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TECHNICAL NOTE

A general solution for 1D consolidation induced by depth- and time-dependent changes in stress

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The paper describes a general analytical solution for obtaining the excess pore pressure in a consolidating layer due to depth and time-dependent changes of total stress, given by a simple product of depth- and time-dependent functions. The solution is validated against three previously published solutions, and the paper ends with analysis of a case history involving settlement and consolidation beneath a periodically loaded silo. Good agreement is observed between the analytical solution and measured settlements, and the transient response is also validated against a finite-element solution.

KEYWORDS: consolidation; finite-element modelling; pore pressures; settlement; time dependence

INTRODUCTION

Terzaghi (1943) provided a classical one-dimensional (1D) consolidation theory based on the assumption that an external surface load is instantaneously applied, is held constant with time and causes a uniform increase in total stress with depth. In practice, external surface loads may be time-dependent, and owing to stress distribution effects, may also cause increases in total stress that vary with depth. To analyse time-dependent loading, a graphical construction method was suggested by Terzaghi (1943), and Schiffman & Stein (1970) presented an analytical solution to the layered consolidation problem for different boundary conditions and an arbitrary load–time history. Olson (1977) developed analytical solutions of 1D consolidation of soil under ramp loading and, more recently, the response of layered soil was re-examined by Huang & Griffiths (2010) using a finite-element approach. Taking compressibility of the pore fluid into account, Rahalt & Vuez (1998) developed a solution for settlement and pore pressure induced by sinusoidal loading and Conte & Troncone (2006, 2008) expanded a general time-dependent loading as a Fourier series, and superposed the solutions of each harmonic. The method was used to obtain the transient response of saturated soil layers to general loading variations at the surface. Building on this, Razouki & Schanz (2011) studied a 1D consolidation process under sinusoidal loading with and without rest periods. Razouki *et al.* (2013) presented and discussed an exact analytical solution of the non-homogeneous partial differential equation governing the conventional 1D consolidation under haversine repeated loading.

The above approaches all assumed that the increase in vertical total stress is uniform with depth. In field situations involving anything other than extensive loading, the increase in vertical total stress varies with depth due to stress distribution effects, and Taylor (1948) described several examples, including both linear and sinusoidal variation. Several solutions for this type of variation with different levels of complexity have been developed. For example, Singh (2008) developed diagnostic methods for simultaneously identifying the consolidation coefficient, final settlement and ratio of top to bottom excess pore-water pressures from observed settlements. Liu *et al.* (2012) studied 1D consolidation of an aquitard whose vertical total stress increased linearly with depth due to water table drawdown at the bottom of the aquitard, with an unchanged water table at the top. Lovisa *et al.* (2012) proposed generalised curve-fitting procedures which can be used to analyse laboratory or field settlement–time data and determine c_v for any case where a non-uniform excess pore-water pressure distribution is encountered.

Less work has been reported on changes in total stress that are both time- and depth-dependent. Zhu & Yin (1998, 1999) presented consolidation analyses for a soil layer subjected to ramp load varying linearly with depth, which was later expanded to include two soil layers. Li *et al.* (2012) used the finite-difference method to consider the change of vertical total stress with depth and time together in 1D consolidation of a double-layered soil with non-Darcian flow. Liu *et al.* (2014) derived analytical solutions for consolidation of marine clay under linear depth-varying and time-dependent load.

In the present paper, the authors present a general solution for the consolidation of soil under general depth-dependent and time-varying load. Validity and accuracy of the solution is verified by comparison with some published special cases of the proposed solutions. Finally, a case history is considered involving settlements generated by cyclic loading of a silo.

DEVELOPMENT OF THE GENERAL SOLUTIONS

Problem description

The general scheme of the 1D consolidation problem considered in this paper with depth- and time-dependent loading is shown in Fig. 1. As shown, H is the thickness of the soil layer, m_v is the coefficient of volume change and k_v is the vertical hydraulic conductivity of clay. The z-axis represents depth with the origin at the ground surface. The base of the layer is either drained or undrained, while the

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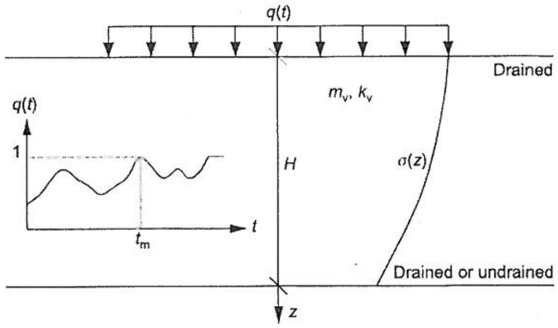


