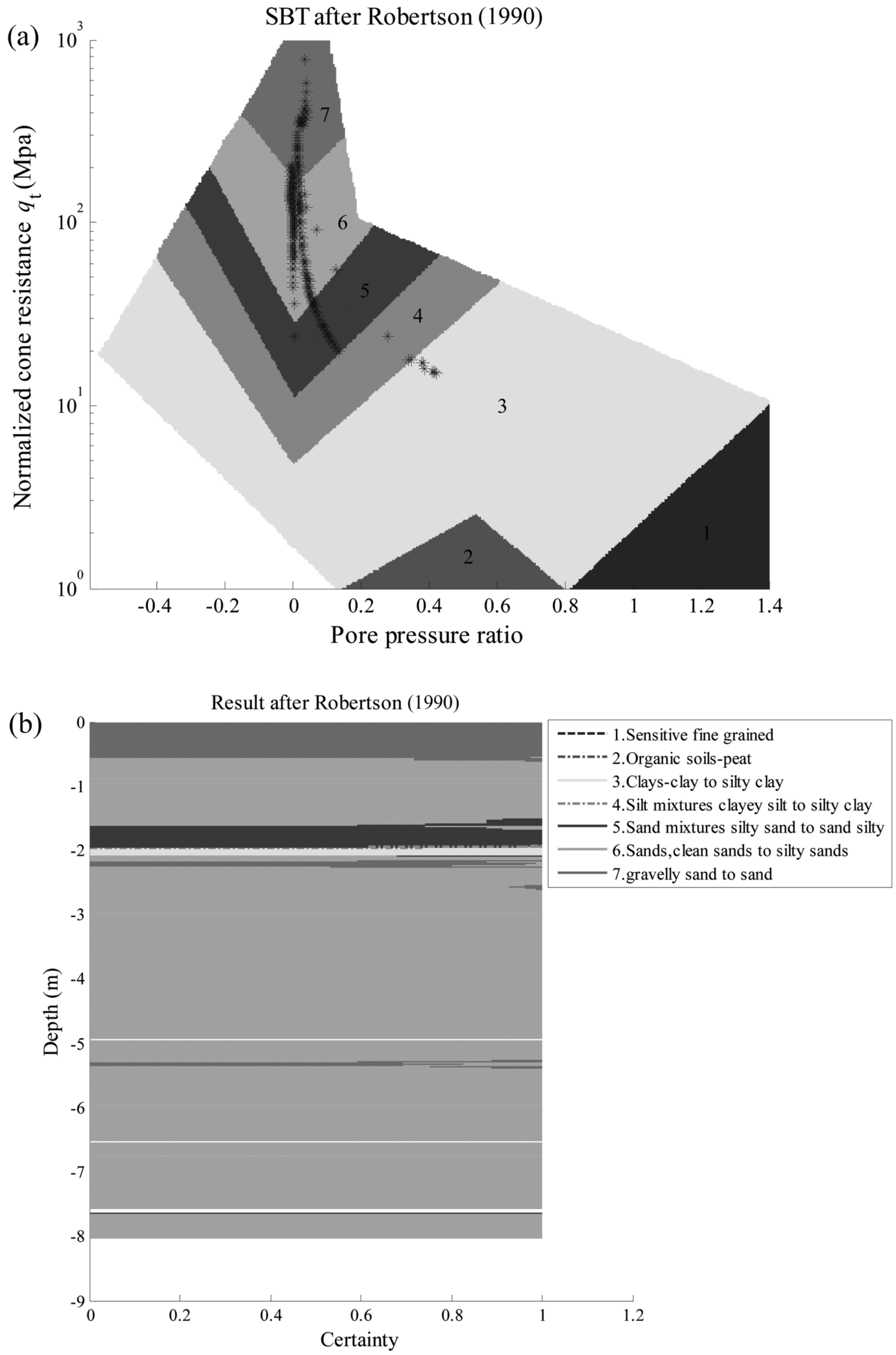
Regarding the certainty with which a classification method (in this case the method proposed by Robertson 1990) identifies a given soil type, a distinction must be made between two aspects. Firstly, a classification method may be more or less accurate in identifying a soil type. This may be regarded as a model uncertainty, and as proposed by Firouzianbandpey et al. (2012) this uncertainty can be identified by comparing the classification made from the CPT from a classification made from soil samples taken within a bore hole. The present work does not consider this kind of uncertainty, but previous studies indicate that the method provides a reliable overall identification of soil types for regions with the considered type of soil deposits.

Fig. 4. Representative CPT profiles obtained at (*a*) Aalborg site and (*b*) Frederikshavn. u_0, hydrostatic pore pressure induced by water level surface of the region.



Secondly, each CPT classification method is based on the idea that combinations of raw CPT data (in this case pore pressure and cone resistance) falling within a "zone" in the classification diagram are related to a particular type of soil. However, the borders between these "zones" are subject to uncertainty in the sense that the engineer has to decide whether a given data point is in one zone or another. One idea would be to define a borderline of no extent such that it is uniquely defined to which zone a given data point belongs. However, in this paper another approach is considered. Thus, the classification diagram is digitalized (in this case a resolution of about 2000 × 2000 pixels is used) and a window is defined around a given data point as illustrated in Fig. 7. The size

