Effect of Spatial Correlation Length on the Interpretation of Normalized CPT Data Using a Kriging Approach

S. Firouzianbandpey, Ph.D.[1]; L. B. Ibsen[2]; D. V. Griffiths[3]; M. J. Vahdatirad, Ph.D.[4]; L. V. Andersen[5]; and J. D. Sørensen[6]

Abstract: In geotechnical engineering analysis and design, the frequency and spacing of borehole information is of great interest, especially when field data are limited. This paper uses random field models to deal with uncertainty in soil properties owing to spatial variability, by analyzing in-situ cone penetration test (CPT) data from a sandy site in northern Denmark. To provide a best estimate of properties between observation points in the random field, a Kriging interpolation approach has been applied. As expected, for small correlation lengths, the estimated field quantities at intermediate locations between data points are close to the mean value of the measured results, and a high uncertainty is associated with the estimate. A longer correlation length reduces the error and implies more variation in the estimated values between the data points. **DOI:** 10.1061/(ASCE)GT.1943-5606.0001358.

Author Keywords: Geostatistics; Kriging; Spatial correlation length; Cone penetration test (CPT); Random field; Normalized cone data.

Introduction

In geotechnical investigations, the scope is often governed by how much the client and project manager are willing to spend, rather than by what is needed to characterize the subsurface conditions (e.g., Jaksa et al. 2005). To design and analyze a foundation, practitioners ideally would like to know the soil properties at many locations; but achieving this goal can be unrealistic and expensive. Researchers are searching for new ways to determine these parameters using statistical approaches. Probabilistic methods have been applied in geotechnical engineering for assessing the effects of uncertainties in geotechnical predictions (e.g., Zhang et al. 2011), and the application of geostatistics to large geotechnical projects has also proved to be a powerful tool, allowing coordination of field data in analysis and design (e.g., Ryti 1993; Rautman and Cromer 1994; Wild and Rouhani 1995; Rouhani 1996).

When uncertainties occur, they may often be attributable to limited sampling, rather than inaccuracies/measurement uncertainties in the soil tests themselves (e.g., Goldsworthy et al. 2007). In-situ tests in particular can provide a good characterization of soil properties at the locations where tests were performed, but inevitable uncertainty remains at locations which have not been examined.

[1]Engineer, Dept. of Civil Engineering, Aalborg Univ., 9000 Aalborg, Denmark (corresponding author). E-mail: sf@civil.aau.dk

[2]Professor, Dept. of Civil Engineering, Aalborg Univ., 9000 Aalborg, Denmark.

[3]Professor, Dept. of Civil and Environmental Engineering, Colorado School of Mines, Golden, CO 80401; and Australian Research Council Centre of Excellence for Geotechnical Science and Engineering, Univ. of Newcastle, Callaghan, NSW 2308, Australia.

[4]Engineer, Dept. of Civil Engineering, Aalborg Univ., 9000 Aalborg, Denmark.

[5]Associate Professor, Dept. of Civil Engineering, Aalborg Univ., 9000 Aalborg, Denmark.

[6]Professor, Dept. of Civil Engineering, Aalborg Univ., 9000 Aalborg, Denmark.

Note. This manuscript was submitted on July 8, 2014; approved on April 29, 2015; published online on June 30, 2015. Discussion period open until November 30, 2015; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Geotechnical and Geoenvironmental Engineering***, © ASCE, ISSN 1090-0241/04015052(9)/$25.00.

A more formal mathematical characterization of spatial variability using random fields (e.g., Fenton and Griffiths 2008), can quantify probabilistically how the variability at one location can be used to represent the variability at another location some distance away. The well-established Kriging method, based on D. G. Krige's empirical work for evaluating mineral resources (Krige 1951), and later formalized by Matheron (1963) into a statistical approach in geostatistics can also be used to perform spatial interpolation between known borehole data. In addition to generating a best, linear unbiased estimate of a random field between known data, Kriging has the added ability of estimating certain aspects of the mean trend by using a weighted linear combination of the values of a random field at each observation point (e.g., Fenton and Griffiths 2008). In environmental and geotechnical engineering, Kriging is commonly applied to the mapping of soil parameters and piezometric surfaces (e.g., Journel and Huijbegts 1978; Delhomme 1978; ASCE 1990).

Kriging has numerous advantages compared with other common interpolation techniques. For example, Kriging can produce site- and variable-specific interpolation schemes by directly incorporating a model of the spatial variability of the data (Rouhani 1996). As a collection of linear regression techniques, Kriging accounts for the stochastic dependence among the data (Olea 1991). The geological processes result in a *stochastic* dependency, which may have acted over a large area across geological time scales (e.g., sedimentation in large basins) or in fairly small domains for only a short time (e.g., turbiditic sedimentation, glacio-fluviatile sedimentation). Geological characteristics that form in a slow and steady geological environment are better correlated to each other than those that result from an often abruptly changing geological process.

The purpose of this study is to interpolate normalized cone data in a sandy site by using the Kriging method and investigate the effect of spatial correlation length on the results. This statistical analysis procedure consists of two main parts:

1. Verification of the method using a random field simulation. In this part the Kriging method has been applied to a simulated 3D Gaussian random field and then at given intermediate points, these simulated values compared with the best estimate

Kriging values and estimates of the standard deviation or the coefficient of variation of the error.
2. Applying Kriging to real values of cone data. Kriged values are also estimated at different depths below the surface based on two different horizontal correlation lengths to analyze the effect of correlation length on the results.

The results indicate that by increasing the horizontal correlation length, the standard deviation of estimated values by the Kriging method decreases, resulting in less uncertainty in prediction of values at intermediate locations. It is worth noting that in this procedure, it is assumed that the data represent samples from a statistical homogeneous domain.