Fig. 1. Scheme of the consolidation problem

upper surface is always assumed to be drained. The increase in vertical total stress in soil is assumed to be the product of a function that depends only on depth and a function that depends only on time, and is assumed to have the following form

$$Q(z, t) = \sigma(z)q(t) \tag{1}$$

where $0 \leq q(t) \leq 1$ is a dimensionless function of time t, and $\sigma(z)$ represents the distribution of total stress with depth corresponding to $q(t) = 1$ occurring at time $t = t_m$. Stresses marked $\sigma_t = \sigma(0)$ and $\sigma_b = \sigma(H)$ represent, respectively, the maximum increases in vertical total stress at the top and bottom of the layer. Except for the depth- and time-varying vertical total stress aspects, the basic assumptions made in this analysis are the same as those made by Terzaghi in his classical 1D theory. (*a*) The soil layer is saturated and homogeneous. (*b*) Darcy's law is valid. (*c*) The coefficient of consolidation (c_v) is constant. (*d*) Compressibility of water and soil grains is negligible. (*e*) The flow of water is in the direction of compression. There are many limitations of the assumptions. For example, secondary consolidation is omitted, and it is only applicable to extensively loaded areas.

Taking the vertical total stress increase into account, the governing equation is given by

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} + \frac{\partial Q}{\partial t} \tag{2}$$

where $u(z,t)$ is the excess pore-water pressure of the soil layer; $c_v = k_v/(m_v\gamma_w)$ is the coefficient of consolidation of the soil layer and γ_w is the unit weight of water.

If the top surface is drained and the bottom surface is undrained (single-drained), the boundary conditions of the problem can be given as

$$u|_{z=0} = 0, \left.\frac{\partial u}{\partial z}\right|_{z=H} = 0 \tag{3}$$

while both surfaces are drained (double-drained), the boundary conditions are

$$u|_{z=0} = 0, u|_{z=H} = 0 \tag{4}$$

The initial condition of the problem is given by

$$u(z, 0) = Q(z, 0) = \sigma(z)q(0) \tag{5}$$

Solution for the single-drained case

By substituting equation (1) into equation (2), the Laplace transform of equation (2) can be rewritten as

$$s\tilde{u}(z, s) = c_v \frac{d^2\tilde{u}(z, s)}{dz^2} + s\tilde{q}\sigma(z) \tag{6}$$

where $\tilde{u}(z, s)$ is the Laplace transform of $u(z, t)$ with respect to time t written as $\tilde{u}(z, s) = L[u(z, t)] = \int_0^\infty u(z, t)e^{-st}dt$; $\tilde{q}$ is the Laplace transform of $q(t)$ with respect to time t written as $\tilde{q}(s) = L(q) = \int_0^\infty qe^{-st}dt$; and s is a complex number representing the frequency domain or Laplace-space variable.

The Laplace transform of equation (3) with respect to time t is given by

$$\tilde{u}|_{z=0} = 0 \tag{7a}$$

$$\left.\frac{d\tilde{u}}{dz}\right|_{z=H} = 0 \tag{7b}$$

In practical engineering, the distribution of total stress with depth $\sigma(z)$ generally satisfies Dirichlet Fourier series conditions. Fourier series expansion of $\sigma(z)$ is given as

$$\sigma(z) = \sum_{k=1}^{\infty} C_k \sin\left(\frac{M_k z}{H}\right) \tag{8}$$

where

$$C_k = \frac{\int_0^H \sigma(z)\sin(M_k z/H)dz}{\int_0^H \sin^2(M_k z/H)dz}, \quad M_k = \frac{2k-1}{2}\pi,$$

$$k = 1, 2, \ldots$$

By substituting equation (8) into equation (6), the Laplace transform of equation (6) can be rewritten as

$$s\tilde{u}(z, s) = c_v \frac{d^2\tilde{u}(z, s)}{dz^2} + s\tilde{q}\sum_{k=1}^{\infty} C_k \sin\left(\frac{M_k z}{H}\right) \tag{9}$$