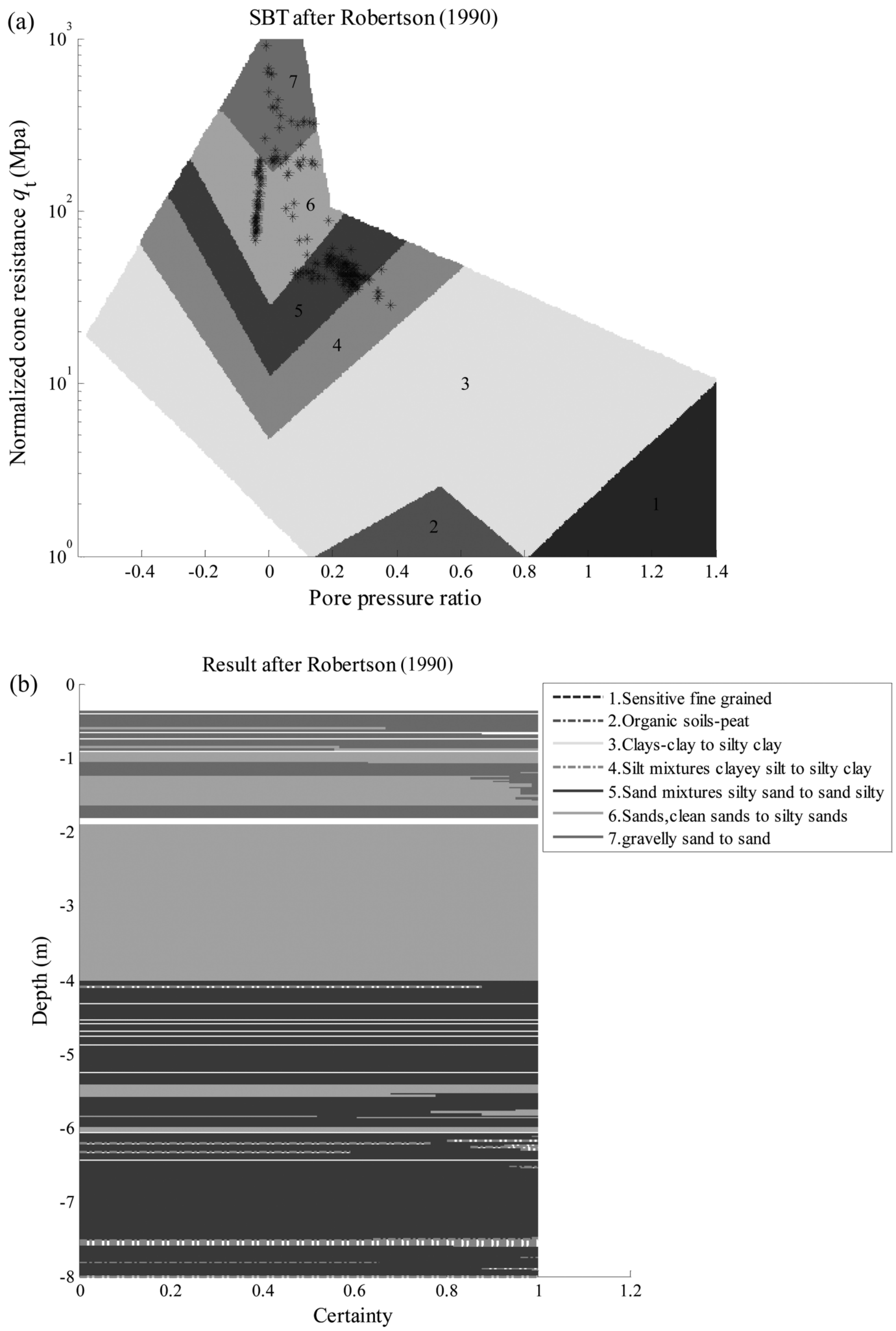
Fig. 5. Soil classification results of a representative CPT sounding (CPT No. 5) performed at the Aalborg site (after Robertson 1990): (*a*) Soil behavior type (SBT) classification from CPTu data, using the method of Robertson (1990). Zones 1 to 7 are: 1, sensitive, fine-grained soils; 2, organic soils and peat; 3, clays, clay to silty clay; 4, silt mixtures, silty clay to clayey silt; 5, sand mixtures, sandy silt to silty sand; 6, sands, silty sand to clean sand; 7, sand to gravelly sand. (*b*) Possible soil type for each measurement versus depth. The shades of grey are the same as the zones in Fig. 5*a*.



of the window is a heuristic choice, however, the basic idea is that it should be chosen such that it reflects the uncertainty related to an engineer performing a visual interpretation of the printed diagram. A larger window implies that the soil types assigned to measurements near zone boundaries are identified with less confidence, and more candidates for the underlying soil type may be proposed by the classification method.

The size of the window is chosen to be 9 × 9 pixels in the present study, but for comparison, Fig. 7 illustrates the principle for a smaller window of the size 5 × 5 pixels as well. The number of pixels in the window occurring in each zone is now counted and compared to the total number of pixels in the window. In this context, the ratio between the number of pixels in the window residing in a given zone and the number of total pixels in the

Fig. 6. Soil classification results of a representative CPT sounding (CPT No. 8) performed in Frederikshavn site (after Robertson 1990): (*a*) Soil behavior type classification from CPTu data, according to Robertson (1990). The soil types corresponding to zones 1 to 7 are the same as defined in the caption of Fig. 5*a*. (*b*) Possible soil type for each measurement versus depth. The shades of grey are the same as the zones in Fig. 6*a*.

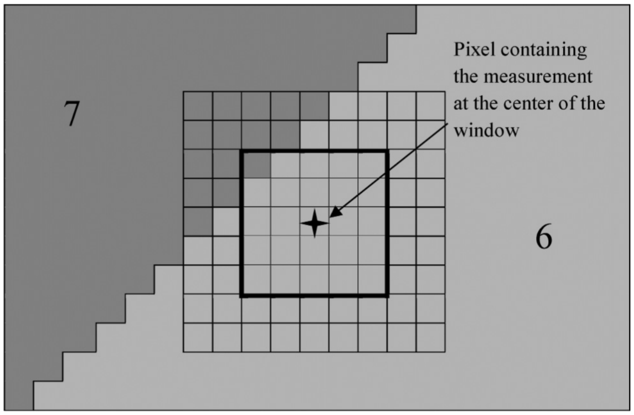


window provides a number that may be regarded as a classification certainty related to a given classification method. The terminology "classification certainty" will be used in what follows in accordance with this definition. As an example, for the data point indicated in Fig. 7, the classification certainty related to the green zone 6 becomes 24/25 = 0.96 for the small window and 66/81 = 0.81 (i.e., the classification certainty related to the zone in wherein the data point resides decreases with increasing size of the window).

Next, Figs. 5*b* and 6*b* show the soil profile resulting from this method. For each depth, a horizontal line is drawn and divided into segments proportional to the certainty of each soil type in the window. The most certain soil type is plotted first, so the left vertical axis represents the most likely form of the soil profile. When the certainty is 1, the input data measurement is located well within a zone, far away from the boundaries.

It is evident from Figs. 5 and 6 that the soil types predicted by the 1990 Robertson chart method are mostly sand, sand mixtures,

Fig. 7. 5 × 5 and 9 × 9 pixel windows for determination of classification certainty related to a CPT data point.



and gravelly sand. These results agree with the classification of soil samples retrieved from the boreholes.

Coefficient of variation (COV) of cone data in vertical direction

Using data from sub layers identified as spatially homogeneous by the soil-behavior-type classification system suggested by Robertson (1990), the coefficients of variation of the CPT data in the Aalborg site and Frederikshavn site have been estimated. Tables 1 and 2 provide the mean and coefficient of variation of the "normalized cone resistance" and "normalized friction ratio" at the two different sites. All CPT data have been classified based on the Robertson (1990) chart. Then the data points that were classified in the same zone were counted. The total number of samples is the number of CPT data obtained from the soundings.

Table 1 shows that the gravelly sand layers of the Frederikshavn site have much higher coefficient of variation (COV) values between the different soundings than other soil types, for both cone resistance and sleeve friction. At the Aalborg site, the sleeve friction is always highly variable, but the COV values for cone resistance are higher in gravelly sands than in other layers.

Silt mixtures exhibit the most consistent cone resistance measurements at both sites (low COV). The soil type with the lowest sleeve friction variability is silty sands and sand mixtures in Frederikshavn, but clayey silt layers in Aalborg.

Since COV is an indicator of the degree of variation in soil properties across the site, in soil layers with relatively high COVs, there is more expectation for variability of the soil parameters.

