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## Vertical spatial correlation length based on standard penetration tests

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### ABSTRACT

The spatial correlation length (SCL), or the scale of fluctuation, is a parameter for describing the spatial variability of soil and one of the important parameters used in random field theory. Studies reporting the spatial correlation length based on real field data of offshore/nearshore sea bottom soils are rather limited in the literature, so in this study, the vertical spatial correlation length is determined using site investigation data from two sites of the southern coast of Turkey. Based on quite extensive data, the vertical spatial correlation length is estimated using four different autocovariance functions. The values are within typical ranges reported in the literature for similar soil groups, both onshore and offshore. It is also noted that the widely-used exponential function almost always gives the lowest value of spatial correlation length. The results of this study add to the database of spatial correlation lengths based on real data and could be useful for future studies on reliability assessment of offshore foundations using random finite element method.

**Abbreviations:** Autoreg: second order autoregressive; CL: low plasticity clay; CH: high plasticity clay; CosExp: cosine exponential; COV: coefficient of variation; CPT: cone penetration test; CPTU: piezocone penetration test;  $c_u$ : undrained shear strength; CU: consolidated undrained; DPL: dynamic probing;  $D_r$ : relative density; Exp: exponential; ML: low plasticity silt; MH: high plasticity silt; Pa: atmospheric pressure; SC: clayey sand; SCL: spatial correlation length; SM: silty sand; SP: poorly graded sand; SPT: standard penetration test; SPT-N: standard penetration test N value; SqrExp: squared exponential; Std: standard deviation; SW: well graded sand; RFEM: random finite element method; UC: unconfined compression; USCS: unified soil classification system; UU: unconsolidated undrained

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Nearshore soils; scale of fluctuation; spatial correlation length; standard penetration test; variability

## Introduction

Variability and uncertainties in soil properties have been a topic of interest for geotechnical engineers, especially in the recent decades. In conventional geotechnical design, characteristic/representative values of soil parameters are used leading often to a Factor of Safety which is unable to give any guidance on variability (e.g., Li and Lumb 1987; Cherubini 2000). Because of the inherent variability of soil properties from site to site (and within a site), Baecher and Christian (2003) caution that it is “neither easy nor wise to apply typical values of soil property variability ... for a reliability analysis”. The key issue here is that a single characteristic value is unable to model variability, which needs at least two numbers (e.g., a mean and a standard deviation). The importance and the effects of determining the variability in soil properties have been illustrated by various researchers with examples from actual case studies (e.g., Lacasse and Nadim 1996; Cho and Park 2009; Cho 2010; Zhang and Chen 2012; Carswell et al. 2013; Sarma, Krishna, and Dey 2014; Liu et al. 2015; Jha 2016). Parameters of soil used in any design, such as foundations, dams, natural slopes, road cuts, embankments, and levees, have significant uncertainties due to limited site investigations and laboratory tests in addition to the

uncertainties and limitations involved in empirical correlations. Furthermore, there is no way to make enough soil investigations to get deterministic values for soil parameters at every point (Vanmarcke 1977). For this reason, in stochastic methods, the variability of soil parameters is defined by a mean, a standard deviation, and a spatial correlation length (SCL). The importance of SCL in soils was brought to the attention of the geotechnical engineering community in the mid 1990's by Griffiths and colleagues (e.g., Griffiths and Fenton 1993; Paice, Griffiths, and Fenton 1996; Griffiths and Fenton 2007; Fenton and Griffiths 2008; Griffiths, Huang, and Fenton 2009) with the development of the Random Finite Element Method (RFEM). The SCL is defined as the distance over which the soil parameters tend to be spatially correlated. The SCL may be anisotropic (e.g., Cherubini 2000) with a higher value in the horizontal direction. In this study, however, only the vertical SCL is considered.

Statistical evaluation of offshore field data is rare in Turkey but has significant potential benefits for reliability-based design of nearshore/offshore structures. In this study, site investigation data of two nearshore sites on the southern coast of Turkey is gathered; properties of these near-shore sediments are presented and the SPT-N data is analyzed to

obtain the vertical SCL using four different autocovariance functions. The results of the present study add to the database of spatial correlation lengths based on real data and could be useful for future studies on reliability assessment of offshore foundations using advanced tools such as the RFEM.

