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Numerical modelling of the trap door problem

N. C. KOUTSABELOULIS* and D. V. GRIFFITHS*

The trap door problem is a useful model for providing a clearer understanding of the stress distribution around civil engineering structures such as anchor plates and tunnels. The passive mode can be used either to compute the uplift force of anchors or any buried structure which may be idealized as an anchor; the active mode can be used to compute the gravitational flow of a granular material between vertical walls or the soil reaction curve for tunnel design. Both modes of displacement are modelled numerically in this Paper using the finite element method. The results are presented in the form of influence charts, which may be used for both the passive and active modes to provide failure loads for a range of geometries and soil properties, using non-dimensionalized parameters. The use of these equations is compared with results obtained from other sources using both physical and numerical models.

KEYWORDS: anchor design; finite element analysis; trap door problem; tunnel design

Le problème de la trappe représente un modèle utile pour l'obtention d'une compréhension plus claire de la distribution des contraintes autour de structures du génie civil telles que les plaques d'ancrage et les tunnels. Le mode passif peut s'employer pour calculer la force d'arrachement des éléments d'ancrage ou d'une construction quelconque ayant la fonction d'un élément d'ancrage, tandis que le mode actif peut être utilisé pour calculer l'écoulement gravitationnel d'une matière granulée entre des murs verticaux ou la courbe de réaction du sol dans le cas d'un tunnel. Dans cet article les deux modes de déplacement sont modelés de façon numérique à l'aide de la méthode aux éléments finis. Les résultats sont présentés dans la forme de graphiques d'influence qui peuvent s'employer pour les modes passif et actif dans le but d'obtenir des chargements de rupture pour une série de géométries et de propriétés du sol en utilisant des paramètres non-dimensionnés. L'emploi de ces équations est comparé aux résultats obtenus à partir d'autres sources en utilisant des modèles physiques et numériques.

The finite element method has become very popular in recent years in academic environments as a design tool in soil engineering, but in soil engineering practice its use for analysis and design purposes is limited. This is due either to uncertainties in the basic finite element techniques used to describe soil behaviour, or to the development of complex numerical algorithms to approximate various aspects of soil behaviour which were not easily understood by engineers.

In this Paper the Authors implement the finite element method on the trap door problem. This problem as described by Terzaghi (1936) has two modes of displacement, depending on whether the trap door is translated into the soil (passive) or away from it (active).

The passive mode may be used for the evaluation of the uplift force of anchors, or any buried structure which may be idealized as an anchor. Applications of this problem occur during exploration and utilization of ocean resources, for which the breakout forces of objects embedded in sediments at the ocean bottom must be computed. This geotechnical problem is encountered in many marine operations (in both shallow and deep waters) which require the use of anchors to transmit upward-directed forces to the ocean bottom. Typically, any mooring system for ocean surface or submerged platforms owes its stability to the ocean-bottom anchors. The problem appears in design and construction of deep-sea habitats as often as in the design of salvage operations or in repositioning of deep-sea platforms. In addition, as demands for distribution of electric power increase, so does the need for the large-scale scale construction of pylons. The calculation of the resistance against breaking out of the foundations of such pylons is essential for their commercial design.

Analyses of the passive mode of the trap door problem have been performed in the past using various types of physical modelling (Vesic, 1971;

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Das & Seeley, 1975; Sutherland, 1965; Balla, 1961; Vardoulakis, Graf & Gudehus, 1981). Failure loads were provided corresponding to certain types of mechanism observed at failure stages. There was a general agreement on the type of failure mechanism formed above the trap door and the way it changed its shape with varying geometric and material properties.

Numerical analyses of the trap door problem have been performed by Koutsabeloulis (1985), where the influence of dilation angle, Young's modulus, Poisson's ratio and the at-rest earth pressure coefficient K_0 was investigated. It was found that only the dilation angle made a difference to the failure loads. A fully associated flow rule is well known to give excessive volume increase, so Griffiths (1986) proposed more realistic dilation angles which were a function of friction angles in the context of the Mohr–Coulomb criterion. Rowe & Davis (1982a, b) used the finite element method to compute uplift forces of strip anchors embedded in clay and in sand, and other numerical results for the trap door problem have been reported by De Borst & Vermeer (1984), and Griffiths & Koutsabeloulis (1985a).

The active mode can be used to study the gravitational flow of granular material between vertical walls (silo problem), and has also applications to tunnel design. The tunnelling aspects relate to the evaluation of the loosening pressure of sands and/or rocks after excavation takes place during the construction of tunnels. From the engineering point of view, an underground structure can be considered to represent a foreign inclusion inside a mass which has definite rheological properties and is subjected to gravity forces. If the rheological properties of the inclusion are different from those of the surrounding mass, a perturbation in the original stress field will occur around the inclusion, decreasing rapidly with distance according to St Venant's principle. The contact forces between the ground and the inclusion, which are of main interest in the design of underground structures, can therefore, in principle, be determined by the methods of continuum mechanics, if the rheological properties of both the inclusion and the ground are known, or if the displacements of the former are prescribed. In many cases it is common for the exact rheological properties not to be known, particularly for those soils that are mixed with oil and gas. For those cases, Harris, Poppen & Morgenstern (1979) suggested that preliminary stability analyses should be performed using the trap door problem. Einstein & Schwartz (1979) reported that in cases of tunnel openings which were supported after the load corresponding to the free-field stresses has been applied, the simple assumption of external loading instead of excavation unloading may lead to support forces that are 50–100% too conservative. The use of the trap door problem in an analysis was fully supported by Szechy (1973). For the problem of a trap door displacing downwards, limited analytical solutions and experimental data exist.