Correlation structure of the field

To characterize a random field, knowledge about how rapidly the field varies in space is needed. This is captured by the second moment of the field's joint distribution, which is expressed by the covariance function,

$$C(t', t^*) = \mathrm{Cov}[X(t'), X(t^*)] = E[X(t')X(t^*)] - \mu_X(t')\mu_X(t^*) \tag{5}$$

where $\mu_X(t)$ is the mean of X at the position t. A more meaningful measure about the degree of linear dependence between $X(t')$ and $X(t^*)$ is the correlation function

$$\rho(t', t^*) = \frac{C(t', t^*)}{\sigma_X(t')\sigma_X(t^*)} \tag{6}$$

where $\sigma_X(t)$ is the standard deviation of X at the position t.

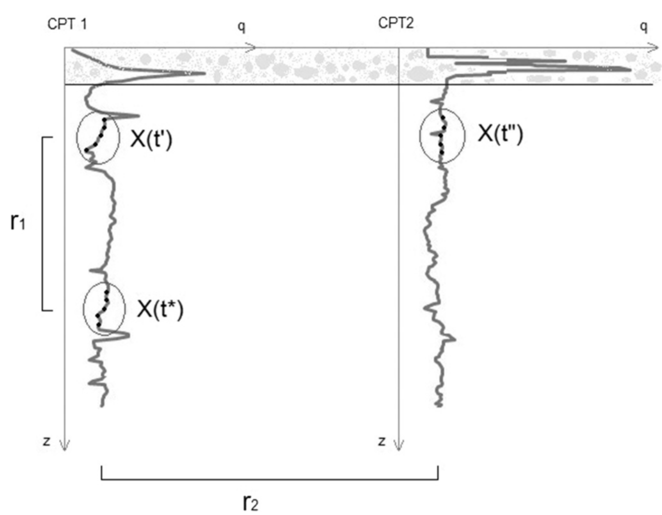
Table 1. Results of spatial variability of cone data for different soil layers at the Aalborg site.

		Soil type (Robertson 1990)			
Cone parameter	Statistical parameter	Gravelly sand	Sands, clean sands	Silty sand, sand mixtures	Clayey silt to silty clay, silt mixtures
Normalized cone resistance	Number of samples	1109	2399	272	229
	COV (%)	61	56	23	11
	Mean	3.2	2.7	0.1	0.04
Normalized friction ratio	Number of samples	1109	2399	272	229
	COV (%)	60	60	81	53
	Mean	0.03	0.01	0.004	0.001

Table 2. Results of spatial variability of cone data for different soil layers at the Frederikshavn site.

		Soil type (Robertson 1990)			
Cone parameter	Statistical parameter	Gravelly sand	Sands, clean sands	Silty sand, sand mixtures	Clayey silt to silty clay, silt mixtures
Normalized cone resistance	Number of samples	382	1999	1110	43
	COV (%)	82	28	15	12
	Mean	2.69	0.6	0.31	0.16
Normalized friction ratio	Number of samples	382	1999	1110	43
	COV (%)	51	22	2	32
	Mean	0.08	0.09	1	0.03

Fig. 8. Schematic view of an ideally horizontal homogeneous sublayer of soil and data points in the vertical and horizontal directions.



A commonly applied model of the correlation function for soil parameters is a single exponential curve (e.g., Vanmarcke 1977; DeGroot 1996; DNV 2010).

In this study an attempt is made to estimate the spatial correlation length of normalized cone data in the horizontal and vertical directions by fitting an exponential model to the results (correlation coefficients of normalized cone data versus distance). This method was preferred to the maximum likelihood method because of the lack of information about the model function of the error.

Fig. 9. Vertical correlation coefficient of q_{c1N} for (*a*) a representative sounding (CPT No. 8) and (*b*) for all soundings in sand layer (Frederikshavn site).

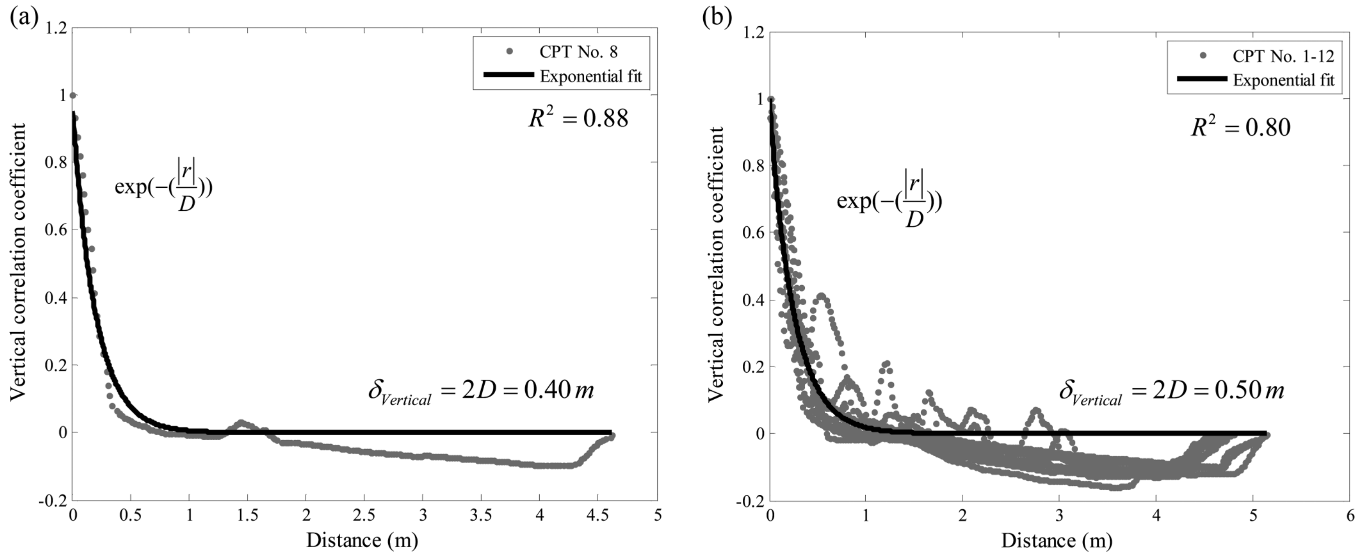
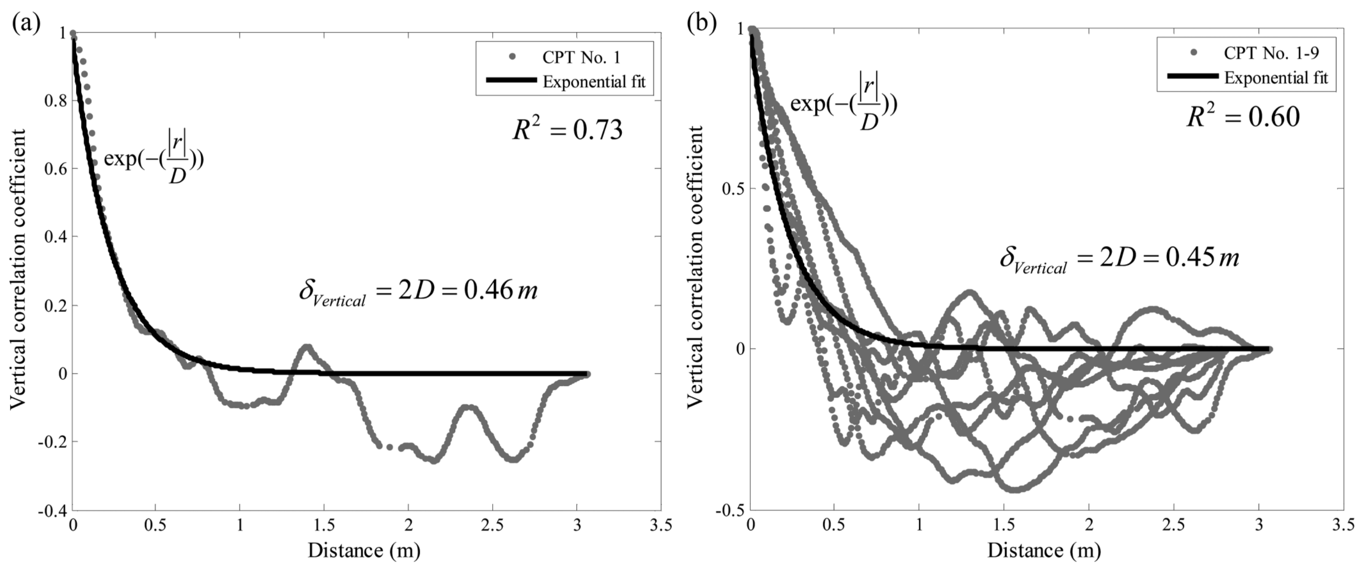