Normalization of Cone Data

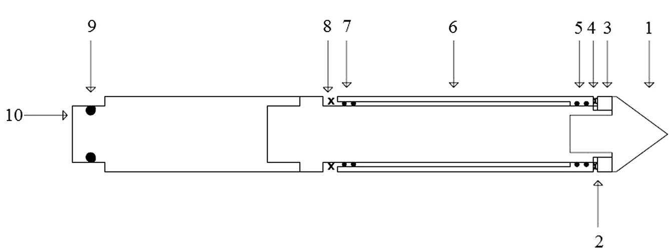
Because of the significant influence of the effective overburden stress on CPT measurements (e.g., Moss et al. 2006), various methods have been proposed for normalizing CPT data to account for this effect. In this study, the technique proposed by Robertson and Wride (1998) has been applied to the measurements of cone tip resistance, i.e.

$$q_{c1N} = \left(\frac{q_c}{P_a}\right) C_Q, \qquad C_Q = \left(\frac{P_a}{\sigma'_{v0}}\right)^n \tag{1}$$

where q_{c1N} is the dimensionless cone resistance normalized by the weight of soil on top of the cone, q_c is the measured cone tip resistance, and C_Q is the correction for overburden stress. The power n takes the values 0.5, 1.0 and 0.7 for cohesionless, cohesive and intermediate soils, respectively, whereas σ'_{v0} is the effective vertical stress and P_a is the reference pressure (atmospheric pressure) in the same units as σ'_{v0} and q_c.

Description of the Site

This study concerns a site close to Aalborg in northern Denmark where a wind turbine blade storage facility is to be constructed. The site is a basin deposit area as it is close to the Limfjord. The soil layers consist of 4 m clay on top and silty sand in the lower layers. Using piezocone penetration test (CPTu) data, the statistical characteristics of the cone tip resistance at the site have been estimated. A total of nine cone penetration tests was conducted using a Geotech NOVA Acoustic system and a 20 t digital piezocone penetrometer, and data was acquired digitally. The CPTu system also consists of a hydraulic pushing and leveling system and 1-m long segmental rods. Fig. 1 shows a schematic cross section of the CPT probe. All CPTu soundings reached a depth of approximately 8 m. The nine soundings were arranged in a cross-shaped pattern with a 10-m separation distance between holes, and the cross was framed by four boreholes (Fig. 2). CPT data were sampled at 20-mm intervals. Fig. 3 illustrates a representative CPT profile obtained in the field. Standard classification test results were carried out on samples retrieved from the boreholes showing that the soil deposit is primarily sand and a sand–silt mixture.



Item	Description	Item	Description
1.	Point/ Tip, 10 cm^2	6.	Friction sleeve
2.	Support ring under the X-ring	7.	Friction sleeve, 2 pcs O-ring
3.	Filter ring brass, 10 cm^2 – Pore pressure	8.	X-ring
4.	X-ring	9.	O-ring, battery pack, 10 cm^2
5.	Friction sleeve, 2 pcs O-ring	10.	Serial number of the probe

Fig. 1. CPT probe, 10 cm^2

Modeling Spatial Variability of the Site Using Kriging

Kriging is essentially a best, linear unbiased estimation with the added benefit of being able to estimate the mean. The main objective is to provide a best estimate of the random field at unobserved points. The Kriging estimate is modeled as a linear combination of the observations

$$\hat{X} = \sum_{k=1}^{n} \beta_k X_k \tag{2}$$

where x is the spatial position of the unobserved value being estimated. The unknown coefficients β_i are determined by considering the covariance between the observations and the prediction point.

To assess the effect of known data at an intermediate position, maps were created by using kriging on the cone resistance data from CPT soundings in the region. This approach provided a best estimate of a random field between known data to estimate the random field at any location using a weighted linear combination of the values of the random field at observation points. The following steps are applied for this procedure:

Assume a correlation length of the site (θ) (in this study two arbitrary correlation lengths have been chosen).

Estimate the correlation coefficient of the data (ρ) assuming a homogeneous random field

$$\rho(\mathbf{x_i}, \mathbf{x_j}) = \exp\left(\frac{-2|\tau_{ij}|}{\theta}\right) \qquad \tau_{ij} = |\mathbf{x_i} - \mathbf{x_j}| \tag{3}$$