NUMERICAL SOLUTION TECHNIQUE

The numerical algorithm used to implement soil plasticity within the finite element method was that of the 'initial stress' method (Zienkiewicz, Valliappan & King, 1969), which has been shown (see e.g. Koutsabeloulis & Griffiths, 1986) to be an efficient and versatile way of solving plasticity problems in geomechanics. The algorithm incorporates iterations using equivalent elastic solutions, until any stresses that originally violated yield have returned to the surface of the specified yield criterion within quite strict tolerances. The influence of these tolerances on predicting failure loads of a trap door problem was investigated by Koutsabeloulis (1985). It was found that they made little difference to collapse loads as long as they were kept below 1%, in conjunction with the implementation of correction factors within the initial-stress approach (Nayak & Zienkiewicz, 1972).

Convergence was said to have occurred when the change in nodal displacements, non-dimensionalized with respect to the largest absolute value, nowhere exceeded 1%. Equilibrium and continuity were also satisfied in the usual way using a displacement finite element formulation.

Fifteen-node triangular, isoparametric elements (Nagtegaal, Parks & Rise, 1974; Sloan & Randolph, 1982) were used throughout the analysis using 12 and 16 integration points under plane strain and axisymmetric strain conditions respectively.

Readers interested in the precise details of the initial-stress algorithm used to obtain the solutions quoted in this Paper, are referred to the text by Smith & Griffiths (1988).

SOIL CONSTITUTIVE RELATIONS

The solution presented here assumes that the soil behaves as an elastic, perfectly plastic material. For the passive mode of the trap door problem both plane strain and axisymmetric conditions were assumed, while for the active mode only plane strain conditions were considered, as plane strain and axisymmetric conditions provided similar results. The Mohr–Coulomb failure surface was used in conjunction with a non-associated flow rule. Assumption of zero plastic volume change is reasonable for loose sands, but not for dense sands which dilate during shearing.

When using the Mohr–Coulomb yield criterion, the amount of dilation can be controlled by varying the dilation angle. If the dilation angle is equal to the friction angle then an associated flow rule is assumed, otherwise the flow rule is non-associated. An associated flow rule is automatically imposed by frictionless materials, i.e. $\phi = \psi = 0°$; for sands, more realistic dilation angles have been used in the present work in the range $0 < \psi < \phi$ (Griffiths, 1986).

However, the assumption of zero volume change in the case of dense sands should provide a conservative estimate of failure loads, especially for the active mode. In addition, Atkinson & Potts (1977), using upper bound theorems, presented collapse mechanisms formed above the roof of a tunnel embedded in cohesionless soils. Upper bound theorems are true only for materials whose flow rule is associated. In every other case, Atkinson & Potts (1977) suggested that the upper bound solution provided by the assumption of an associated flow rule will be an upper bound for any other solution for which the dilation angles are lower than the friction angle. However, this can lead to overestimating collapse loads for frictional materials, as Zienkiewicz, Humpheson & Lewis (1975) indicated. They computed differences up to 30% in collapse loads between associated and non-associated flow rules under axisymmetric conditions.

As the aim of the calculations was to estimate collapse loads necessary to cause general shear failure rather than the settlements before failure, the stress–strain behaviour assumed to act within the failure surface was considered relatively unimportant.

The properties assigned to the soil within the failure surface were chosen to be $E = 1{\cdot}3 \times 10^5$ kN/m^2, $\gamma = 20$ kN/m^3, $\nu = 0{\cdot}3$ and $K_0 = 1{\cdot}0$. The shear strength of the soil was governed by a Mohr–Coulomb failure criterion with effective parameters C and ϕ.

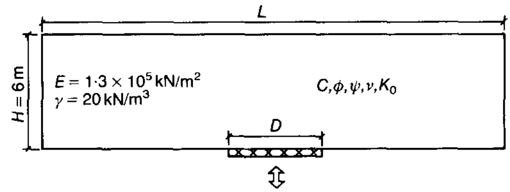

Fig. 1. Definition of the trap door problem

NUMERICAL MODELLING OF THE TRAP DOOR PROBLEM

The trap door problem as defined by Terzaghi (1936) is shown in Fig. 1. It was found (Rowe & Davis, 1982a) that for $L/D = 5$, the modelling effects were insignificant; thus the L/D ratio for the present analysis was fixed at $L/D = 5$. The height H was fixed at 6 m, and to obtain different H/D ratios, the trap door width D was altered keeping H and L/D constant. Although the analysis performed by Rowe & Davis (1982) was for plane strain conditions, Koutsabeloulis (1985) showed that $L/D = 5$ also represented a suitable limit under axisymmetric conditions. The numerical model of the problem is shown in Fig. 2, as defined by De Borst & Vermeer (1984). To avoid singularities, the lower boundary of the first element beside the trap door was given a linear displacement distribution, with the leftmost node of it remaining fixed. The trap door itself received a uniform set of prescribed displacements.