## Background

Probabilistic models considering spatially varying soil properties are being used in studies on general foundations of structures (Paice, Griffiths, and Fenton 1996; Griffiths and Fenton 2000; Griffiths and Fenton 2001; Griffiths, Fenton, and Tveten 2002; Popescu, Deodatis, and Nobahar 2005; Griffiths, Fenton, and Ziemann 2006; Cassidy, Uzielli, and Tian 2013) as well as in offshore foundations, especially in the recent years (Andersen, Vahdatirad, and Sørensen 2011; Vahdatirad et al. 2011; Andersen et al. 2012; Vahdatirad et al. 2013; Liu et al. 2015; Nadim 2015; Overgard 2015). Significant economic and risk-associated benefits, and/or optimized design in terms of higher reliability index, and lower probability of failure for offshore foundations are provided with the use of spatial correlation length approach (Lacasse and Nadim 1996; Cho and Park 2009; Cho 2010; Zhang and Chen 2012; Carswell et al. 2013; Sarma, Krishna, and Dey 2014; Liu et al. 2015; Jha 2016). For example, Liu et al. (2015) compared the annual probability of failure obtained for axial pile capacity with and without accounting for the vertical SCL for undrained shear strength of clays and relative density of sands. Based on CPT cone tip resistance at an offshore piled jacket foundation site in Western Australia, Liu et al. (2015) calculated the vertical SCL in the range of 0.1–0.5 m for sands, and 0.05–1.0 m for clays. Taking into account the vertical SCL gave higher annual reliability index and a lower probability of failure, which led to a more optimal and cost-effective pile penetration depth. The reduction is reported to be by a factor of 2 or 3 on the annual probability of failure (Liu et al. 2015). Therefore, the quantification of the vertical SCL is important and useful for the reliability-based design of offshore structures (Cho and Park 2009; Carswell et al. 2013; Liu et al. 2015; Jha 2016). Although there exist numerous studies investigating the value of vertical SCL of soil properties (Chiasson et al. 1995; Jaksa, Kaggwa, and Brooker 1999; Akkaya and Vanmarcke 2003; Firouziandbandpey et al. 2014), their number is rather limited for offshore/nearshore sediments (Phoon, Quek, and An 2003; Huber 2013; Liu et al. 2015; Zhang et al. 2016).

The degree of spatial correlation can be represented by an autocovariance function,  $C(r)$ , where  $r$  is the vector of the separation distance between two points. The normalized form of the autocovariance function  $\{C(r)/C(0)\}$  is known as the autocorrelation function, where  $C(0)$  is the autocovariance function at a distance  $r=0$  (i.e., the variance of the data) (Lacasse and Nadim 1996). Exponential and squared-exponential equations (Table 1 and Figure 1) are examples of the most commonly used autocovariance functions in modelling soil properties (DeGroot and Baecher 1993; Akkaya and Vanmarcke 2003; Huber 2013; Firouziandbandpey et al. 2014; Zhang et al. 2016; Peng et al. 2017). In the exponential models, the distance at which the autocovariance function  $[C(r)]$

**Table 1.** Autocovariance functions used in this study (Vanmarcke 1977).

Autocovariance function	Scale of fluctuation
Exponential: $e^{-(r/a)}$	$2a$
Squared exponential: $e^{-(r/b)^2}$	$\sqrt{\pi}b$
Cosine exponential: $e^{-\frac{r}{c}} \cos(\tau/c)$	$c$
Second order autoregressive: $e^{-(\tau/d)}[1 + (\tau/d)]$	$4d$

$\tau$  is the lag distance (spacing).