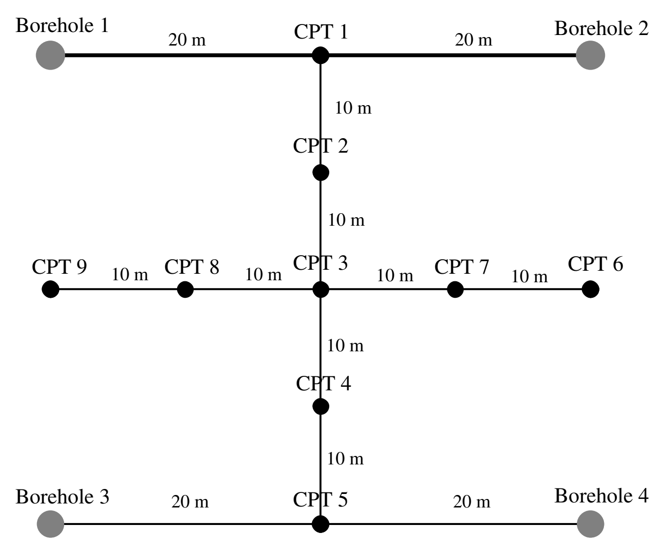


Fig. 2. Plan of boreholes and CPTu positions

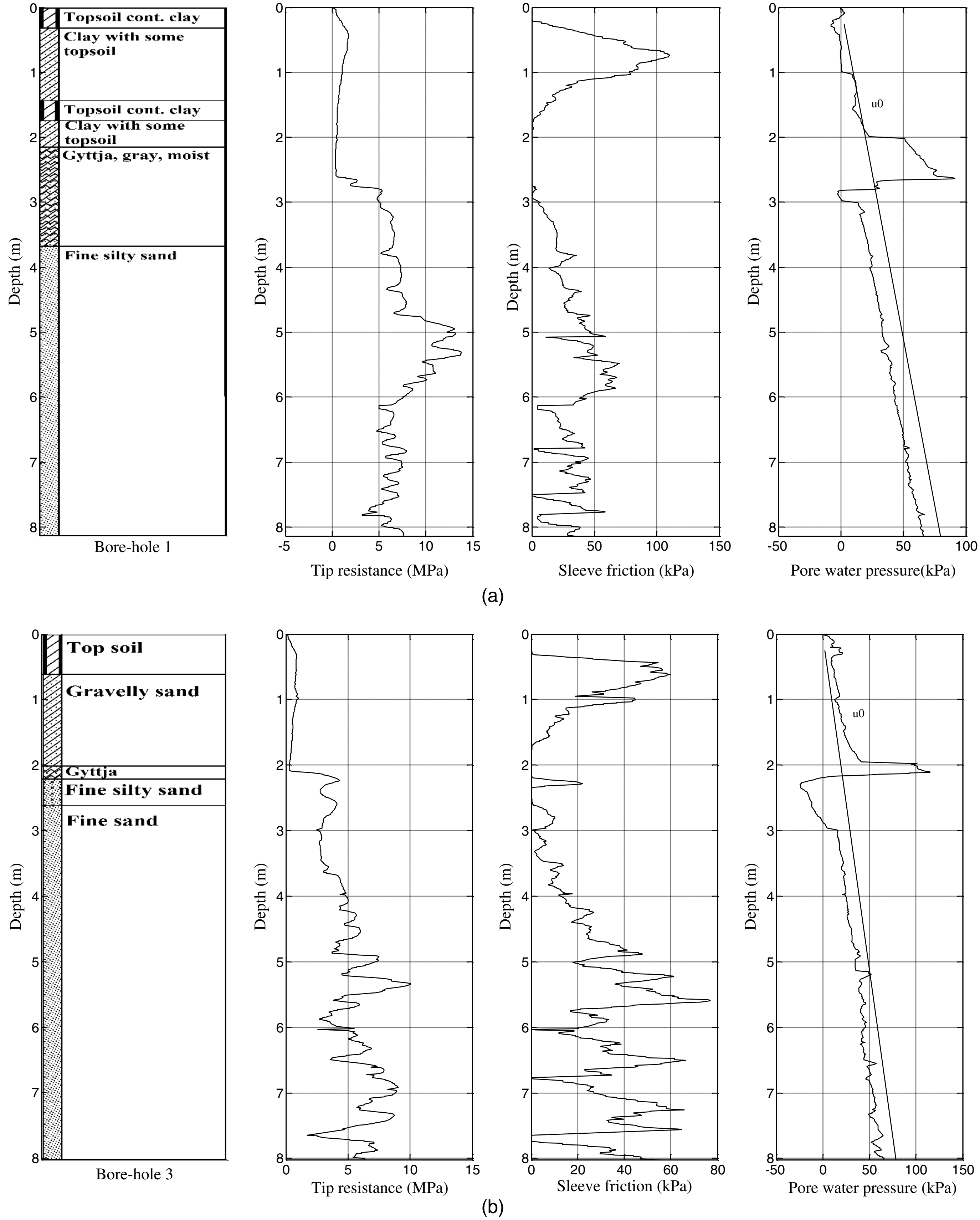


Fig. 3. Two representative CPT profiles obtained from the site. u_0 is the hydrostatic pore pressure induced by the phreatic level of the region (CPT 3 and 5)

Calculate the covariance between data (between $\mathbf{x}_i$ and $\mathbf{x}_j$)

$$C_{ij} = \sigma_x^2 \exp\left(\frac{-2|\tau_{ij}|}{\theta}\right), \qquad \left(\text{Note:} \rho(\tau) = \frac{C(\tau)}{\sigma_x^2}\right) \tag{4}$$

In Kriging it is assumed that the mean can be expressed as in a regression analysis

$$\mu(X) = \sum_{i=1}^{m} a_i g_i(x) \tag{5}$$

where a_i is an unknown coefficient and $g_1(x) = 1$, $g_2(x) = x$, $g_3(x) = x^2$, and so on (in a one-dimensional case). A similar approach is used in higher dimensions (Fenton and Griffiths 2008).

Estimate of the Kriging matrix $\mathbf{K}$

$$\mathbf{K} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} & g_1(x_1) & g_2(x_1) & \cdots & g_m(x_1) \\ C_{21} & C_{22} & \cdots & C_{2n} & g_1(x_2) & g_2(x_2) & \cdots & g_m(x_2) \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} & g_1(x_n) & g_2(x_n) & \cdots & g_m(x_n) \\ g_1(x_1) & g_1(x_2) & \cdots & g_1(x_n) & 0 & 0 & \cdots & 0 \\ g_2(x_1) & g_2(x_2) & \cdots & g_2(x_n) & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ g_m(x_1) & g_m(x_2) & \cdots & g_m(x_n) & 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{6}$$

Because $\mathbf{K}$ is a function of the observation point locations and covariance between them, it could be inverted and used repeatedly at different spatial points to build up the best estimate of the random field.

Calculate the covariance between the ith observation point and a given, intermediate spatial point $\mathbf{x}$

$$\mathbf{M} = \left\{ \begin{matrix} C_{1\mathbf{x}} \\ C_{2\mathbf{x}} \\ \vdots \\ C_{n\mathbf{x}} \\ g_1(\mathbf{x}) \\ g_2(\mathbf{x}) \\ \vdots \\ g_m(\mathbf{x}) \end{matrix} \right\} \tag{7}$$

By definition, the so-called best linear unbiased predictor $\hat{X}$ of X implies that it is linear. So the n unknown weights β_k in Eq. (2) have to be determined to find the best estimate at the point $\mathbf{x}$

$$\mathbf{K}\boldsymbol{\beta} = \mathbf{M}, \qquad \boldsymbol{\beta} = \left\{ \begin{matrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \\ -\eta_1 \\ -\eta_2 \\ \vdots \\ -\eta_m \end{matrix} \right\} \tag{8}$$

For each specific point, $\mathbf{M}$ changes, as does the vector of weights, β. The quantities η_i are a set of Lagrangian parameters used to solve the variance minimization problem subject to the unbiased conditions.

Estimate unknown values at the desired location

$$\hat{X}(x) = \sum_{k=1}^{n} \beta_k x_k \tag{9}$$

where the hat indicates that this is an estimate, and $x_1, x_2, \ldots, x_k$ are observation points.