Fig. 10. Vertical correlation coefficient of q_{c1N} for (*a*) a representative sounding (CPT No. 1) and (*b*) for all soundings in sand layer (Aalborg east site).



In the vertical direction, each sounding is considered as a set of data with a number of CPT points. Then five measurements in each homogeneous sublayer were taken and the correlation coefficient of them was calculated by eq. (6). The distance *r* was the difference between two sequences of depth in the position of mean value of each data set (Fig. 8). By fitting an exponential model to these points, the *D* parameter was estimated to give the value of spatial correlation length. In the horizontal direction, the same approach was employed, but where *r* was the distance between two sounding locations.

By assuming two sampling zone with the same length (i.e., x_i, $i = 1, 2, \ldots, n$ and y_i, $i = 1, 2, \ldots, n$), the sample means and sample variances have been estimated and sample correlation coefficients are found with eq. (6).

To convert the data set to a random field with a stationary mean and variance, eq. (7) is used:

$$x_i' = \frac{x_i - \overline{X}}{S_X} \quad y_i' = \frac{y_i - \overline{Y}}{S_Y} \tag{7}$$

The correlation coefficients are calculated between one fixed cone penetration test and the remaining tests. To incorporate as many different intervals between the cone penetration tests, all data sets are fixed in turn (with no repetition).

This was also done for the horizontal direction except that each 5 data points were taken from each sounding, and as shown in Fig. 8, the distance between data points was the distance between CPT soundings (r_2).

Spatial correlation length (vertical direction)

The spatial correlation length, also known as the scale of fluctuation, is a concise indicator of the variability of a strongly correlated domain. There are various techniques available in the geotechnical literature for the estimation of the spatial correlation length. Vanmarcke (1977) approximated correlation functions and corresponding correlation lengths of residuals by use of these common models. For example, using the exponential model, DeGroot and Baecher (1993) estimated the horizontal correlation length of undrained shear strength in a soft marine clay

Fig. 11. Vertical correlation coefficient of F_R for (*a*) a representative sounding (CPT No. 2) and (*b*) for all soundings in the sand layer (Frederikshavn site).

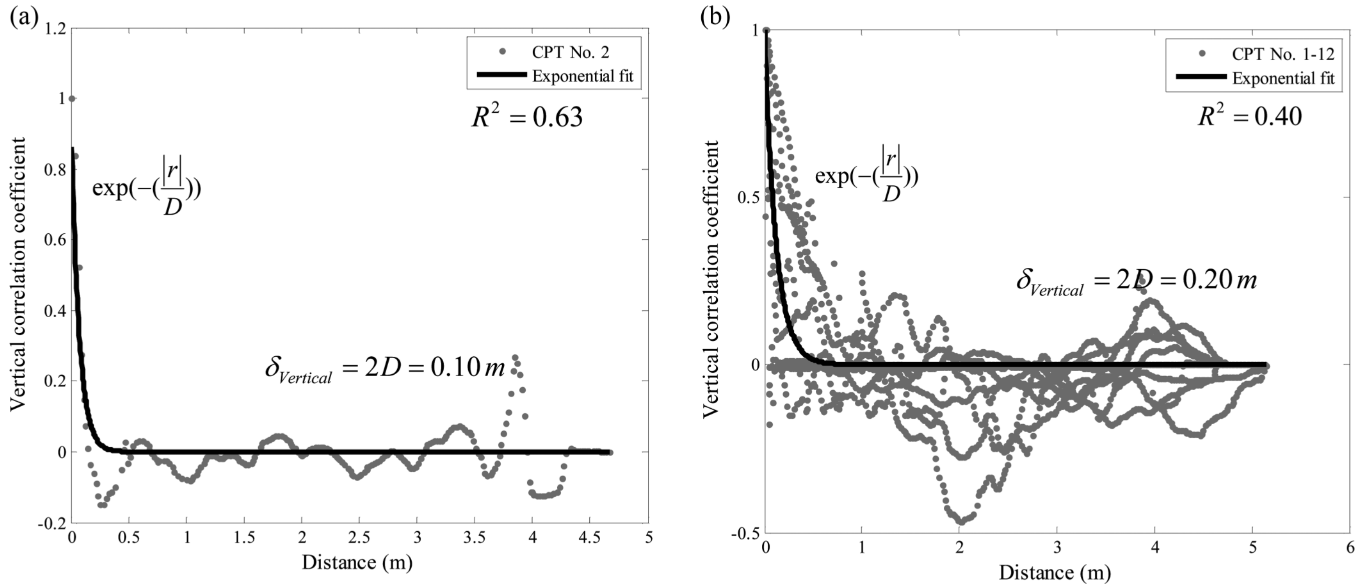
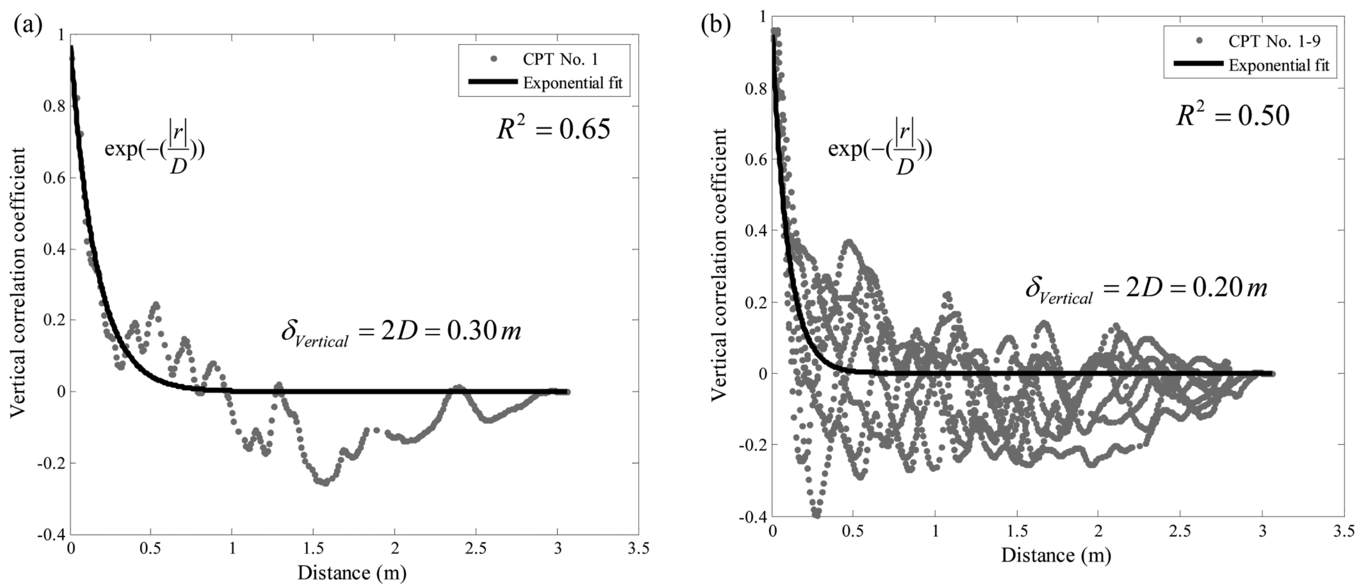