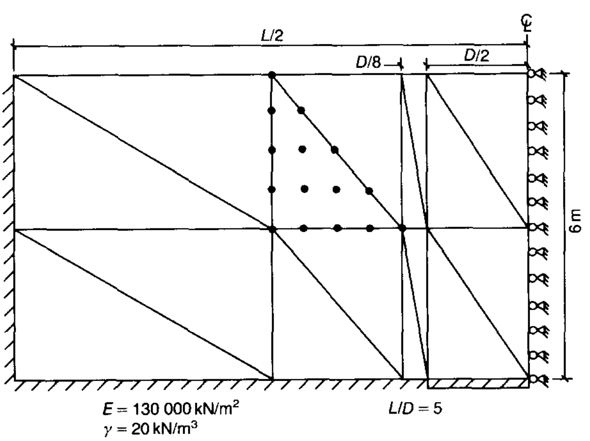

Fig. 2. Finite element discretization of the trap door problem; $L/D = 5$

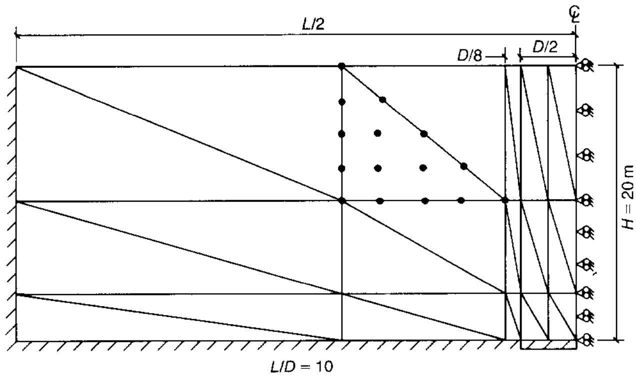

Fig. 3. Finite element discretization of the trap door problem; $L/D = 10$

The rationale for using displacement rather than load control was explained by Griffiths (1982). Failure under displacement control is indicated by a levelling out of the averaged stresses above the trap door; having reached the ultimate capacity, these remain at that value despite further displacement increments.

To eliminate possible numerical errors, some test cases were first analysed using both frictional and frictionless material types, in conjunction with the passive and active modes. The finite element mesh shown in Fig. 2 and a finer mesh shown in Fig. 3, which had an L/D aspect ratio of 10, were used for these verification test cases.

First, a problem was analysed using the active mode of the trap door problem. The soil was assumed to be frictionless, with self-weight: for this type of problem, Davis (1968) has proposed both upper and lower bound solution. Since $\phi = 0°$, the numerical solution of a finite element analysis incorporates an associated flow rule and thus should coincide with Davis's (1968) solution. According to this solution the residual stress above the trap door for $H/D = 1$ is given by

$$(\gamma H - P)/C = 2 \tag{1}$$

where γ is the unit weight, C is the soil cohesion and P is the vertical stress above the trap door.