Concerning step 1 in the procedure listed above, there are different techniques available in the literature for the estimation of the correlation length using geotechnical data of the field (Vanmarcke 1977; DeGroot and Baecher 1993; Tang 1979; DNV 2010). If sufficient data are available, then it is possible to use one of those techniques. The correlation coefficient between each pair of data can be calculated and plotted versus the spatial distance between the corresponding positions. Then an admissible type of autocorrelation function is fitted to them and using the regression analysis, the *best* values for the model parameters (incl. correlation length parameter) can be estimated (JCSS-C1 2006). For example, in the quadratic exponential model, the correlation length is the double of the D factor (Firouzianbandpey et al. 2014).

With regard to step 4 of the procedure, an assumption that the mean is either constant (i.e., $m = 1,\ g_1(x) = 1,\ a_1 = \mu(X)$ or linearly varying ($m = 2,\ \mu(X) = a_1 + a_2 x$) is usually sufficient. The correct form of the mean trend can be determined by plotting the results and visually checking the mean trend. The trend can also be found by performing a regression analysis or performing a more complex structural analysis (Journel and Huijbergts 1978).

Method Verification Using Random Field Simulation

In this study, the Kriging method has been applied to a generated 3D Gaussian random field using the correlation matrix decomposition method. The procedure was as follows:

1. Simulate a realization of the random field

Using the simulated values in the soundings positions, establish a Kriging model.

The best estimate Kriging values of cone data are then compared with the simulated values at given intermediate points. This was also undertaken for the purpose of verification of the procedure in assuming a constant mean trend and ignoring Lagrangian parameters.

The mean and standard deviation of the normalized cone data from the field q_{c1N} (e.g.) were used to generate the random field. A Markovian correlation function

$$\rho_{\text{field}} = \exp\left(-\frac{2|\Delta x|}{\delta_x} - \frac{2|\Delta y|}{\delta_y} - \frac{2|\Delta z|}{\delta_z}\right) \qquad (10)$$

has been used for modeling the random field (for example, Vahdatirad et al. 2014) and the correlation between points in the field was modeled as an exponentially decaying function of the absolute distance between the points. In Eq. (10), Δx, Δy and Δz are the spatial distances in the horizontal and vertical directions, respectively, and δ_x, δ_y and δ_z are correlation lengths in the appropriate directions. The real correlation lengths of cone data in the region (δ_x, $\delta_y = 2$ m and $\delta_z = 0.45$ m) have been estimated and used in the model (Firouzianbandpey et al. 2014).

For each realization, a vector of standard Gaussian random seeds, U_x, is generated for each random field with the same size as the number of integration points. The correlation matrix $\tilde{R}$ is constructed with the correlation function specified in Eq. (10) and decomposed as

$$\tilde{L} \times \tilde{L}^T = \tilde{R} \qquad (11a)$$

$$G(x) = \tilde{L} \times U(x) \qquad (11b)$$

where $\tilde{L}$ is the lower triangular matrix used for transferring U_x to the correlated field with zero mean $G(x)$.

For each random variable, transformation to the random fields with real distribution is:

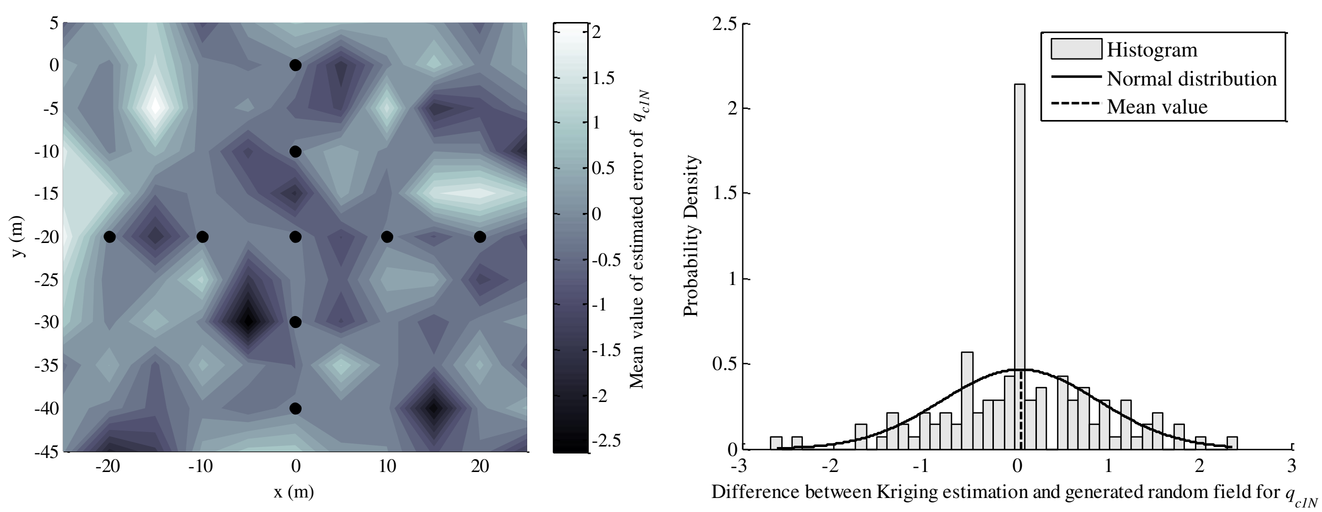
$$Y = \exp(\mu_{\ln} + \sigma_{\ln} G)$$

where $\mu_{\ln}$ and $\sigma_{\ln}$ are lognormal mean value and lognormal standard deviation for q_{c1N}, respectively.

After 1,000 Monte Carlo simulations, the mean values of the random field at the same position as the soundings were used to estimate the Kriging values at intermediate positions in the field. Then the difference between the Kriging estimations and those generated by the random fields has been calculated and by fitting a normal distribution, the mean value of the error has been found. As shown in Fig. 4(a), the mean value of the error is very small, and it indicates that the method is quite acceptable in estimating values. Fig. 4 illustrates the results in the simulated random field. The black circles identify the location of the CPT soundings.