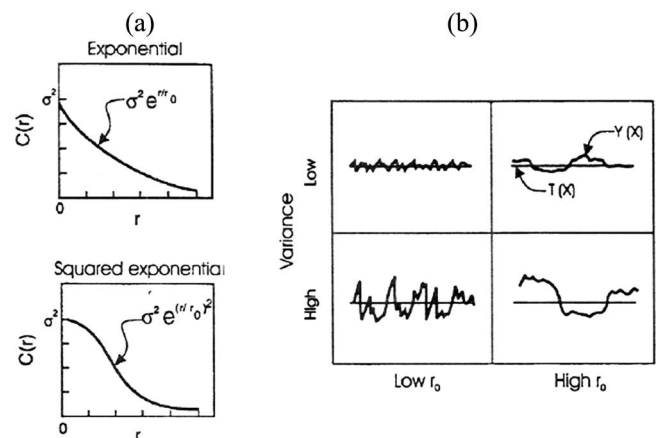
$a, b, c, d$  are constants of the best fit equation.

decays to a value of  $1/e$  (where  $e$  is the base of the natural logarithm) is called the autocorrelation distance,  $r_0$ . This length is a measure of the extent of the spatial correlation. Fluctuation of a soil property is shown in Figure 1 to illustrate the autocorrelation distance and the variance (DeGroot and Baecher 1993).

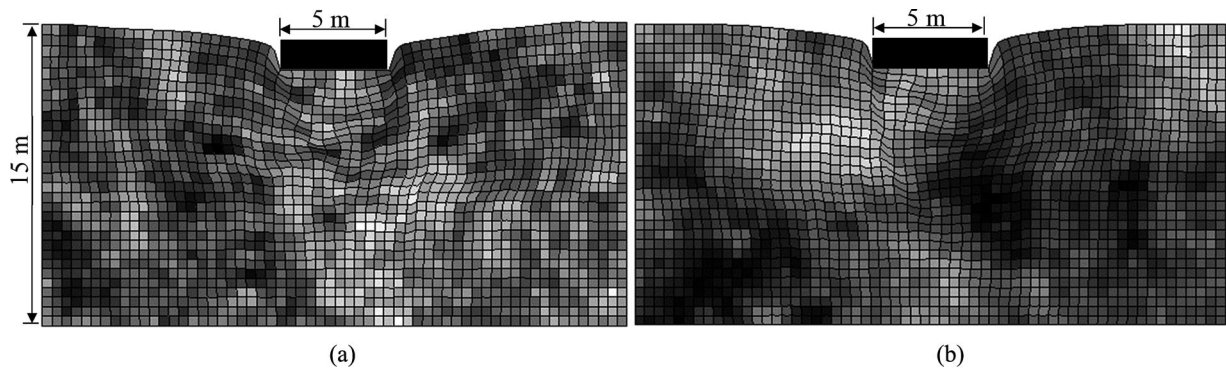
When calculating the SCL, some important considerations are noted in the literature: (1) subdivision of a soil profile into similar layers and detrending inside each layer, have a significant impact on the correlation length (Overgard 2015), (2) the correlation length might be dependent on the sample size, and it is recommended to sample with a distance between the measurement points that is at least smaller than  $1/5$  to  $1/4$  of the correlation length (Huber 2013), (3) spatial correlation in soil properties is somewhat dependent on the soil type and testing method, and is extremely site-dependent (Uzielli et al. 2007).

Large values of SCL indicate that the soil parameters vary smoothly over large distances and within which the soil parameters tend to be close to each other (highly correlated). However, small values indicate that the field is rough and the parameters change rapidly in space with short correlations. Figure 2 shows the visual representation of different vertical SCL values in a random field for a bearing capacity problem, where the random field is generated assuming log-normal distributed cohesion values, and using RBEAR2D software (Fenton and Griffiths 2008), where horizontal and vertical SCL are assumed to be the same value.

It may be noted that an increasing body of literature (e.g., Griffiths et al. 2016) using RFEM has demonstrated the existence of a “worst case” spatial correlation length for failure problems, indicating that an infinite SCL may not always be



**Figure 1.** (a) Commonly used autocovariance functions, (b) effect of autocovariance distance and variance (DeGroot and Baecher 1993; Lacasse and Nadim 1996).