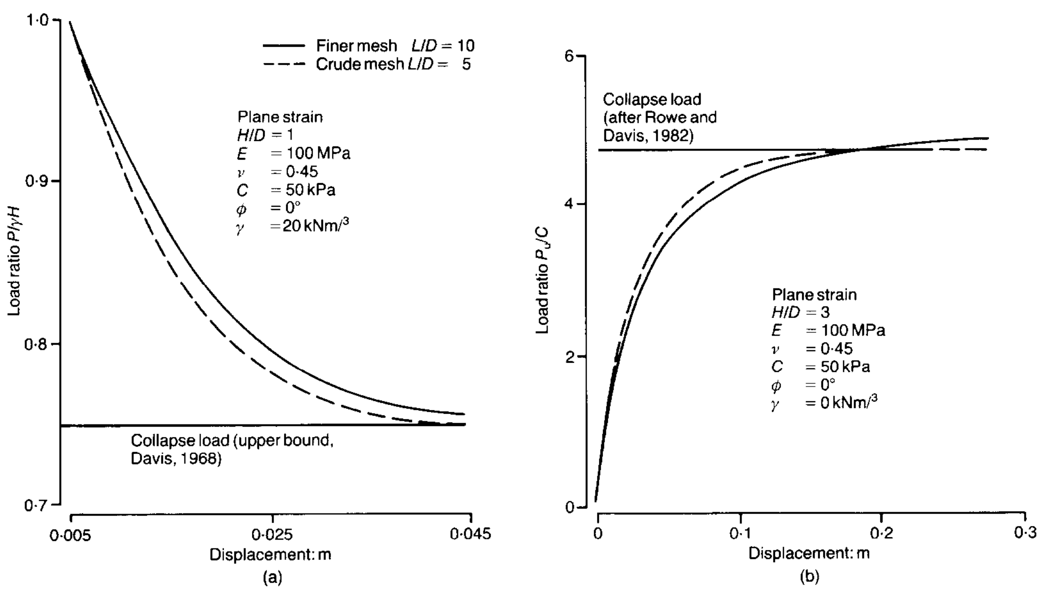

Fig. 4. Active mode: load ratio plotted against imposed displacement

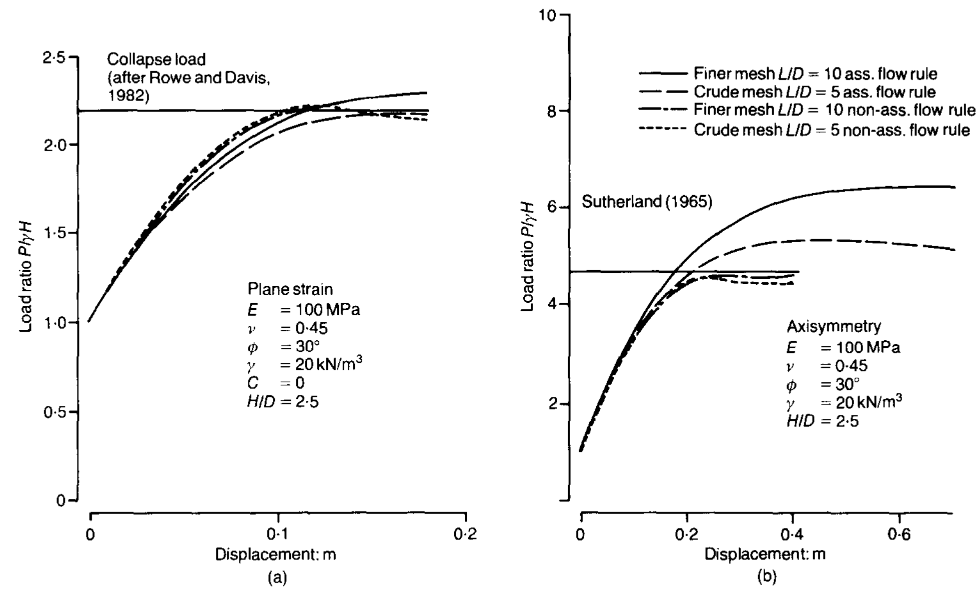

Fig. 5. Passive mode: load ratio plotted against imposed displacement

Setting $C = 50$ kPa and $H = 20$ m, equation (1) becomes $P = 0{\cdot}75\gamma H$. The numerical solutions obtained using the finite element meshes of Figs 2 and 3 are shown in Fig. 4(a). As can be observed, the numerical results are independent of the L/D ratio and the refinement of the mesh, and are similar to the solution proposed by Davis (1968).

In all analyses, a displacement-controlled approach was used, with the vertical stresses adjacent to the door noted after each displacement increment. Failure was deemed to have occurred when the gradient of the load–deformation curve became sufficiently small.

In a similar fashion, a problem of a frictionless material was analysed using the passive mode of the trap door problem. In this case, the trap door model implies that there will be an immediate breakaway of the trap door from the soil below it. In most realistic cases this is true, as most soils have limited tensile strength, as pull-out forces apply. Assuming $H/D = 3$, this problem was also analysed by the meshes of both Fig. 2 and Fig. 3: its numerical solution as compared with that proposed by Rowe & Davis (1982b) is shown in Fig. 4(b). If P_u/C is the dimensionless collapse load, the agreement between the two results and Rowe & Davis's (1982b) results is good. In addition, refinement of the mesh and different L/D aspect ratios made little difference to the numerical solution.

To investigate the influence of the association of the flow rule, the passive mode of the trap door problem was used assuming the soil to be cohesionless. Under these conditions an immediate breakaway will occur between the trap door and the soil below it, as zero tensile stresses are implied. The problem was analysed by both meshes shown in Figs 2 and 3, assuming both plane strain and axisymmetric conditions, with $\phi = 30°$ and $H/D = 2{\cdot}5$.

The results obtained under plane strain conditions are shown in Fig. 5(a) and indicate that neither the ratio L/D nor the flow rule had much influence on the collapse load. Maximum discrepancy of the four results did not exceed 5%, and the collapse load computed for a non-associated flow rule was similar to that computed by Rowe & Davis (1982b).

However, under axisymmetric conditions, the maximum discrepancy between an associated flow rule and a non-associated flow rule was approximately 30%, as high as that noticed by Zienkiewicz *et al.* (1975) when they analysed a triaxial test problem. They suggested that such discrepancies are due to the confinement of the problem, which increases as axisymmetric conditions are approached. However, in practice, as Rowe & Davis (1982b) explained, the soil will exhibit friction and dilatancy up to the peak values, which reduce to the critical values of ϕ_{cr} and $\psi = 0°$ at large strains. As different points in the soil mass will reach peak values at different times, there will be a variation in ϕ and ψ through the soil mass. In consequence, any solution using peak values will tend to overestimate the pull-out load. This implies that, in reality, the

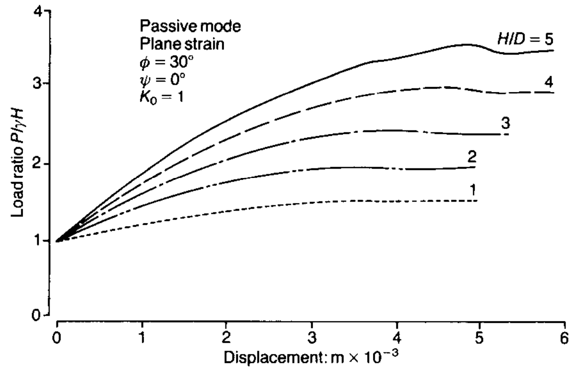

Fig. 6. Passive mode, plane strain: load ratio plotted against displacement; ϕ constant

collapse load might reach a peak value, which subsequently reduces to an ultimate value corresponding to zero volume change condition. The collapse loads obtained using a non-associated flow rule by both meshes of Figs 2 and 3 were in good agreement (Fig. 5(b)), and they compared favourably with that suggested by Sutherland (1965).

In the following analysis, cohesionless soil types will be considered, assuming a non-associated flow rule and using the mesh of Fig. 2.

PASSIVE MODE

Plane strain conditions

The average applied pressure P required to cause failure of the soil above the trap door, as shown in Fig. 1, can be defined as a dimensionless load ratio as follows

$$P/\gamma H = f(H/D, \phi) \tag{2}$$

where γ is the unit weight of the cohesionless soil, H is depth of the soil above the trap door, D is width (diameter) of the trap door and ϕ is soil friction angle.

For $\phi = 30°$, $\psi = 0°$ and $K_0 = 1$, five H/D aspects ratios were tested and the results obtained are shown in Fig. 6.