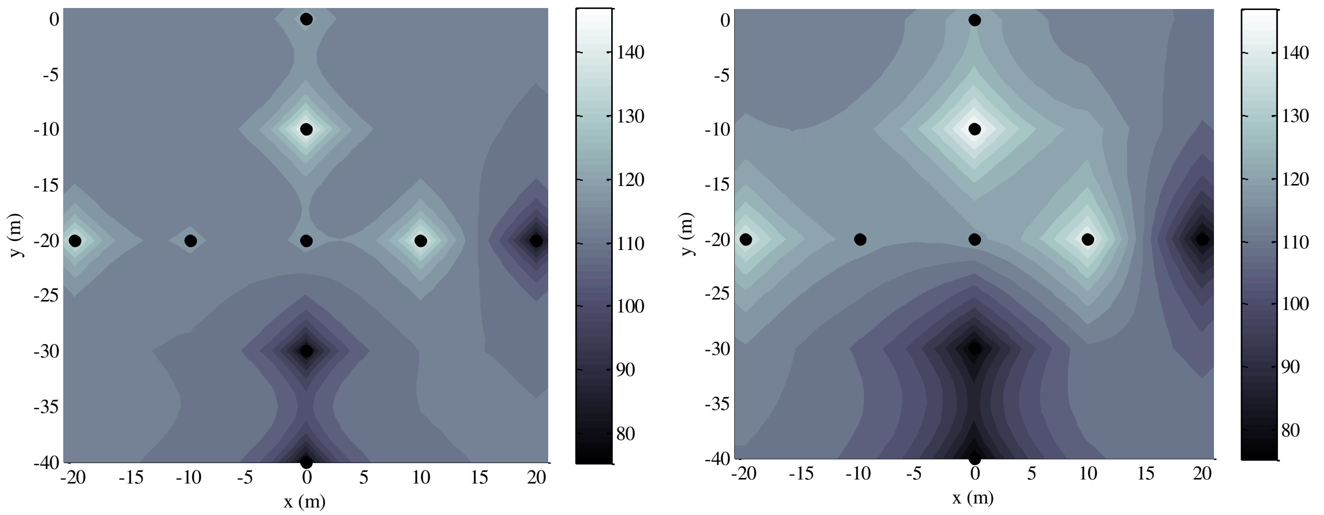
Applying Kriging to the Real Values of Cone Data

Kriged values were also estimated at a depth of 2 m below the surface based on two different horizontal correlation lengths: 5 and 10.5 m (half of the mean distance between soundings which was



(a) (b)

Fig. 4. Kriging estimation: (a) estimated random field at 2 m depth; (b) difference between Kriging estimation and generated random field ($\mu_{qc1N} = 129.7$, $\sigma_{qc1N} = 25$)



(a) (b)

Fig. 5. Normalized cone resistances estimated by Kriging at a depth of 2 m with $\theta_v = 0.45\,m$: (a)$\theta_h = 5\,m$; (b)$\theta_h = 10.5\,m$

21 m), are used to examine the effect of the correlation length on the kriged values. For the vertical direction, the real value of vertical correlation length of the region ($\theta_v = 0.45$ m) was applied. Because the real estimated horizontal correlation length of data was too small, it was preferred in the study to employ two representative values of this parameter for the purpose of comparison and inference.

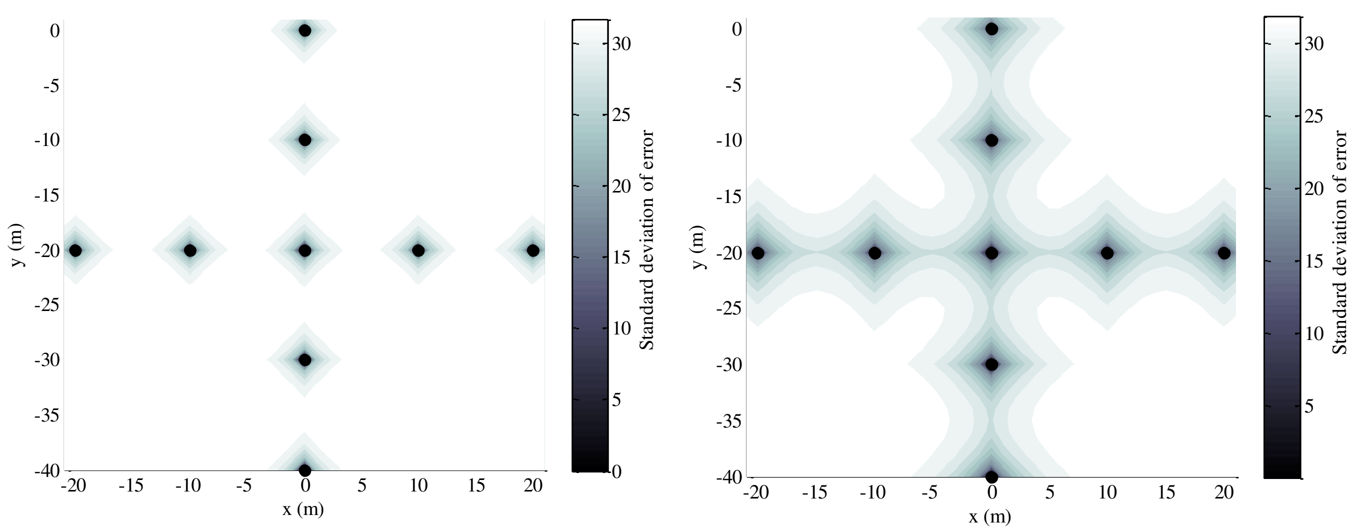
Fig. 5 shows a map of the estimated normalized cone resistances with two different correlation lengths at the chosen depths. To reflect the variability of this parameter in two directions, the normalized cone resistance values are shown as contours throughout the site plan. The black circles again identify the locations of the CPT soundings. As the distances between these points are increased, the correlation between the values of normalized cone resistances decreases. In other words, the values are increasingly different as the distances between the points increase. When the correlation length is large, the data are highly correlated, and the values are much closer to each other for a greater distance. This fact can be seen by comparing two plots with different horizontal correlation lengths. When the correlation length increases, points with the same color are distributed in a wider separation distance from fixed known locations, which implies a higher dependency in space. In the figures, θ_h and θ_v denote horizontal and vertical correlation lengths, respectively.