**Figure 2.** Effect of small and large SCL values on the random field for a bearing capacity of a foundation generated by RBEAR2D software (Fenton and Griffiths 2008) (horizontal and vertical SCL are the same) (a) SCL = 2 m, (b) SCL = 10 m. (darker colors indicate larger values of elastic modulus).

conservative. Therefore, evaluation of SCL has significant importance for geotechnical design problems, especially for high-cost and high-risk offshore/nearshore designs. In this study, SCL based on SPT-N data in the vertical direction is evaluated for nearshore sea bottom soils. In the literature, the vertical SCL based on various data sets (both field test data and laboratory test data) have been studied by many researchers for different type of soils, and a summary is presented in Table 2.

### Description of the sites

This study uses site investigation data at two sites in nearshore soils in the Mediterranean Sea of the southern coasts of Turkey (Figure 3). Summary of the available data used in this study is presented in Table 3.

Both sites are located at the intersection of Arabian, African and Anatolian Plates and their geological formations are similar, which are mainly composed of the weathered Mesozoic Limestone, Ophiolitic Rocks and Eocene-aged limestones of Amanos Mountain and sediments transported by alluvial rivers, consisting of gravel, sand, and clay that settled in the Holocene Epoch (Derinsu 2011a, 2011b, 2014, 2015). All soil units in the coastal regions are mixtures of various materials transported and weathered. The majority of the sea-bottom sediments at both sites are composed of “mixture soils”, that is silty, clayey and sandy materials with different proportions, which are classified as CL, CH, ML, MH, SM, SC, SW, SP according to Unified Soil Classification System (USCS). Only in some boreholes, clay layers (CL) of varying thicknesses were identified. Therefore, to determine the vertical SCL, the soil layers are grouped into two: (1) clay layers (Figure 4b) and (2) mixture layers, composed of silty, clayey and sandy materials with different proportions (Figure 4a), gravelly parts are not included in the SCL evaluation in this study. Results of Atterberg limits tests and sieve analyses at both sites are illustrated in Figure 4.

The SPT-N data, site description of the soils in borehole logs and laboratory classification tests (sieve analysis, hydrometer data, fines content, USCS classification and Atterberg limits), are used to identify sublayers that can be described as a relatively homogeneous soil layer. At both sites, the standard penetration tests are conducted at 1.5 m vertical spacing.

The data in the same sublayer are counted and presented in Table 4, which also provide the mean and coefficient of variation of the SPT-N data from two sites. Identification of relatively homogeneous sublayers and the need for studying the vertical SCL of each sublayer within itself are also noted by Phoon and Kulhawy (1999a, 1999b), Uzielli et al. (2007), Overgard (2015) and Firouziandbandpey et al. (2014), among others. After identifying the layers which tend to be sufficiently homogeneous, the raw SPT-N data are analyzed to estimate the mean value and standard deviation of vertical SCL. If the measured data shows a trend, trend analyses can be conducted by separating the random process into a deterministic trend and a residual variability around the trend (Overgard 2015).

The inherent variability of SPT-N measurements is reported in Phoon, Kulhawy, and Grigoriu (1995) by COV value in the range of 25–49% in sandy and silty soils, whereas this value is 37–57% in clayey soils. In this study, COV of SPT-N data of both sites varies between 71 and 88%.

A very limited number of laboratory shear strength (UU triaxial) tests are available on undisturbed samples in cohesive soils. Therefore, undrained shear strength ( $c_u$ ) is determined by utilizing the relationship between SPT-N blowcount and  $c_u$  (Eq. 1) depending on plasticity index (Stroud 1974), acknowledging the limitations of the method. Figure 5 shows that the  $c_u$  of clay layers increases linearly with depth below mudline and Table 5 shows the rate of increase of  $c_u$  with depth at sites 1 and 2.

$$c_u = f_1 N_{60} \quad (1)$$

where  $N_{60}$  is the SPT-N value corrected for 60% energy efficiency and field procedures, and  $f_1$  is a coefficient depending on the plasticity index of clay (Stroud 1974). At site 1, soft to stiff clay layers exist having an undrained shear strength ( $c_u$ ) in the range of 5–100 kPa. At site 2, clays can be classified as soft to medium stiff clays with max  $c_u$  values of 50 kPa. The rate of increase in  $c_u$  with depth is found as 2.1–2.2 kPa/m (for both sites) by utilizing the relationship between SPT-N blowcount and undrained shear strength (Eq. 1). The rate of increase of  $c_u$  with depth, at both sites in this study, are within reported values in the literature (Table 5).