As can be observed, the displacement control method employed in the present work clearly indicated the load ratio, as defined by equation (2) at failure, for all five H/D ratios. Some numerical instability was observed at $H/D = 4$ and $H/D = 5$, but that was due mainly to the convergence tolerance used in the analysis. Lower tolerances could refine the mode of failure, but could not improve the collapse load ratio by more than 2% (Koutsabeloulis, 1985).

For $H/D = 4$, $\psi = 0°$ and $K_0 = 1$, three different friction angles were tested, and the results are plotted in Fig. 7. As in Fig. 6, failure was clearly indicated for all three friction angles. Further

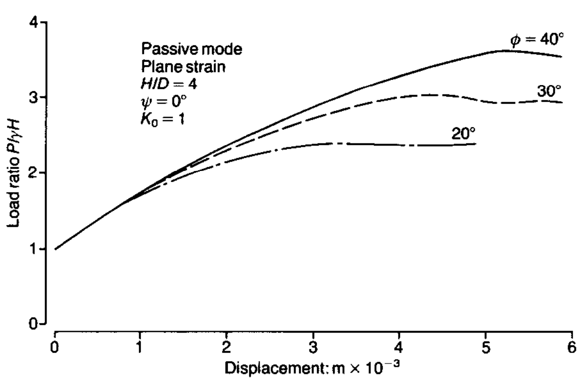

Fig. 7. Passive mode, plane strain: load ratio plotted against displacement; H/D constant

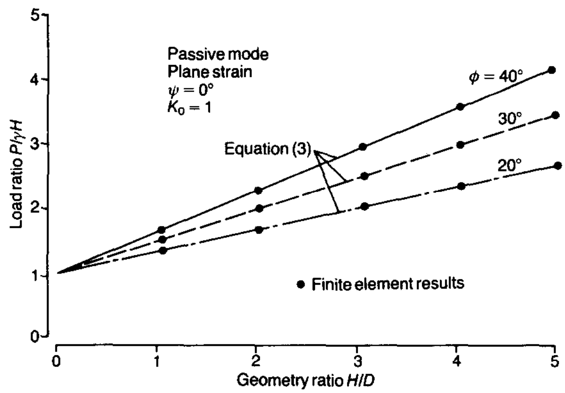

Fig. 8. Passive mode, plane strain: ultimate load ratio plotted against H/D

parametric studies for different friction angles indicated a linear variation between $P/\gamma H$ at failure and H/D. The Authors found that this variation was closely modelled by the expression

$$P/\gamma H = H/D \sin \phi + 1 \tag{3}$$

The solution of equation (3) is shown in Fig. 8.

Using equation (3), a comparison was made against numerical and laboratory results obtained by Rowe & Davis (1982b), as shown in Fig. 9. The agreement with equation (3) was generally good, providing results that tended to be slightly conservative.

In order to account for the contribution of dilation to passive resistance, the following modification to equation (3) is proposed

$$P/\gamma H = H/D \sin (\phi + \psi) + 1 \tag{4}$$

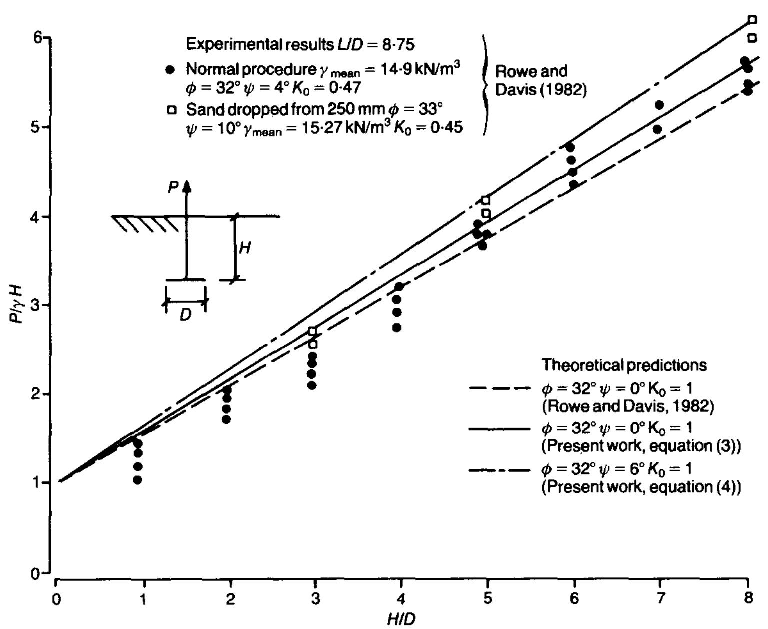

Fig. 9. Passive mode, plane strain: comparison of experimental and theoretical collapse loads

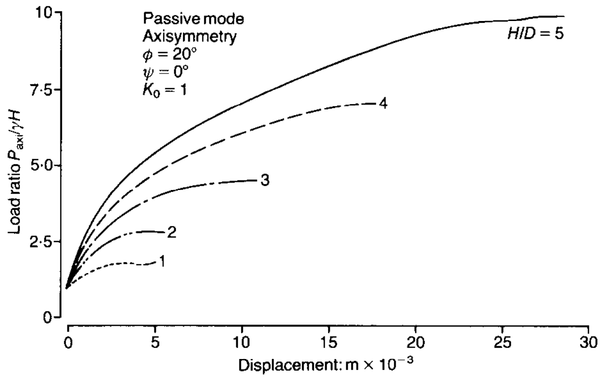

Fig. 10. Passive mode, axisymmetry: load ratio plotted against displacement; ϕ constant

The expression given by equation (4) was found to fit well to computed results, in which ψ was varied, with all other parameters kept constant. This procedure was repeated for a number of different H/D ratios and friction angles. It is suggested that the value of ψ used in equation (4) is that proposed by Griffiths (1986).