Hence a general solution of equation (9) can be written as

$$\tilde{u} = A_1 \exp\left(\sqrt{\frac{s}{c_v}}z\right) + A_2 \exp\left(-\sqrt{\frac{s}{c_v}}z\right) + s\tilde{q}\sum_{k=1}^{\infty} D_k \sin\left(\frac{M_k z}{H}\right) \tag{10}$$

where A_1 and A_2 are coefficients to be determined and

$$D_k = \frac{C_k}{s + c_v(M_k/H)^2}$$

By substituting equation (10) into equation (7), the boundary conditions can be expressed as follows

$$0 = A_1 + A_2 \tag{11a}$$

and

$$0 = A_1\sqrt{\frac{s}{c_v}}\exp\left(\sqrt{\frac{s}{c_v}}H\right) - A_2\sqrt{\frac{s}{c_v}}\exp\left(-\sqrt{\frac{s}{c_v}}H\right) \tag{11b}$$

The coefficients A_1 and A_2 can be obtained from equations (11) as

$$A_1 = 0, \quad A_2 = 0 \tag{12}$$

By substituting equations (12) into equation (10), $\tilde{u}$ can be derived in generalised form as

$$\tilde{u} = \sum_{k=1}^{\infty} \tilde{G}_k(s)s\tilde{q}\sin\left(\frac{M_k z}{H}\right) \tag{13}$$

where

$$\tilde{G}_k(s) = \frac{C_k}{s + c_v(M_k/H)^2} \tag{14}$$

Substituting equation (14) into equation (13) and applying an analytical inversion of Laplace transform gives

$$u = \sum_{k=1}^{\infty} \int_0^t \frac{dq(\tau)}{d\tau} G_k(t-\tau)d\tau \sin\left(\frac{M_k z}{H}\right) + q(0)\sum_{k=1}^{\infty} G_k(t)\sin\left(\frac{M_k z}{H}\right) \quad (15)$$

where

$$G_k(t) = C_k \exp(-M_k^2 T_v) \quad (16)$$

$$T_v = \frac{c_v t}{H^2} \quad (17)$$

Solution for the double-drained case

A similar derivation procedure as that for the single-drained case can be used to solve the solution for the double-drained case. The solution of the double-drained case is the same as that of the single-drained case, namely, equation (15). However, M_k should be replaced by N_k, which is expressed as equation (18). Correspondingly, C_k is expressed as equation (19).

$$N_k = k\pi, \quad k = 1, 2, \ldots \quad (18)$$

$$C_k = \frac{\int_0^H \sigma(z)\sin(N_k z/H)dz}{\int_0^H \sin^2(N_k z/H)dz} \quad (19)$$

SOLUTION VERIFICATION AGAINST SOME SPECIAL CASES

Three special cases of Q(z, t) (Table 1)

In case 1, Zhu & Yin (1998) considered a linearly increasing time-dependent 'ramp' load with a linearly varying total stress distribution with depth. The ramp reached a maximum at $t_m = t_0$ and remained constant thereafter. In case 2, Rahalt & Vuez (1998) assumed a sinusoidal time-dependent cyclic load with a uniform total stress distribution with depth, and in case 3, Liu *et al.* (2014) considered a triangular time-dependent cyclic load with a linearly varying total stress distribution with depth.

Solutions of special cases

Substituting $\sigma(z)$ and $q(t)$ from Table 1 into equation (13) for single-drained cases (cases 1 and 3) or equation (16) for the double-drained case (case 2) yields the solutions listed in Table 2. It can be seen that solutions obtained by the present method are exactly the same as those developed by the previous studies as shown in Fig. 2. It may be noted that the spatial coordinate z used in the case 2 solution here is equivalent to $D - z$ (D is drainage path) in the paper written by Rahalt & Vuez (1998).

CASE STUDY OF CYCLIC SILO LOADING

In this section, the solution developed previously is applied to a case history presented originally by Favaretti & Mazzucato (1994). The case involves the time-dependent loading and unloading with corn of a rectangular silo of dimensions 45 × 71 m in the town of Ca' Mello near Porto Tolle, Italy. The subsoil consists of a layer of loose, silty sand overlying a thick layer of soft to very soft, silty clay. Below the clay is dense sand. In order to limit the absolute settlement and differential settlement of the silo during its lifetime, a preloading embankment wider than the silo was built on the site and maintained in place for 15 months. The silo was constructed soon after the embankment was removed. Settlement was measured during subsequent loading and unloading cycles of the silo.