By using empirical equations (Eqs. 2 and 3) based on the SPT-N blowcount, the effective friction angle (Kulhawy and Mayne 1990; Schmertmann 1975) and relative density (Gibbs

**Table 2.** Summary of spatial correlation length in the vertical direction from the literature (number in parenthesis is the mean value).

Reference	Vertical SCL (m)	Soil type	Remarks
Alonso and Krizek (1975) and Lumb (1975), reported by Huber (2013)	0.3–4	Clean sand and sand fill	SPT-N value
Vanmarcke (1977)	2.4	Sandy	SPT-N value
Keaveny, Nadim, and Lacasse (1990)	0.3–1.0	Offshore cohesive soils	Undrained shear strength, CU triaxial
Phoon et al. (1995)	0.1–2.2	Sandy silty	Cone tip resistance
	0.7–1.1	Clay	Cone tip resistance
	2.0–6.2	Clay	Undrained shear strength obtained by vane test,
	0.8–6.1	Clay	Undrained shear strength obtained by various lab tests
Chiasson et al. (1995)	2 m autocorrelation distance	Lightly overconsolidated and highly sensitive clay deposit	Piezocone cone resistance and in-situ vane
Phoon and Kulhawy (1999a, 1999b)	0.8–6.1 (2.5)	Clay	Undrained shear strength
	0.1–2.2 (0.9)	Sand, clay	Cone tip resistance
	0.2–0.5 (0.3)	Clay	Corrected cone tip resistance
	2.0–6.2	Clay	Undrained shear strength from vane shear test
Jaksa et al. (1999)	1.6–12.7 (5.7)	Clay, loam	Natural water content
	0.63–2.55	Relatively homogeneous, stiff, overconsolidated clay known as Keswich Clay	Detrended residuals of cone tip resistance measurements
Cafaro and Cherubini (2002)	0.19–0.72	Clay	Cone tip resistance
Valdez-Llamas, Auvinet, and Núñez (2003)	0.8–2.0	Superficial soft clay	Natural water content
	21	Deep deposits with alternating clayey and sandy soils	Natural water content
Akkaya and Vanmarcke (2003)	0.61–3.72	Sand	Cone tip resistance
	0.36–3.53	Sand	CPT sleeve friction
	0.26–3.14	Clay	Cone tip resistance
	0.30–3.62	Clay	CPT sleeve friction
Phoon et al. (2003)	0.38–0.8	Offshore sediments	CPT data, lab-measured shear strength (UC etc) data
Uzielli, Vannucchi, and Phoon (2005)	0.13–1.11 (0.70)	Sand, Clay, Silt (Mixture)	Cone tip resistance
	0.12–0.60 (0.36)	Sand, Clay, Silt (Mixture)	CPT friction ratio
Schweiger, Peschl, and Pöttler (2007)	1.0–10.0	for “materials such as keuper and middle trias formations”	Reports literature values
Liu and Chen (2010)	1.86	onshore alluvial deposits (loose sandy soils, cohesive soils, medium dense to dense sands and clay layers)	CPT cone tip resistance
	0.82		CPT sleeve friction
Akbas and Kulhawy (2010)	4.0–6.2	Ankara Clay	Liquid limit, $w_L$
	2.5–5.5	Ankara Clay	Natural water content, $w_n$
	1.0–3.0	Ankara Clay	Undrained shear strength, $s_u$
	3.0–3.8	Ankara Clay	SPT-N value
Zhang and Chen (2012)	1.36–3.01	Sandy	SPT-N value
Lloret-Cabot et al. (2014)	0.40–0.44	Filled sand in artificial island	Cone tip resistance
Firouziandbandpey et al. (2014)	0.45–0.50	Clayey silty sand	Normalized cone resistance
	0.2	Clayey silty sand	Normalized friction ratio
Liu et al. (2015)	0.1–0.5	Offshore sands	CPTU cone tip resistance
	0.05–1.0	Offshore clays	
Nadim (2015)	0.18–0.39	Different soil units	Cone tip resistance
Overgard (2015)	0.4–3.0	Offshore sand and clay sublayers	CPT cone tip resistance
Shuwang and Linping (2015)	0.16–0.32 (0.23)	Very soft clay (sand inclusion)	Static cone penetration test
	0.14–1.00 (0.37)	Mud and very soft clay	
	0.16–0.57 (0.37)	Very soft clay and clay	
	0.13–0.32 (0.24)	Clay	
	0.10–0.43 (0.23)	Silty clay	
Bouayad (2017)	0.32–1.32 (0.78)	Onshore sandy soils (loose to medium dense sands, dense fine sands and silty sands)	CPT cone tip resistance
Pantelidis and Christodoulou (2017)	0.11–0.29	Onshore two clay sites	UC tests and light dynamic probing (DPL) in-situ tests