The format of equations (3) and (4) is similar to that of an equation proposed by Ladanyi & Hoyaux (1969) for the same problem.

As a benchmark, the example given by Rowe & Davis (1982b) has been re-analysed. An anchor of width $D = 2$ m is buried at a depth of 6 m in a granular material, with strength properties $C = 0$ kN/m^2, $\phi = 35°$, $\psi = 12°$, $K_0 = 0{\cdot}5$, $\gamma = 20$ kN/m^3, and subjected to a vertical pull. From equation (3) for $H/D = 6/2 = 3$ and $\phi = 35°$

$$P_1/\gamma H = \sin 35° + 1 = 2{\cdot}72$$

Hence

$$P_1 = 2{\cdot}72 \times 20 \times 6 = 326 \text{ kN/m}^2 \qquad (5)$$

Using $\psi = 12°$, equation (3) gives

$$P_2/\gamma H = 3 \sin (35° + 12°) + 1 = 3{\cdot}19$$

Hence

$$P_2 = 3{\cdot}19 \times 20 \times 6 = 383 \text{ kN/m}^2 \qquad (6)$$

However, for $\phi = 35°$ Griffiths (1986) provided $\psi = 6°$, and thus equation (3) gives

$$P_3/\gamma H = 3 \sin (35° + 6°) + 1 = 2{\cdot}97$$

Hence

$$P_3 = 356 \text{ kN/m}^2 \qquad (7)$$

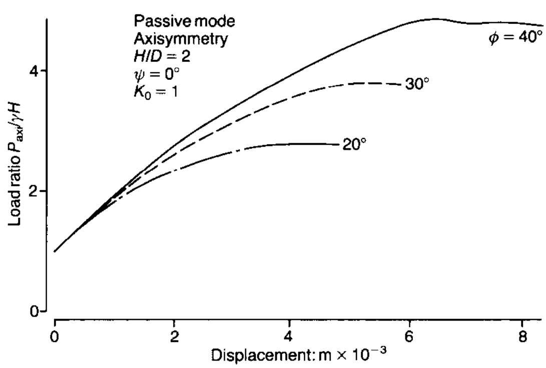

Fig. 11. Passive mode, axisymmetry: load ratio plotted against displacement; H/D constant

The laboratory test gave $P = 346$ kN/m^2, while the numerical results of Rowe & Davis (1982) gave $P = 363$ kN/m^2.

Compared with the laboratory results, the use of equation (4) with the modified dilation angle came closest, but was slightly conservative.

Axisymmetric conditions

In a similar manner to that used under plane strain conditions, the trap door problem was analysed under axisymmetric conditions. Although Rowe & Davis (1982) emphasized the influence of the anchor type, they did not extend their work to the case of a circular anchor. However, the problem has received some attention from Balla (1961), Sutherland (1965) and Vésic (1971, 1972), who considered the breakout resistance of objects embedded in the ocean bottom as a cavity expansion problem.

Figure 10 shows the load–displacement characteristics with $\phi = 20°$, $\psi = 0°$ and $K_0 = 1$ for various H/D values, where D is the diameter of the trap door. It was noted that the displacement to failure varied considerably depending on the value of H/D. Fig. 11 shows similar plots with $H/D = 2$, $\psi = 0°$ and $K_0 = 1$, for friction angles $\phi = 20°$, 30° and 40°. The results indicated good numerical convergence and stability.

As in the case of plane strain, an equation was developed to fit results obtained for five values of the H/D ratio and three values of the friction angle ϕ. The equation which provided the best fit was

$$P_{axi}/\gamma H = P/\gamma H[R_{\phi}^{-(H/D)R_{\phi\theta}}] \qquad (8)$$

where R_θ and $R_{\phi\theta}$ are parameters which depend on ϕ as shown in Fig. 12, and $P/\gamma H$ represents the dimensionless failure load under plane strain conditions.

Equation (8) has similar format to that proposed by Vardoulakis *et al.* (1981) and it is more straightforward than that proposed by Vésic (1971).

Included in the variation of $R_{\phi\theta}$ shown in Fig. 12 is the angle b, which was taken to vary linearly with the friction angle ϕ. At $\phi < 20°$, b was set equal to zero; at $\phi = 40°$, b was set equal to 6·5°. This angle b was introduced to account for an arching shear band forming above the trap door.

Results obtained using equation (8) are plotted against the finite element solutions in Fig. 13. As can be seen, equation (8) overestimates the load ratio at high H/D ratios and high friction angles, but this overestimation never exceeds 5%.

Results obtained using equation (8) are plotted in Fig. 14 against other solutions obtained by Balla (1961) and Sutherland (1965). Balla (1961) described the formation of an arching type of failure mechanism above the trap door, which occurred irrespective of the H/D value and the friction angle. In respect of his theory, b was set equal to $\phi/2$. In situ tests reported by Sutherland (1965) showed load ratio values lower than those predicted by Balla's theory, but higher than Sutherland's experimental results. The numerical results, for $H/D \leq 3$, showed very good agreement with the results obtained by Sutherland, but for $H/D > 3$ the results showed a tendency to