Fig. 12. Vertical correlation coefficient of F_R for (*a*) a representative sounding (CPT No. 1) and (*b*) for all soundings in the sand layer (Aalborg site).



layer, and Tang (1979) employed the quadratic exponential model to estimate the horizontal correlation length of cone resistance of CPT data in a marine clay layer.

Figures 9 and 10 display the correlation coefficient functions and spatial correlation lengths of vertical CPT tip resistance data in all sand layers at a given site. The resistance data are normalized by the vertical effective stresses, as described in eq. (1). The best-fit exponential models for each function are determined using regression analysis. The result of a single representative CPT is shown in Figs. 9*a* and 10*a*, while the exponential model that best fits all correlation coefficient functions from a site is shown in Figs. 9*b* and 10*b*.

The vertical spatial correlation lengths of the cone tip resistance are similar at the two sites. At the Frederikshavn site, the parameter D of the best-fit exponential model is 0.25 m, so the spatial correlation length is 0.5 m. At the Aalborg site, the spatial correlation length is 0.45 m.

The R^2 parameter is also estimated in each case, showing that the model is a reasonable fit to the data, although for the Aalborg site, the scatter in the data are greater.

The spatial correlation length of the normalized friction ratio in the vertical direction in each site is also estimated by fitting the exponential model proposed by Vanmarcke (1977) in Table 3. The vertical spatial correlation length of the friction ratio is 0.2 m at both sites, as illustrated in Figs. 11 and 12. The low values of the R^2 parameter show a greater scatter of data in the sand layer.

Figure 13 plots the spatial correlation lengths obtained by fitting exponential models to each individual CPT sand layer profile against the corresponding mean values of q_{c1N} and F_R. Figure 13*a* shows that the spatial correlation lengths and mean values of cone tip resistance (q_{c1N}) fall into similar ranges at the two sites. However, Fig. 13*b* shows that the mean values of F_R in the Frederikshavn site are greater than in the Aalborg site. The latter is mostly of a clean sand type while the former also contains silty

Fig. 13. Vertical spatial correlation lengths of (*a*) q_{c1N} and (*b*) F_R, plotted against the mean, for the Frederikshavn and Aalborg sites.

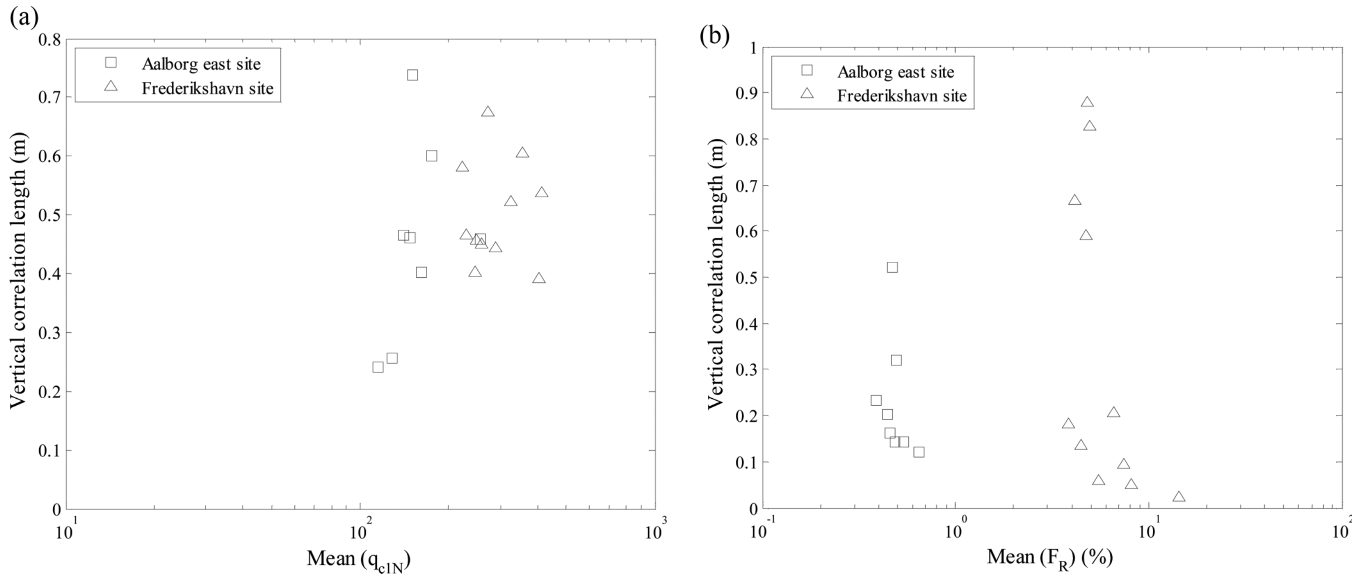


Table 3. Results of spatial variability of cone data for sand layer in the horizontal direction (Aalborg site).