Estimator Error

Owing to a finite number of observations, there is always an error associated with any estimate of a random process. This error should be calculated to achieve the accuracy of the estimate. The difference between the estimated $\hat{X}(x)$ and its true (but unknown and random) value $X(x)$ can be given by

$$\mu_E = E[X(x) - \hat{X}(x)] = 0 \tag{12}$$

$$\sigma_E^2 = E[X(x) - \hat{X}(x)^2] = \sigma_X^2 + \beta_n^T(K_{n\times n}\beta_n - 2M_n) \tag{13}$$



(a) (b)

Fig. 6. Estimated standard deviation of the error at a depth of 2 m with $\theta_v = 0.45\,m$: (a)$\theta_h = 5\,m$; (b) $\theta_h = 10.5\,m$

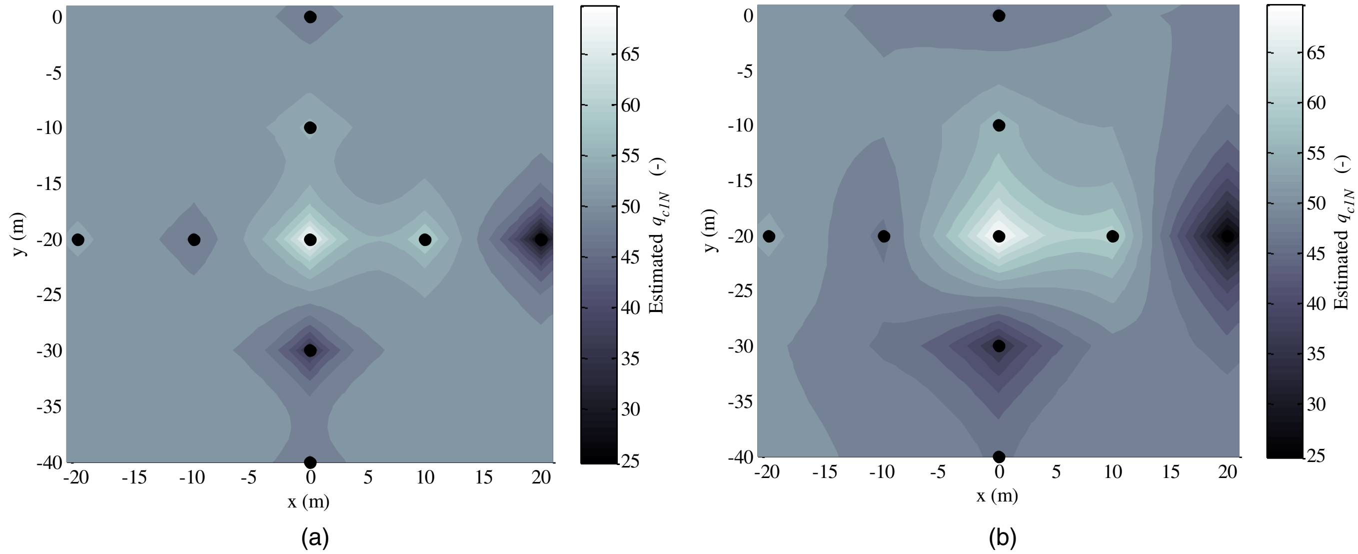


Fig. 7. Estimated normalized cone resistance by Kriging. Depth = 4 m, $\theta_v = 0.45\,m$: (a) $\theta_h = 5\,m$; (b) $\theta_h = 10.5\,m$

where β_n and M_n are the first n elements of $\boldsymbol{\beta}$ and $\mathbf{M}$, and $K_{n\times n}$ is the $n \times n$ upper left submatrix of K containing the covariances. Also β_n^T is the transpose of β_n. The individual standard deviation of the error has been estimated for two different correlation lengths (Fig. 6). As shown in Fig. 6 (left), the standard deviation of the error is small when it is close to the observation points and increases by increasing the distance. In Fig. 6 (right), with a higher horizontal correlation length, the standard deviation of error is obviously smaller in a larger domain around each observation point.

This procedure was applied to a different depth (4 m) to illustrate how the normalized cone resistance varied in the horizontal and vertical directions with correlation length. By understanding the correlation structure of the field, the values of a desired parameter of the soil can be estimated at intermediate locations. Fig. 7 provides information about the variation of normalized cone