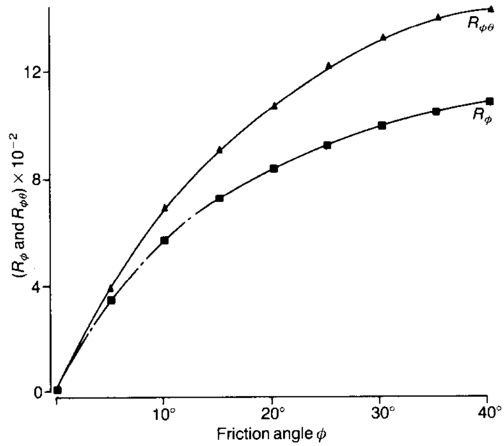

Fig. 12. Passive mode, axisymmetry: variation of R_ϕ and $R_{\phi\theta}$ with friction angle ϕ

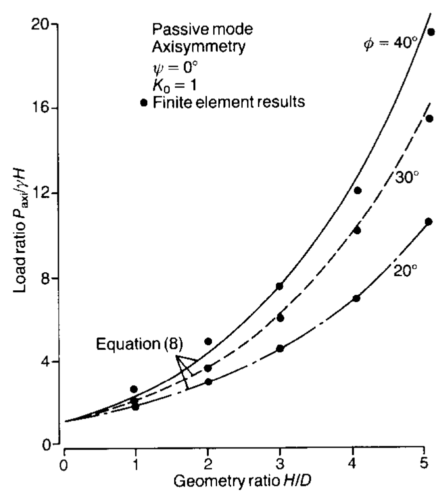

Fig. 13. Passive mode, axisymmetry: ultimate load ratio plotted against H/D

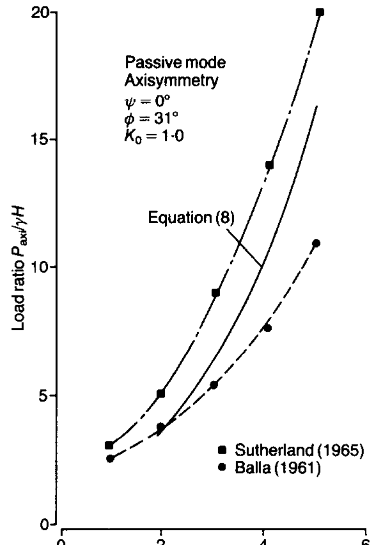

Fig. 14. Passive mode, axisymmetry: variation of failure load with H/D

approach Balla's prediction. This was in accordance with comments made by Vésic (1971), in that there is a characteristic relative depth H/D beyond which anchor plates start behaving as 'deep' anchors. After this stage the failure load increases faster as H/D increases. Thus, the use of b was justified.

Under axisymmetric conditions, higher H/D values were not considered, as they required large amounts of displacement to reach collapse. Analyses of this type would probably benefit from a 'large deformation' approach. Similar observations were made by Rowe & Davis (1982) for plane strain conditions.

ACTIVE MODE

For the active mode of displacement, only the plane strain problem is considered, because as Vardoulakis *et al.* (1981) observed, the analysis does not differ very much from that of axisymmetry.

In general, the active mode of the trap door problem could be used on two types of problem

(*a*) to model the gravitational flow between two parallel vertical walls, along which two thin boundary layers are formed (Vardoulakis *et al.*, 1981)

(*b*) to model the 'ground reaction curve' (Pells, 1978) for the soil–structure interaction problem of tunnel design.

For the first type of problem, Vardoulakis *et al.* (1981) showed that with increase of the overburden pressure, the ratio $P/\gamma(D/2)$ tends to a constant value, and this problem is meaningful only for friction angles $\phi < 30°$.

However, for the second type of problem, monitoring the ratio $P/\gamma H$, the 'ground reaction curve' can be obtained for a particular soil in which a tunnel is to be constructed. Superimposing on this curve the stiffness of the tunnel lining, an equilibrium point can be computed in the sense of Brown, Bray, Ladanyi & Hoek (1983), which will define the soil–structure interaction load. If this is

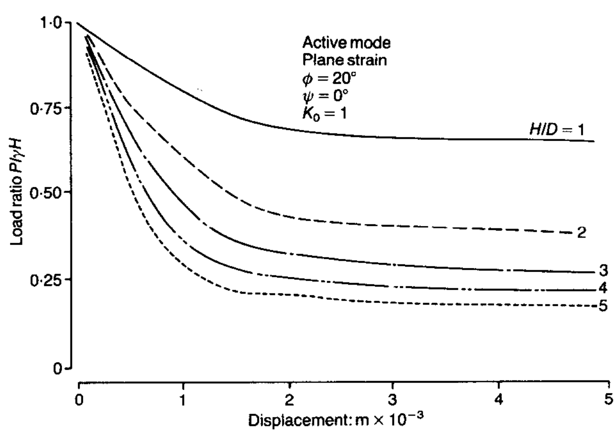

Fig. 15. Active mode, plane strain: load ratio plotted against displacement; ϕ constant

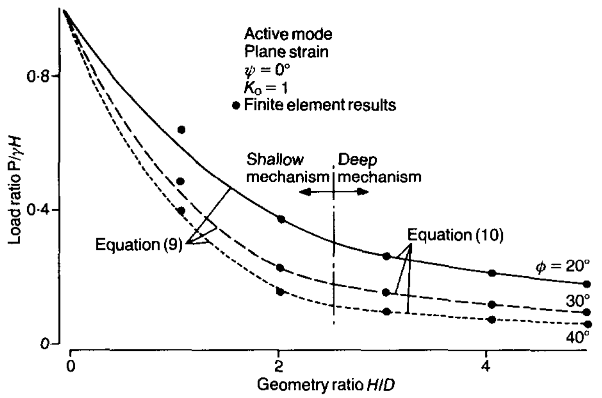

Fig. 16. Active mode, plane strain: ultimate load ratio plotted against ***H/D***

not done, support forces that are 50–100% too conservative may be obtained, as pointed out by Einstein & Schwartz (1979).