Rahalt & Vuez (1998) assumed double-drained conditions (maximum drainage path $D = 7{\cdot}5$ m, total thickness of the consolidating layer, $H = 15$ m) and soft soil properties $c_v = 1{\cdot}5 \times 10^{-6}$ m²/s and $m_v = 296$ kPa^{-1}. Both c_v and m_v were subsequently assumed to remain constant. Owing to silo loading, the variation of total stress with depth was assumed to be linear with maximum values at the top and bottom of the layer given by $\sigma_t = 32$ kPa and $\sigma_b = 28$ kPa respectively, and given by

Table 1. Special cases for different $Q(z, t)$

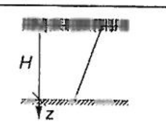
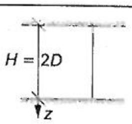
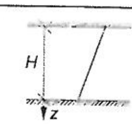
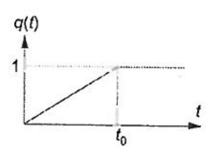
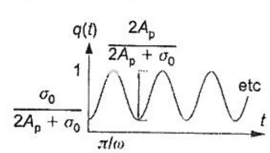
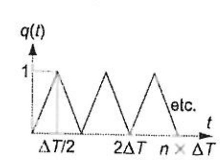
Case	1	2	3
$\sigma(z)$	 $\sigma(z) = \sigma_t + (\sigma_b - \sigma_t)z/H$	 $\sigma(z) = 2A_p + \sigma_0$	 $\sigma(z) = \sigma_t + (\sigma_b - \sigma_t)z/H$
$q(t)$	 $q(t) = \begin{cases} t/t_0 & t \leqslant t_0 \\ 1 & t > t_0 \end{cases}$	 $q(t) = \dfrac{A_p(1 - \cos\omega t) + \sigma_0}{(2A_p + \sigma_0)}$	 $q(t) = \begin{cases} t/\Delta T - (n-1) \\ (n-1)\times\Delta T < t < (2n-1)\Delta T/2 \\ n - t/\Delta T \\ (2n-1)\Delta T/2 < t < n\times\Delta T \end{cases}$
Drainage condition	Single-drained	Double-drained	Single-drained
Authors	Zhu & Yin (1998)	Rahalt & Vuez (1998)	Liu *et al.* (2014)

Table 2. Solutions of special cases using equations (13) and (16)

Case	Solution
1	Single-drained $u=\begin{cases}\sum_{k=1}^{\infty}\frac{2}{M_k^3T_0}B_k\sin\left(\frac{M_kz}{H}\right)[1-\exp(-M_k^2T_\mathrm{v})] & t\leq t_0\\ \sum_{k=1}^{\infty}\frac{2}{M_k^3T_0}B_k\sin\left(\frac{M_kz}{H}\right)[\exp(M_k^2T_0-M_k^2T_\mathrm{v})-\exp(-M_k^2T_\mathrm{v})] & t>t_0\end{cases}$ where $T_0=c_\mathrm{v}t_0/H^2$, $B_k=\sigma_\mathrm{t}+\frac{\sigma_\mathrm{t}-\sigma_\mathrm{b}}{M}(-1)^k$,
2	Double-drained $u=\sigma_0+2A_p\sum_{k=1}^{\infty}\frac{\omega^2D^4}{\omega^2D^4M_k+c_\mathrm{v}^2M_k^5}\sin\left(\frac{M_kz}{D}\right)W_k$ where $D=H/2$, $W_k=\frac{c_\mathrm{v}M_k^2}{\omega D^2}\sin(\omega t)-\cos(\omega t)+\exp(-M_k^2T_\mathrm{v})$
3	Single-drained 1. For $(n-1)\times\Delta T<t<(2n-1)\Delta T/2$, $u=\sum_{k=1}^{\infty}R(k)\left\{1-\exp\left\{-c_\mathrm{v}\frac{M_k^2}{H^2}[t-(n-1)\times\Delta T]\right\}+P(n-1)\right\}\sin\left(\frac{M_k}{H}z\right)$ where $R(k)=\left[\frac{2\sigma_\mathrm{t}}{M_k}+\frac{2(\sigma_\mathrm{b}-\sigma_\mathrm{t})}{M_k^2}(-1)^{k+1}\right]\frac{2}{\Delta T}\frac{H^2}{c_\mathrm{v}M_k^2}$, $P(n-1)=\sum_{i=1}^{n-1}\exp\left(-c_\mathrm{v}\frac{M_k^2}{H^2}t\right)\left\{2\exp\left[c_\mathrm{v}\frac{M_k^2}{H^2}(i-1/2)\Delta T\right]-\exp\left[c_\mathrm{v}\frac{M_k^2}{H^2}(i-1)\Delta T\right]-\exp\left[c_\mathrm{v}\frac{M_k^2}{H^2}i\Delta T\right]\right\}$ 2. For $(2n-1)\Delta T/2<t<n\times\Delta T$, $u=\sum_{k=1}^{\infty}R(k)\left(2\exp\left\{-c_\mathrm{v}\frac{M_k^2}{H^2}[t-(n-1/2)\Delta T]\right\}-1-\exp\left\{-c_\mathrm{v}\frac{M_k^2}{H^2}[t-(n-1)\Delta T]\right\}+P(n-1)\right)\sin\left(\frac{M_k}{H}z\right)$