Cone parameter	Statistical parameter	Layer thickness (m) 0–1	1–2	2–3	3–4	4–5
Normalized cone resistance	Number of samples	50	50	50	50	50
	COV (%)	28	32	26	31	36
	Mean	52.75	56.94	42.93	48.04	46.93
Normalized friction ratio	Number of samples	50	50	50	50	50
	COV (%)	24	16	24	35	30
	Mean	0.5	0.5	0.46	0.51	0.46

Table 4. Results of spatial variability of cone data for sand layer in the horizontal direction (Frederikshavn site).

Cone parameter	Statistical parameter	Layer thickness (m) 0–1	1–2	2–3	3–4	4–5
Normalized cone resistance	Number of samples	50	50	50	50	50
	COV (%)	29	30	26	45	95
	Mean	14.91	15.64	13.56	25.82	62.25
Normalized friction ratio	Number of samples	50	50	50	50	50
	COV (%)	53	37	70	27	100
	Mean	3.96	4.32	5.54	4.48	20.07

sands, and this may not come as a surprise given that the friction ratio increases by increasing the fine content.

In both sites, the lower and upper bounds of the spatial correlation length in F_R are greater than the bounds in q_{c1N}. The absence of a strong correlation structure in F_R is justified by the fact that sleeve friction measurements are inherently much more erratic than tip resistance measurements. The sleeve friction is affected only by adjacent soil, while q_{c1N} is influenced by a volume of soil around the cone tip that is larger than the sampling interval. Therefore, a few consecutive values of q_{c1N} are affected by the same volume of soil as the cone penetrates.

Spatial correlation length (horizontal direction)

This section estimates horizontal correlation coefficient structures using the CPT datasets collected at Aalborg and Frederikshavn. Tables 3 and 4 summarize the horizontal variability of normalized CPT measurements in one-metre interval layers obtained from the last five metres of soundings, which consist mostly of sand and silty sand at both sites. The COV values of normalized cone resistance and friction ratio measurements taken at the Aalborg site are all in a fairly narrow range. In Frederikshavn, however, the COV values are much higher and more variable. In particular, the last metre has a COV larger than 95% for both CPT measurements, testifying to the high variability of this region.

Note that each sublayer with one metre thickness is considered a separate statistical domain with limited data. The correlation coefficient of data in each layer was estimated with a distance as described earlier and regression analysis was employed to determine the *D* parameter and spatial correlation length as proposed by Vanmarcke (1977) in the exponential model. Figures 14–17 show the correlation coefficient of normalized cone data versus relative distance in two sites and the best fit exponential curves. By estimating the *D* parameters of the model, the average spatial correlation length of the normalized cone resistance is 2 m in horizontal direction for the Aalborg site, and 1.2 m for the Frederikshavn site. The average horizontal spatial correlation lengths of the normalized friction ratio are 1.2 and 1.4 m for the Aalborg

Fig. 14. (*a*) Horizontal correlation coefficient of q_{c1N} for all soundings in the sand layer. (*b*) Fitting an exponential curve to the mean values (Aalborg site).

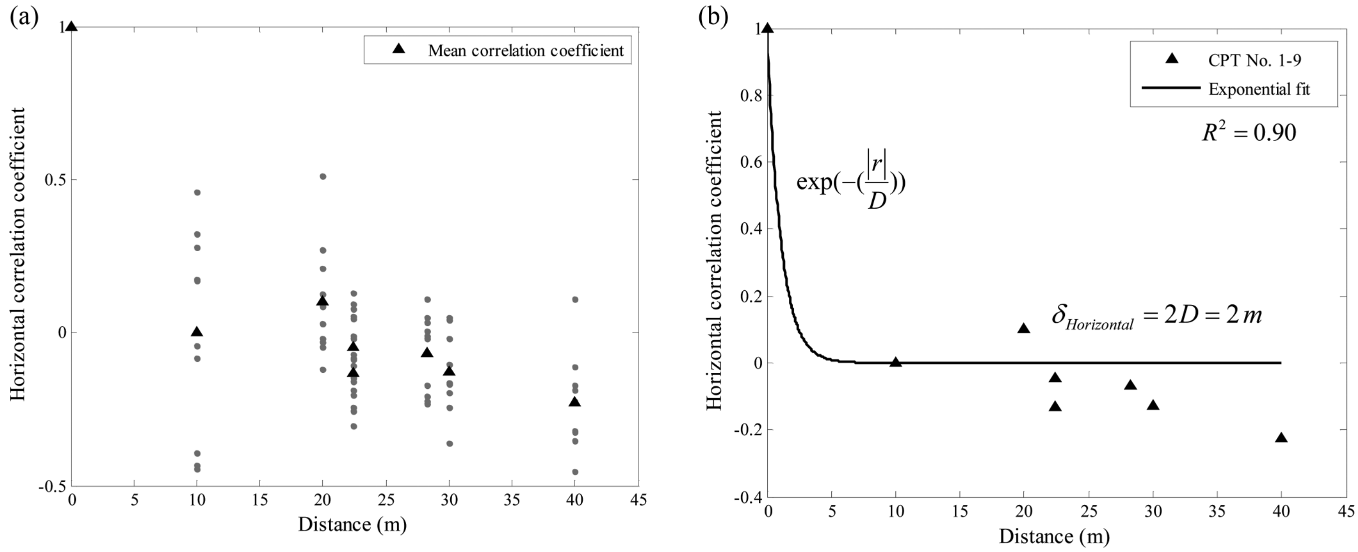
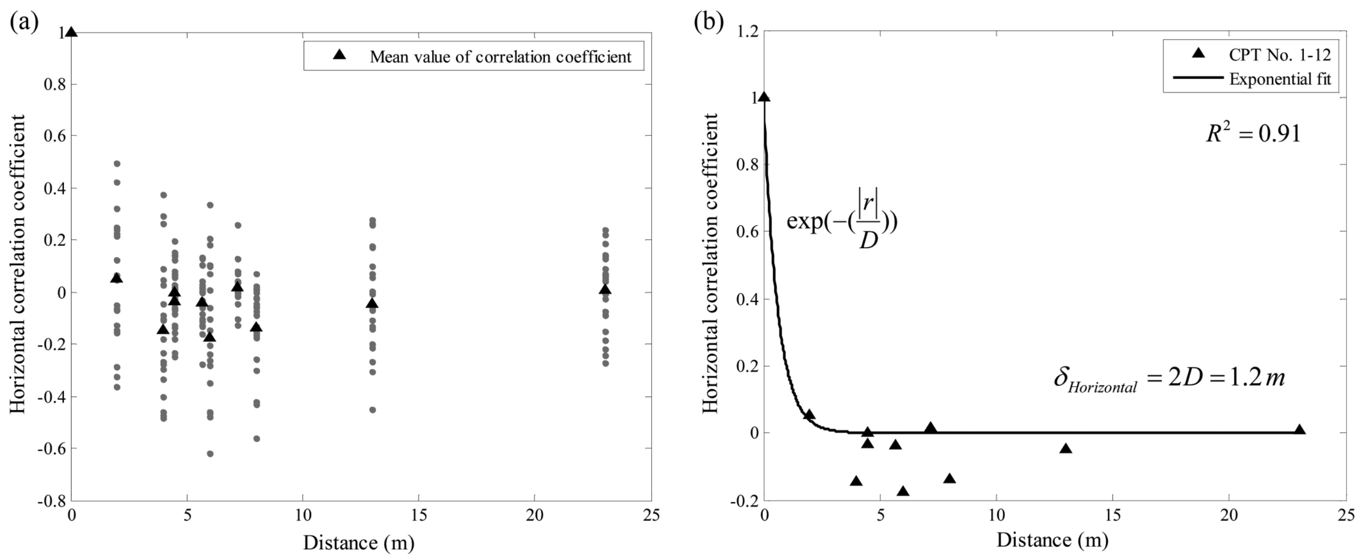