Figure 15 shows the computed failure loads for different H/D ratios considering $\phi = 20°$, $\psi = 0°$ and $K_0 = 1$. It was observed that increasing the H/D ratio resulted in a lower failure load ratio $P/\gamma H$. However, for this particular friction angle the load ratios did not become zero, which implies that for a tunnel design there may exist an ultimate load ratio which must be carried by the tunnel lining.

This load ratio tended to zero as the friction angle increased (Fig. 16). This implies that the tunnel lining at high displacements may take no loading from the soil, as can happen for a tunnel constructed in 'rock' material.

The Authors found that the soil behaviour of Fig. 15 can be closely approximated by the consideration of two types of failure mechanism: a 'shallow' failure mechanism and a 'deep' failure mechanism. The change from one mechanism to another occurs at a depth ratio of around $H/D = 2{\cdot}5$.

The following two expressions are proposed on the basis of the computed results for the active case. For shallow mechanisms (i.e. $H/D \leqslant 2{\cdot}5$)

$$P/\gamma H = (R_{G_1})^{G_1 H/D \tan\phi} \tag{9}$$

and for deep mechanisms (i.e. $H/D > 2{\cdot}5$)

$$P/\gamma H = G_2(R_{G_2})^{G_3 H/D \tan\phi} \tag{10}$$

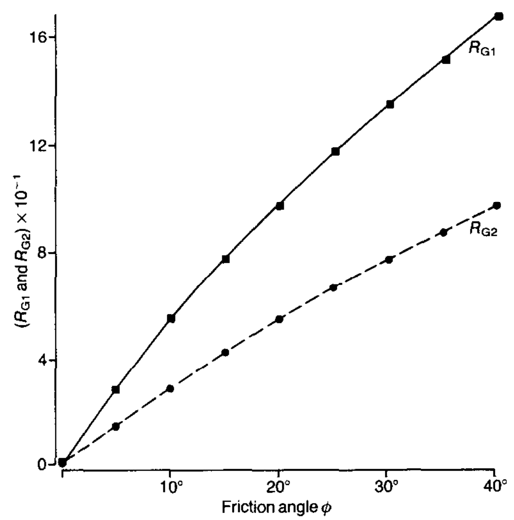

Fig. 17. Active mode, plane strain: variation of R_{G_1} and R_{G_2} with friction angle ϕ

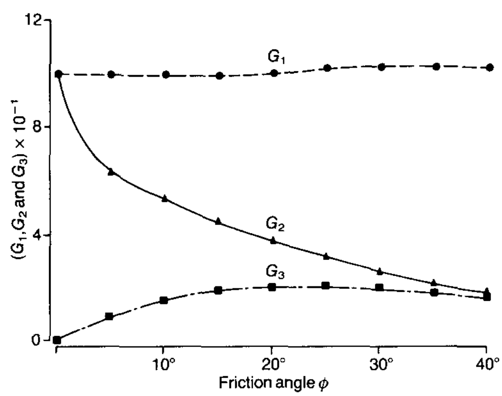

Fig. 18. Active mode, plane strain: variation of G_1, G_2 and G_3 with friction angle ϕ

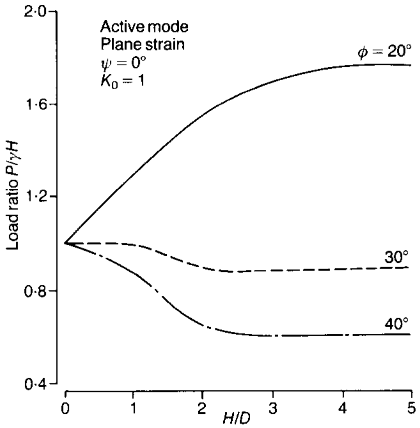

Fig. 19. Active mode, plane strain: ultimate load ratio plotted against H/D

where R_{G_1}, R_{G_2}, G_1, G_2 and G_3 are parameters which depend on ϕ as shown in Figs 17 and 18.

The format of equations (9) and (10) is similar to that of the equation proposed by Ladanyi & Hoyaux (1969) for the same problem. These equations, though, take into account additional factors, such as change of failure mechanism for $H/D = 2{\cdot}5$ and the formation of an arched band.

If a different dimensionless grouping is taken, namely the ratio $P/\gamma(D/2)$, then the problem of gravitational flow between two parallel vertical walls can be considered, as discussed by Vardoulakis *et al.* (1981). Fig. 19 shows the variation of $2P/\gamma D$ with H/D, leading to the following observations

(*a*) for $\phi = 30°$, $2P/\gamma D$ becomes less than one, confirming the findings of Vardoulakis *et al.* (1981)
(*b*) $2P/\gamma D$ tends to a constant value with increasing H/D ratio
(*c*) the change of mechanism is justified for $H/D > 2{\cdot}5$.

CONCLUSIONS

The finite element method has been implemented in both passive and active modes of displacement in the trap door problem. Results obtained have been presented as expressions obtained empirically using curve-fitting methods. The expressions take account of the geometry, the material properties and the type of failure mechanism developed above the trap door.

The results have been validated against existing solutions obtained both physically and numerically. Practical applications of the solutions for the passive case include the prediction of pull-out resistance of anchor plates assuming that the soil possesses zero tensile strength. The solutions for the active case are relevant in the area of tunnel loading and may lead to the use of less conservative design procedures.

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