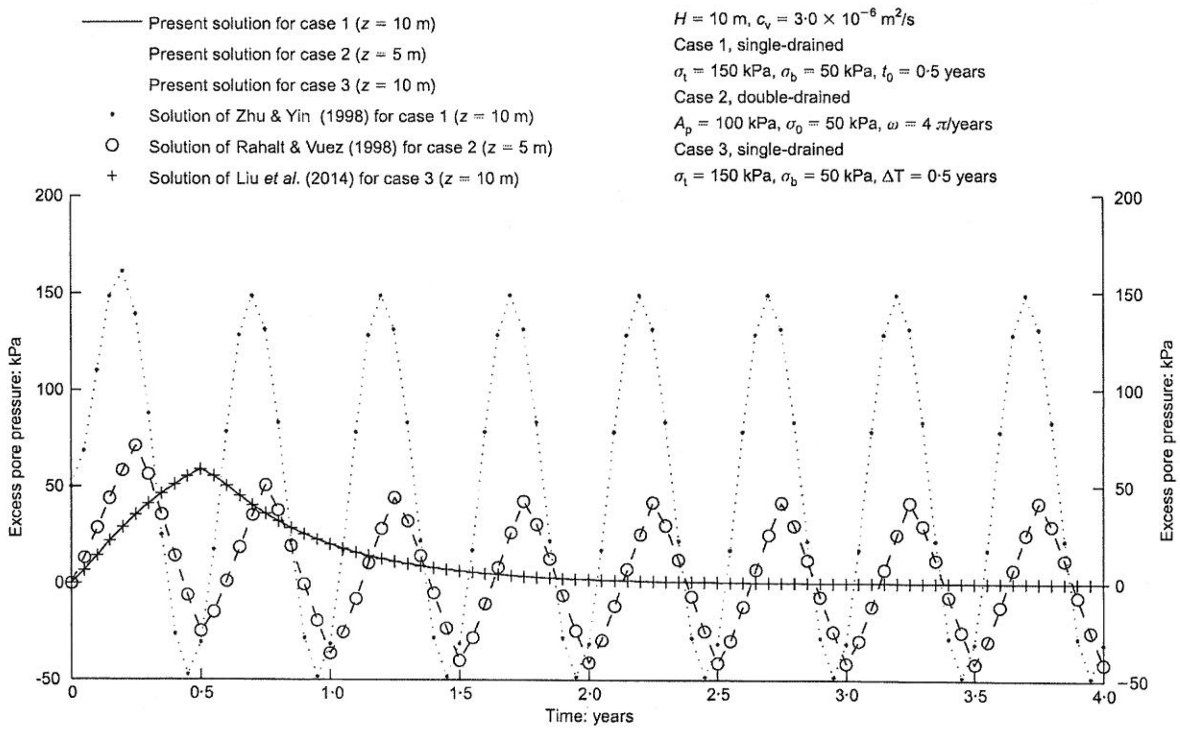


Fig. 2. Comparison of solutions for special cases

$$\sigma(z) = 32 - 4z/H \tag{20}$$

The load time function $q(t)$ was piecewise linear and is shown in Table 3 (Favaretti & Mazzucato, 1994). In the ith segment (going from station i to $i+1$), the time function $q(t)$ can be expressed as

$$q(t) = \frac{t - t_i}{t_{i+1} - t_i}[q(t_{i+1}) - q(t_i)] + q(t_i) \quad t_i \leqslant t \leqslant t_{i+1} \tag{21}$$