Fig. 15. (*a*) Horizontal correlation coefficient of q_{c1N} for all soundings in the sand layer. (*b*) Fitting an exponential curve to the mean values (Frederikshavn site).



and Frederikshavn sites respectively. The values of R^2 near 0.90 indicates that the regression curve fits the data quite well in all charts.

Summary of results and discussions

For both sites, the values of correlation length for normalized cone resistance and friction ratio are summarized in Table 5. What we have in Table 5 is the correlation length estimated after fitting the model to all soundings (in the vertical direction) and all layers in the horizontal direction. These values are the mean of the correlation length in each direction. Table 5 indicates anisotropic spatial correlation lengths, with the vertical spatial correlation length being anywhere from two to seven times shorter than that in the horizontal direction.

Comparing the values of correlation length and distance between sample points in both directions reveals a high variability in the random field. This can be expressed in terms of inhomogeneities of the soil deposit because of the inclination of soil layers. Under the simplifying assumptions that the soil layers are ideally horizontal and homogenous, we did not consider the big influence of inclined soil layers on the results. If there is an inclination in the soil layers, then the data points taken from different layers show a poor correlation. This can happen in the horizontal as well as the vertical directions. As shown in Fig. 18, the measured data at the border between two layers of sand and clay may show more correlation than points that are in the same level as they are. The consideration of inclined layers will be let for future work.

Conclusion

This paper considered soil variability in CPT test data from two different sites in the north of Denmark. The cone resistance and sleeve friction measurements were first normalized for vertical stress, and then used to identify homogonous sublayers following the soil classification system proposed by Robertson (1990). First, the COV values of normalized cone data in both directions were determined. For both sites, the vertical COV values of normalized cone resistance are higher in coarse-grained sands than in fine soil. This is also the case for the normalized sleeve friction in the

Fig. 16. (*a*) Horizontal correlation coefficient of F_R for all soundings in the sand layer. (*b*) Fitting an exponential curve to the mean values (Aalborg site).

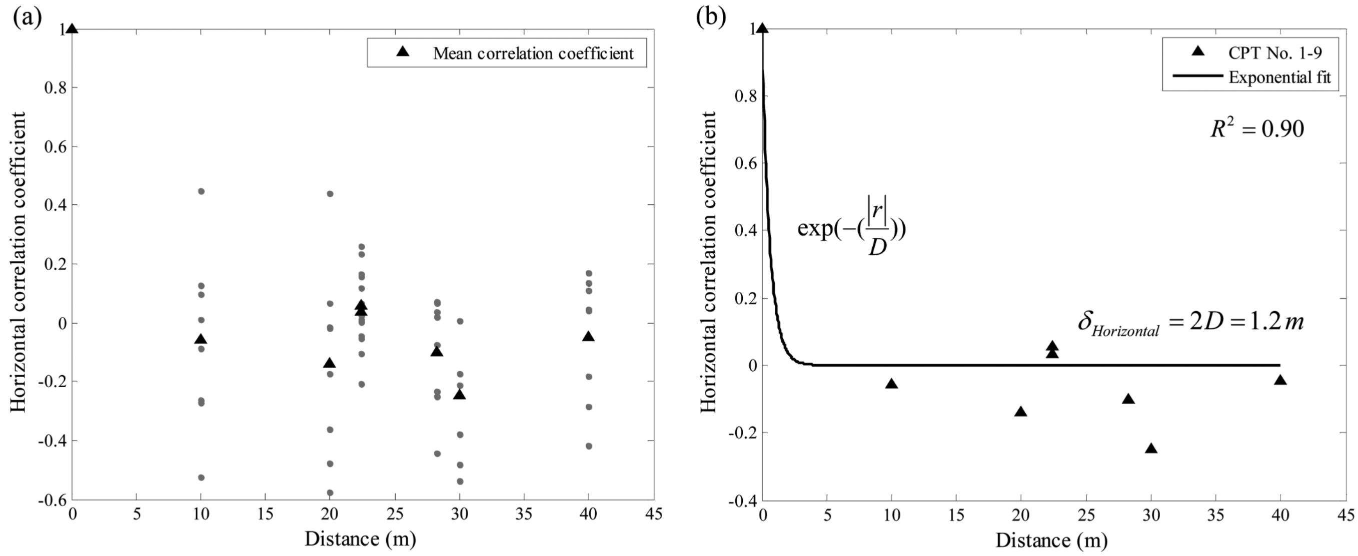


Fig. 17. (*a*) Horizontal correlation coefficient of F_R for all soundings in the sand layer. (*b*) Fitting an exponential curve to the mean values (Frederikshavn site).