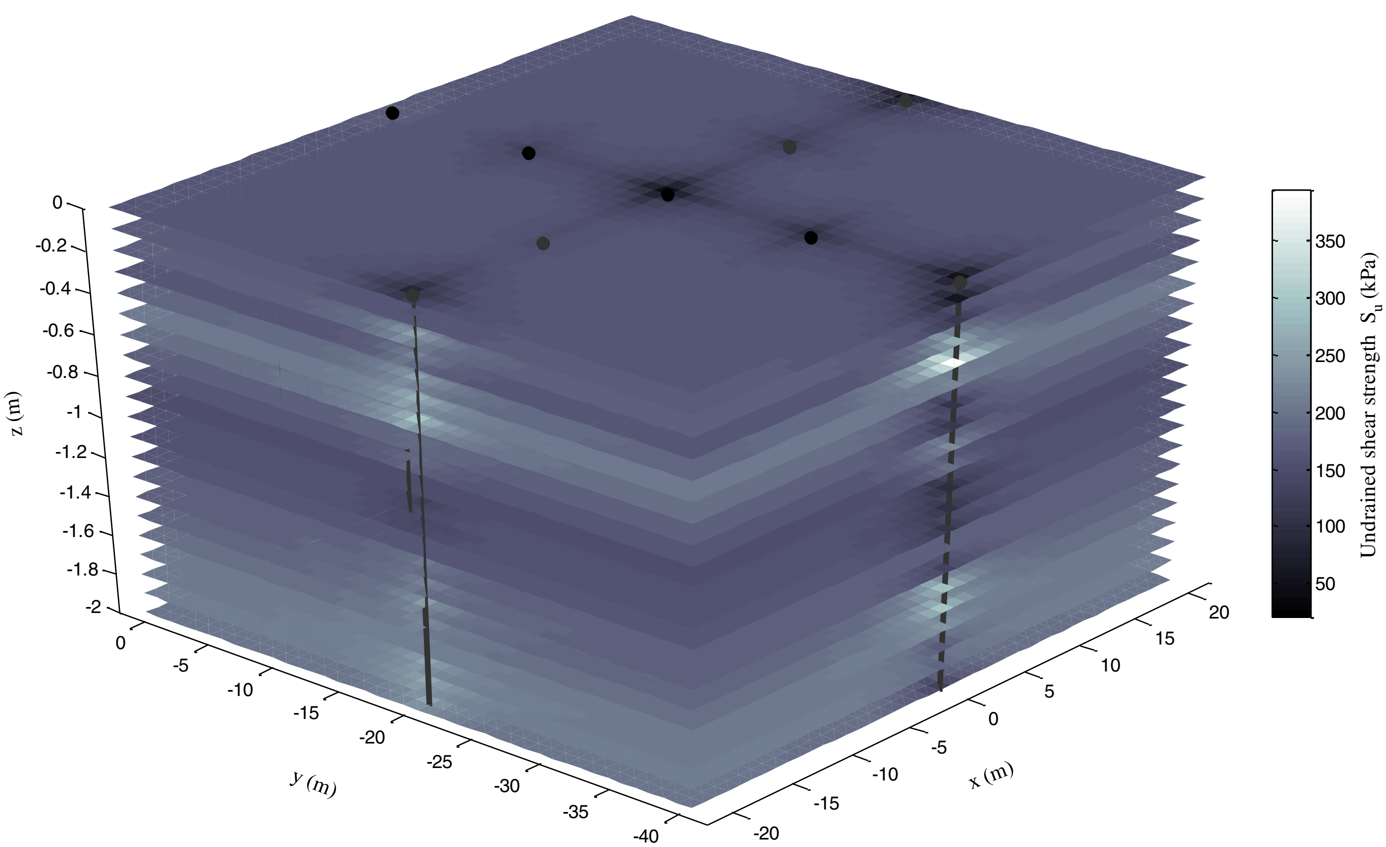


Fig. 8. Estimated undrained shear strength by Kriging. Depth = 0–2 m

J. Geotech. Geoenviron. Eng.

resistance in the region. If one could estimate the correlation length of the site by probabilistic analysis, the value of the cone resistance could easily be determined at any point of the site.

Application of Kriging—Illustrative Example

From a consideration of soil type, drainage conditions, and initial stress state, CPTu data can be used to estimate numerous geotechnical parameters, e.g., friction angle, relative density, small strain shear modulus, undrained shear strength, and OCR. An example is now presented to explain how Kriging can be used to estimate the undrained shear strength of a clayey layer in the region under consideration in this paper.

Studies for predicting the undrained shear strength using CPT have progressed empirically and theoretically. The results of these studies show that the correlation between cone resistance and undrained shear strength of clays can make use of the following equation (Baligh et al. 1980)

$$s_u = \frac{(q_c - \sigma_{v0})}{N_{kt}} \tag{14}$$

where N_{kt} is an empirical cone factor and σ_{v0} is the total in-situ vertical stress. A considerable amount of data has been reported on this equation (e.g., Lunne et al. 1997), indicating N_{kt} of approximately 15–20. Previous studies on Danish clay showed N_{kt} varying from 8.5 to 12, with 10 as an average [Luke (1992)]. For larger projects, site-specific correlations should be developed. By having Kriged q_c values, σ_{v0} and $N_{kt} = 10$, the values of s_u can be determined at any point through the layer by Eq. 14. Fig. 8 shows estimated Kriged values of s_u of the clayey layer at 10-cm intervals in the vertical direction. The approach allows limited CPT data to be fully exploited over a much wider volume of the site. The estimated strength parameters might then be available for use in a comprehensive numerical model of foundation performance on the heterogeneous soil layer.

Conclusion

A Kriging approach has been applied to the normalized cone resistance of a sandy site in Denmark to interpolate between known borehole data. First, a verification process has been performed by generating a 3D random field using statistical parameters of cone data. Some values at discrete borehole locations were sampled as known observation points and then Kriging was used to interpolate between the discrete values and compared with the original random field. These estimated Kriging values are compared with the simulated values at given intermediate points. This procedure was performed to verify some assumptions as a constant mean trend and ignoring Lagrangian parameters. After calculating the difference between the Kriging estimation and the generated random field, known values of the cone data at the location of the sounding were taken as observation points to estimate the values of cone resistance at any point within the field by the Kriging method. Because changes in the correlation length have an inevitable effect on the map of soil variation by Kriging, two values of horizontal correlation length were applied at two depth levels by the Kriging method, to examine the effect of correlation length on estimated values at intermediate locations between known field values. This was undertaken for two depths of 2 and 4 m through the deposit. The results showed that when the correlation length was increased, a good (accurate) estimate could be obtained at a greater number of intermediate points. In contrast, when the correlation length was smaller, these values could not be estimated precisely by Kriging. In the latter case, the estimated values at intermediate locations are approximately equal to the mean values of the data at the observation points, which implies a higher uncertainty. When the correlation length was increased, the data were more correlated with each other, and the values were closer at a greater distance. This observation was clear from the contours of cone values, in which the colors varied more gradually.

This study has used a Kriging technique based on measured field values, to provide a map of normalized cone resistances at a site with known or estimated spatial correlation properties. By having the values of normalized cone data at any desired location, the values of strength parameters for the soil needed for the design and analysis of any type of earth structure can be estimated and, consequently, can highly reduce the expenses of future site investigations. Studies such as this can be further developed to reduce the cost of site investigation by providing more reliable interpolated information for sites possessing limited CPT data.