The excess pore pressure is obtained by substituting equations (20) and (21) into equation (15).

$$u = \sum_{k=1}^{\infty} \frac{2}{N_k} E_k \sin\left(\frac{N_k z}{H}\right) \exp\left(-N_k^2 \frac{c_v t}{H^2}\right)(V_i + V_{i-1}) \quad t_i \leqslant t \leqslant t_{i+1} \tag{22}$$

where

Table 3. Piecewise linear function $q(t)$

Station	t_i: years	$q(t_i)$
1	0·0	0·0
2	0·1	0·8
3	0·4	0·8
4	0·6	0·0
5	1·0	0·0
6	1·1	0·9
7	1·3	0·9
8	1·6	0·0
9	1·7	0·0
10	2·1	1·0
11	2·4	1·0
12	2·8	0·0
13	2·9	0·0
14	3·0	1·0
15	3·3	1·0
16	3·4	0·7

$$V_{i-1} = \sum_{j=1}^{i-1} \left\{ \frac{q(t_{j+1}) - q(t_j)}{t_{j+1} - t_j} \frac{H^2}{N_k^2 c_v} \times \left[\exp\left(N_k^2 \frac{c_v}{H^2} t_{j+1}\right) - \exp\left(N_k^2 \frac{c_v}{H^2} t_j\right)\right] + q(t_j)\exp\left(N_k^2 \frac{c_v}{H^2} t_j\right) - q(t_{j+1})\exp\left(N_k^2 \frac{c_v}{H^2} t_{j+1}\right) \right\} \tag{23}$$

$$V_i = \frac{q(t_{i+1}) - q(t_i)}{t_{i+1} - t_i} \frac{H^2}{N_k^2 c_v} \times \left[\exp\left(N_k^2 \frac{c_v}{H^2} t\right) - \exp\left(N_k^2 \frac{c_v}{H^2} t_i\right)\right] + q(t_i)\exp\left(N_k^2 \frac{c_v}{H^2} t_i\right) \tag{24}$$

$$E_k = \sigma_t - (-1)^k \sigma_b \tag{25}$$

The silo settlement can be given by

$$S(t) = m_v H \left\{ q(t)\frac{\sigma_b + \sigma_t}{2} - \sum_{k=1}^{\infty} \frac{2}{N_k^2} E_k [1 - (-1)^k] \times \exp\left(-N_k^2 \frac{c_v t}{H^2}\right)(V_i + V_{i-1}) \right\} \tag{26}$$

in the range $t_i \leqslant t \leqslant t_{i+1}$

The silo settlement is a piecewise function, whose value can be obtained from equation (26) for each time segment. A computer program has been developed for the silo settlement problem and compared with a finite-element solution modified from program 8·2 in Smith *et al.* (2014). The analytical and numerical results are essentially identical as shown in Fig. 3. Owing to variations in soil layer thickness, both upper and lower bounds on measured settlements are also shown in Fig. 3 and are generally in very good agreement with the theoretical results, although the measured settlement is somewhat underestimated in the first 1–2 years.

Loads assumed for the analysis also had some uncertainty, since the amount of corn stored in the silo at any given time was only estimated. Fig. 4 shows a modified calculation