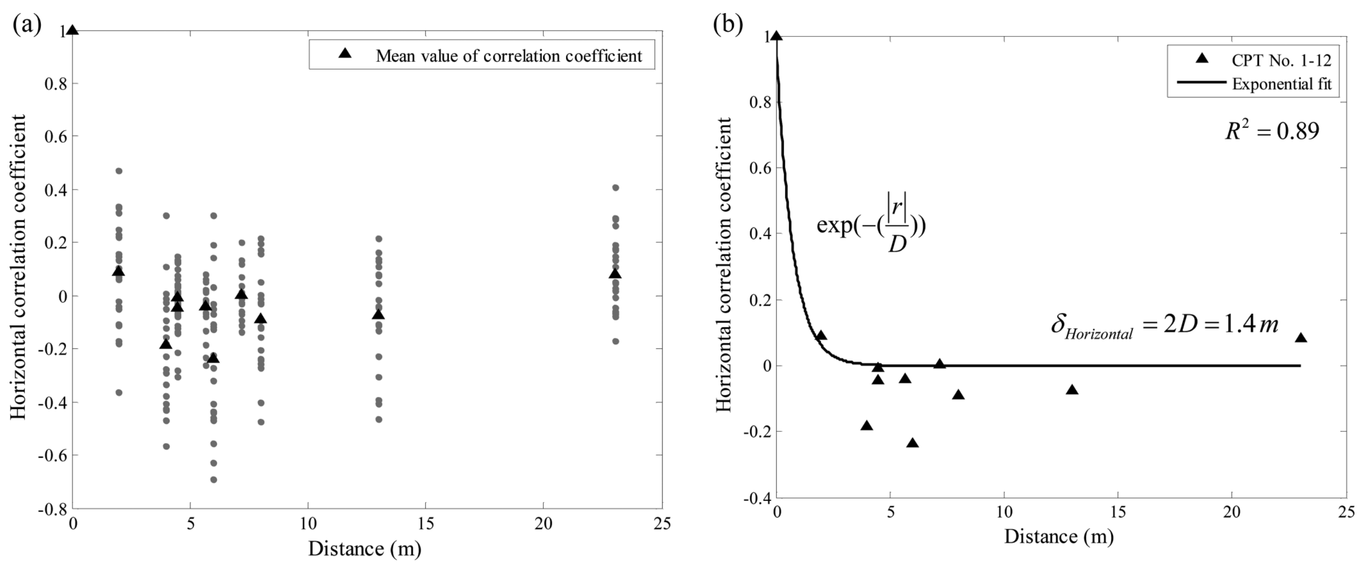


Table 5. Mean values of spatial correlation length in vertical and horizontal direction at both sites.

Site	Cone parameter	Vertical correlation length (mean value)	Horizontal correlation length (mean value)
Frederikshavn	Normalized cone resistance (q_{c1N})	0.5	1.2
	Normalized friction ratio (F_R)	0.2	1.4
Aalborg	Normalized cone resistance (q_{c1N})	0.45	2.0
	Normalized friction ratio (F_R)	0.2	1.2

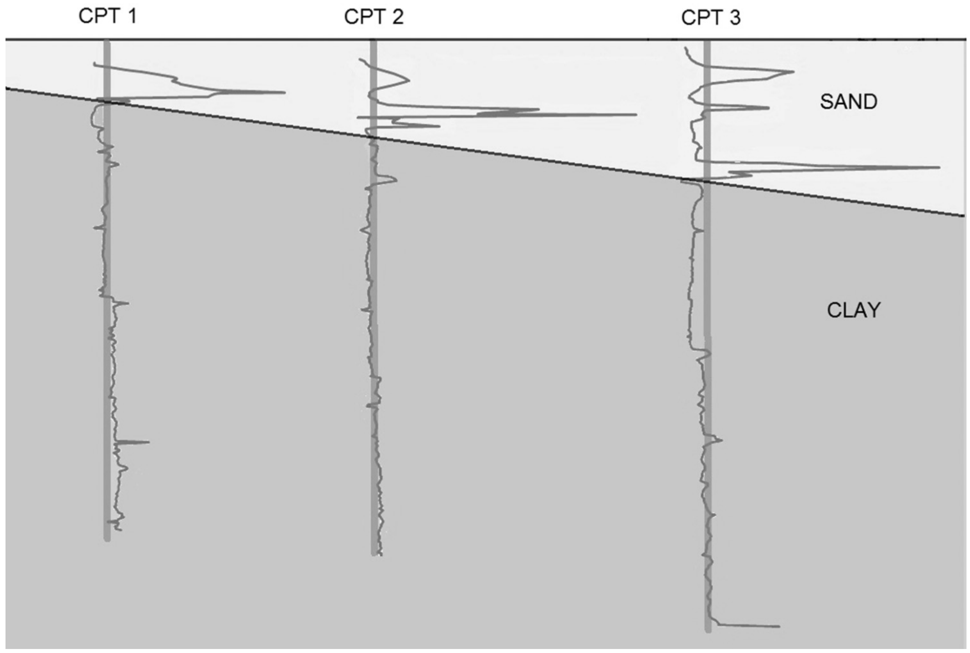
Aalborg site. However, no such trend exists in the vertical COV values of normalized sleeve friction in Frederikshavn. The expectation that very fine-grained soil layers tend to have less variation of soil parameters than granular soil layers is consistent with this finding.

In the horizontal direction, the maximum COVs occur in the last layer of sand deposits in the Frederikshavn site. This effect may be because of thinly interbedded silt mixtures within the layer (Fig. 7*b*).

To more precisely characterize the spatial variability of the cone resistance and sleeve friction, the autocorrelation function with respect to physical distance was calculated on measurements from sand layers. The spatial correlation distance for each variable was estimated by fitting a simple exponential model to these data.

To describe a random field concisely in the second moment sense, the spatial correlation length and the coefficient of variation in both directions were estimated. For this purpose, the average coefficient of variation of cone data in the vertical and

Fig. 18. Example of CPT soundings in an inclined soil layer.



horizontal directions was calculated in both sites, and regression analysis was employed to estimate the spatial correlation length.

Because of natural deposition and soil formation processes, the vertical and horizontal correlation structures from the cone results indicated significant anisotropy, with the vertical length being anywhere from two to seven times shorter than that in the horizontal direction. Also in the vertical direction, it was observed that q_{c1N} is more spatially correlated than F_R, with spatial correlation lengths estimated in the range of 0.5 m and 0.2 m respectively. The same is true in the horizontal direction. The physical meaning of this observation is that the volume of soil around the cone tip that influences q_{c1N} is larger than the sampling interval. As the cone penetrates, several consecutive values of q_{c1N} are affected by almost the same volume of soil, while F_R is affected only by the local soil adjacent to the cone.

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List of symbols

A_c projected area of the cone
A_s surface area of the friction sleeve

B_q pore water pressure ratio
COV coefficient of variation
C_Q correction for overburden stress
$C(t)$ covariance function of variable t
D model parameter
F_R normalized friction ratio
F_s sleeve friction
f_s measured sleeve friction
i data point
n correction value for overburden pressure (0.5, 0.7, 1.0 for cohesionless, intermediate, and cohesive soils, respectively)
P_a reference pressure in compatible unit in the equation
Q_c total force acting on the cone
Q_t normalized tip resistance
q_c measured cone tip resistance
q_{c1N} normalized cone resistance
q_t measured resistance
R^2 regression parameter (fit-goodness)
r_1 vertical separation distance between set of data in each sounding
r_2 horizontal separation distance between soundings
S_X, S_Y variance of the sample zones
u_0 initial pore water pressure
u_2 pore water pressure measurement at the back of the cone penetometer
$\overline{X}$, $\overline{Y}$ mean of the sample zones
$X(t)$ observation point at the position X
x_i, y_i sample zones
x'_i, y'_i conversion of sample zones to a stationary random field
$\delta_{\mathrm{Vertical}}$, $\delta_{\mathrm{Horizontal}}$ vertical and horizontal correlation length
$\mu_X(t)$ mean of X at the position t
$\rho(t)$ correlation function
$\sigma_X(t)$ standard deviation of X at the position t
σ_{v0} total vertical stress
σ'_{v0} effective vertical stress