Notation

The following symbols are used in this paper:

a_i = unknown coefficient;
C_{ij} = covariance between data;
C_Q = correlation for overburden stress;
$g_i(x)$ = regression function;
$G(x)$ = correlated random field with zero mean;
K = Kriging matrix (a function of observation point locations and their covariance);
$\tilde{L}$ = lower triangular matrix;
M = covariance between the observation point and the intermediate point;
N_{kt} = empirical cone factor;
n = results from the correction for overburden pressure (0.5, 0.7, 1.0 for cohesionless, intermediate and cohesive soils, respectively);
P_a = atmospheric pressure;
q_c = measured cone tip resistance;
q_{c1N} = normalized cone resistance;
$\tilde{R}$ = correlation matrix;
s_u = Undrained shear strength;
$U(x)$ = standard Gaussian random seeds;
x = spatial position of unobserved value;
$\tilde{X}$ = estimation of x;
β_i = Kriging coefficient or unknown Kriging weight;
$\delta_{x,y,z}$ = correlation length in x, y and z direction;
η_i = Lagrangian parameter;
θ_h = horizontal correlation length;
θ_v = vertical correlation length;
μ_E = mean value of the estimator error;
$\mu_{\ln}$ = Lognormal mean value of q_{c1N};
μ_X = mean function;
ρ = correlation length;
σ_E = standard deviation of the estimator error;
$\sigma_{\ln}$ = Lognormal standard deviation of q_{c1N};
σ'_{v0} = effective vertical stress;
σ_{v0} = total vertical stress; and
τ = lag distance between observation points.

References

ASCE Task Committee on Geostatistical Techniques in Geohydrology of the Ground Water Hydrology Committee of the ASCE Hydraulics Division. (1990). "Review of geostatistics in geohydrology. I: Basic concepts." *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1990)116:5(612), 612–658.

Baecher, G. B., and Christian, J. T. (2003). *Reliability and statistics in geotechnical engineering*, Wiley, West Sussex, England.

Baligh, M. M., Azzouz, A. S., and Martin, R. T. (1980). "Cone penetration tests offshore the Venezuelan Coast." *M.I.T. Rep. No. R80–21*, Massachusetts Institute of Technology, Cambridge, MA.

DeGroot, D. J., and Baecher, G. B. (1993). "Estimating autocovariance of in-situ soil properties." *ASCE J. Geotech. Eng.*, 119(1), 147–166.

Delhomme, J. P. (1978). "Kriging in the hydrosciences." *Adv. Water Resour.*, 1(5), 251–266.

DNV (Det Norske Veritas). (2010). "Statistical representation of soil data." *DNV-RP-C207*, Oslo, Norway.

Fenton, G. A., and Griffiths, D. V. (2008). *Risk assessment in geotechnical engineering*, John Wiley and Sons, Hoboken, NJ.

Firouzianbandpey, S., Griffiths, D. V., Ibsen, L. B., and Andersen, L. V. (2014). "Spatial correlation length of normalized cone data in sand: A case study in the North of Denmark." *Can. Geotech. J.*, 51(8), 844–857.

Goldsworthy, J. S., Jaksa, M. B., Fenton, G. A., Kaggwa, W. S., Griffiths, D. V., and Poulos, H. G. (2007). "Effect of sample location on the reliability based design of pad foundations." *Georisk Assess. Manage. Risk Eng. Syst. Geohazards*, 1(3), 155–166.

Jaksa, M. B.,, et al. (2005). "Towards reliable and effective site investigations."*Géotech.*, 55(2), 109–121.

JCSS (Joint Committee on Structural Safety). (2006). "JCSS-Cl: Probabilistic model code. Section 3.7: Soil properties." Technical Univ. of Denmark, Denmark.

Journel, A. G., and Huijbregts, C. (1978). *Mining geostatistics*, Academic, London, 600.

Krige, D. G. (1951). "A statistical approach to some basic mine valuation problems on Witwatersrand." *J. Chem. Metall. Mining Soc. South Africa*, 52(6), 119–139.

Luke, K. (1992). "Measuring undrained shear strength using CPT and field vane." *NGM-92: Proc., from 11. Norwegian Geotechnical meeting, Aalborg*, Vol. 1, 95–100.

Lunne, T., Robertson, P. K., and Powell, J. J. M. (1997). "Cone penetration testing in geotechnical practice." Blackie Academic/Chapman & Hall, E&FN Spon, London, 312.

Matheron, G. (1963). "Principles of geostatistics." *Econ. Geol.*, 58(8), 1246–1266.

Moss, R. E., Seed, R. B., and Olsen, R. (2006). "Normalizing the CPT for overburden stress." *J. Geotech. Geoenviron. Eng.*, 10.1061/(ASCE)1090-0241(2006)132:3(378), 378–387.

Olea, R. A. (1991). *Geostatistical glossary and multilingual dictionary*, Oxford Univ. Press, New York.

Rautman, C. A., and Cromer, M. V. (1994). *Three-dimensional rock characteristics models study plan: Yucca mountain site characterization plan SP 8.3.1.4.3.2*, U.S. Dept. of Energy, Office of Civilian Radioactive Management, Washington, DC.

Robertson, P. K., and Wride, C. E. (1998). "Evaluating cyclic liquefaction potential using the cone penetration test." *Can. Geotech. J.*, 35(3), 442–459.

Rouhani, S. (1996). "Geostatistical estimation: Kriging." *Geostat. Environ. Geotech. Appl.*, 1283, 20–31.

Ryti, R. (1993). "Superfund soil cleanup: Developing the piazza remedial design." *J. Air Waste Manage. Assoc.*, 43(2), 197–202.

Tang, W. H. (1979). "Probabilistic evaluation of penetration resistance." *ASCE J. Geotech. Eng. Div.*, 105(GT10), 1173–1191.

Vahdatirad, M. J., Griffiths, D. V., Andersen, L. V., Sørensen, J. D., and Fenton, G. A. (2014). "Reliability analysis of a gravity based foundation for wind turbines: A code-based design assessment." *Géotechn.*, 64 (8), 635–645.

Vanmarcke, E. (1977). "Probabilistic modeling of soil profiles." *ASCE J. Geotech. Eng. Div.*, 103(11), 1227–1246.

Wild, M., and Rouhani, S. (1996). "Effective use of field screening techniques in environmental investigations: A multivariate geostatistical approach." *ASTM Special Technical Publication 1283*, 88–101.

Zhang, J., Zhang, L. M., and Tang, W. H. (2011). "Kriging numerical models for geotechnical reliability analysis." *Soils Found.*, 51(6), 1169–1177.