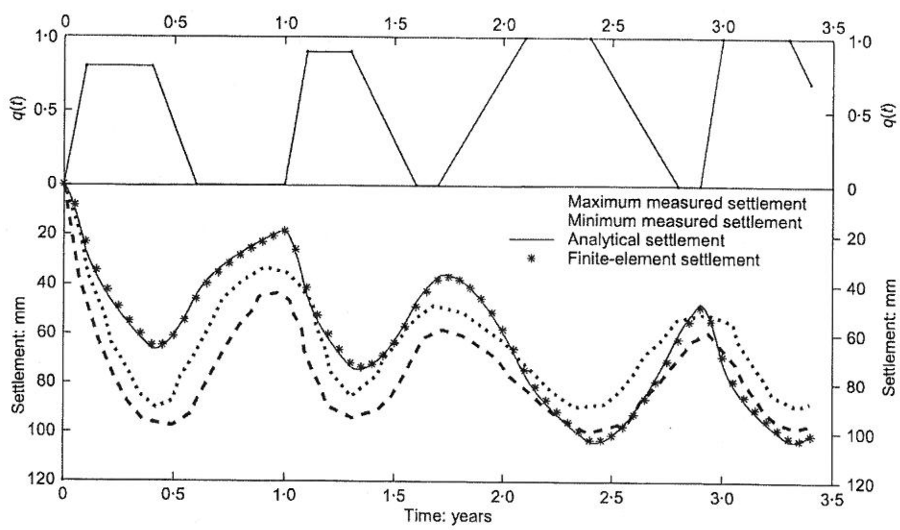


Fig. 3. Comparison between theoretical, finite-element and measured settlements under $q(t)$

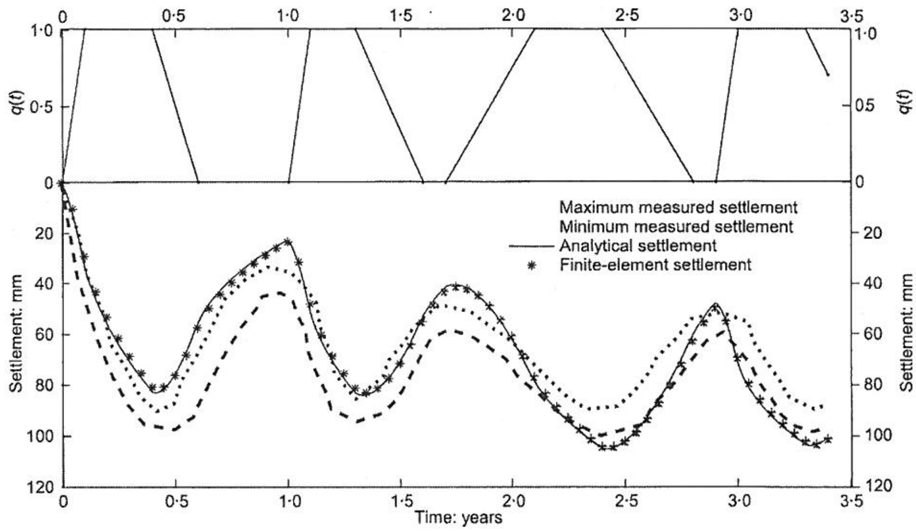


Fig. 4. Comparison between theoretical, finite-element and measured settlements under increased loading

based on higher loads where $q(t) = 1$ at peak loading in both of the first 2 years. Better agreement was observed between the theoretical and measured settlement. It was also noted that settlement of the silo reached a steady state after three filling and emptying cycles.

CONCLUSIONS

An analytical solution for calculation of excess pore pressure in 1D consolidation induced by depth- and time-dependent changes in total stress has been derived using Laplace transforms. The solution was verified by comparison with some available analytical solutions for special cases. The solution was then applied to a case history involving settlement of a silo subjected to time-dependent load filling and emptying. Theoretical and measured settlements agreed very well, especially when the load condition was adjusted up to assume full filling on each cycle.

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NOTATION

A_1, A_2	coefficients to be determined
A_p, ω, σ_0	parameters of sinusoidal time-dependent cyclic load
B_k	series coefficient listed in Table 2
C_k	Fourier coefficient
c_v	coefficient of consolidation
D	drainage path
D_k	series coefficient
$G_k(t)$	time function given as equation (16)
$\bar{G}_k(s)$	Laplace transform of $G_k(t)$
H	thickness of soil layer
i, j, k	counters, 1, 2, 3...
k_v	vertical hydraulic conductivity
M_k	$M_k = (2k-1)\pi/2$, $k = 1, 2, \ldots$
m_v	coefficient of volume change
N_k	$N_k = k\pi$, $k = 1, 2, \ldots$
$P(n-1)$	time function listed in Table 2
$Q(z,t)$	total stress as a product of a function of depth and a function of time
$q(t)$	time function
$\bar{q}$	Laplace transform of $q(t)$
$R(k)$	coefficient listed in Table 2
$S(t)$	settlement with time
s	frequency domain or Laplace-space variable
T_v	time factor, $T_v = c_v t/H^2$
T_0	time factor, $T_0 = c_v t_0/H^2$
t	time
t_m	time t to reach maximum load $q(t_m) = 1$
t_0	construction time of one-step loading
u	excess pore-water pressure
$\bar{u}$	Laplace transform of u
V_{i-1}	time function given as equation (23)
V_i	time function given as equation (24)
W_k	time function listed in Table 2
z	vertical coordinate
γ_w	unit weight of water
σ_t, σ_b	maximum increases in vertical total stress at top and bottom of the soil layer, respectively (linear case)
$\sigma(z)$	maximum increases in vertical total stress as a function of depth z
τ	integration variable
ω	angular frequency

REFERENCES

Conte, E. & Troncone, A. (2006). One-dimensional consolidation under general time-dependent loading. *Can. Geotech. J.* **43**, No. 11, 1107–1116.

Conte, E. & Troncone, A. (2008). Soil layer response to pore pressure variations at the boundary. *Géotechnique* **58**, No. 1, 37–44, http://dx.doi.org/10.1680/geot.2008.58.1.37.

Favaretti, M. & Mazzucato, A. (1994). Settlements of a silo subjected to cyclic loading. *Proceedings of Settlement'94*, Texas, vol. I, pp. 775–785. Reston, VA, USA: ASCE.

Huang, J. & Griffiths, D. V. (2010). One-dimensional consolidation theories for layered soil and coupled and uncoupled solutions by the finite-element method. *Géotechnique* **60**, No. 9, 709–713, http://dx.doi.org/10.1680/geot.08.P.038.

Li, C.-X., Xie, K.-H., Hu, A.-F. & Hu, B.-X. (2012). One-dimensional consolidation of double-layered soil with non-Darcian flow described by exponent and threshold gradient. *J. Cent. South Univ* **19**, No. 2, 562–471.

Liu, J. C., Lei, G. G. & Mei, G. X. (2012). One-dimensional consolidation of visco-elastic aquitard due to withdrawal of deep-groundwater. *J. Central South University* **19**, No. 1, 282–286.

Liu, J. C., Lei, G. H. & Wang, X. D. (2014). One-dimensional consolidation of visco-elastic marine clay under depth-varying and time-dependent load. *Mar. Georesources & Geotechnol.*, http://dx.doi.org/10.1080/1064119X.2013.877109.

Lovisa, J., Read, W. W. & Sivakugan, N. (2012). Calculating c_v based on non-uniform initial excess pore pressure. *Géotechnique* **62**, No. 8, 741–748, http://dx.doi.org/10.1680/geot.11.P.055.

Olson, R. E. (1977). Consolidation under time-dependent loading. *J. Geotech. Engng Div., ASCE* **103**, No. GT1, 55–60.

Rahalt, M. A. & Vuez, A. R. (1998). Analysis of settlement and pore pressure induced by cyclic loading of silo. *J. Geotech. Geoenviron. Eng* **124**, No. 12, 1208–1210.

Razouki, S. S. & Schanz, T. (2011). One-dimensional consolidation under haversine repeated loading with rest period. *Acta Geotechnica* **6**, No. 1, 13–20.

Razouki, S. S., Bonnier, P., Datcheva, M. & Schanz, T. (2013). Analytical solution for 1D consolidation under haversine cyclic loading. *Int. J. Numer. Analyt. Methods Geomech.* **37**, No. 14, 2367–2372.

Schiffman, R. L. & Stein, J. R. (1970). One-dimensional consolidation of layered systems. *J. Soil Mech. Found. Div., ASCE* **96**, No. SM4, 1499–1504.

Singh, S. K. (2008). Identifying consolidation coefficient: Linear excess pore-water pressure. *J. Geotech. Geoenviron. Engng* **134**, No. 8, 1205–1209.

Smith, I. M., Griffiths, D. V. & Margetts, L. (2014). *Programming the finite element method*, 5th edn. New York, NY, USA: Wiley.

Taylor, D. W. (1948). *Fundamentals of soil mechanics*. New York, NY, USA: Wiley.

Terzaghi, K. (1943). *Theoretical soil mechanics*. New York, NY, USA: Wiley.

Zhu, G. & Yin, J. H. (1998). Consolidation of soil under depth-dependent ramp load. *Can. Geotech. J.* **35**, No. 2, 344–350.

Zhu, G. & Yin, J. H. (1999). Consolidation of double soil layers under depth-dependent ramp load. *Géotechnique* **49**, No. 3, 415–421, http://dx.doi.org/10.1680/geot.1999.49.3.415.