and Holtz 1957) are estimated for all borehole soundings where mixture layers are identified.

$$D_r = \left( \frac{N}{12\sigma'_{vo} + 17} \right)^{0.5} \quad [2]$$

$$\phi' = \tan^{-1} \left[ \frac{N}{12.2 + 20.3 \left( \frac{\sigma'_{vo}}{P_a} \right)} \right]^{0.34} \quad [3]$$

where  $N$  is the SPT-N blowcount,  $P_a$  is atmospheric pressure, 100 kPa and  $\sigma'_{vo}$  is the in-situ vertical effective stress (saturated unit weight is taken as 17.5 kN/m<sup>3</sup>). In addition, the friction angle is also obtained using the NC (normally consolidated) curve provided by Stroud (1988).

The estimated effective friction angle and relative density are provided in Figure 6 and the results are tabulated in Table 6. The relative density results show that the upper parts of the seabed profile have greater  $D_r$  than deeper layers which



**Figure 3.** Locations of sites 1 and 2 in the southern coast of Turkey (Google Earth images are dated 2017, boreholes are done in 2010–2011).

**Table 3.** Information about the data used in this study.

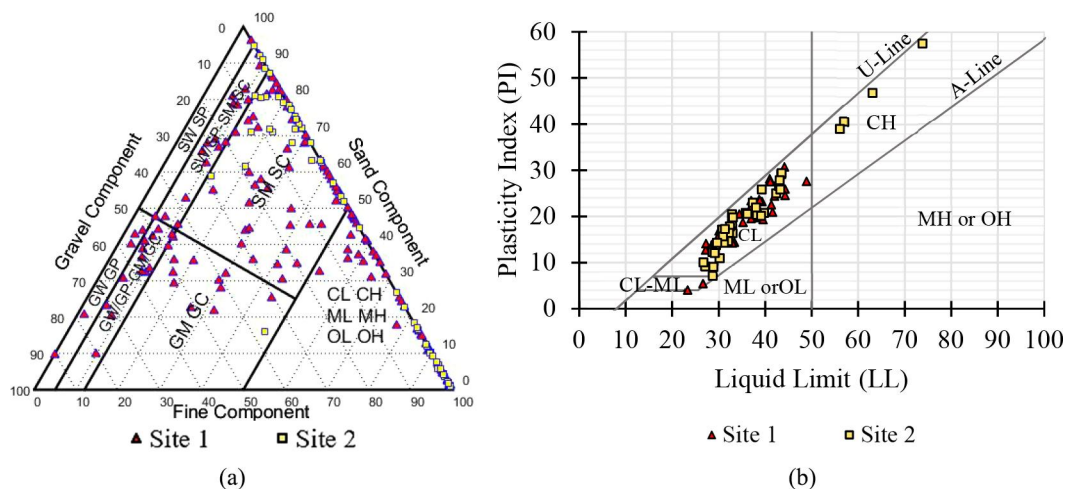
	Number of boreholes	Water depth (m)	Depth of boreholes from seabed (m)
Site 1	27	2.8 to 18.2 (average 8.9 m)	16 to 50.5 (average 30.5 m)
Site 2	14	5.2 to 25.7 (average 16.1 m)	13.8 to 35.4 (average 25.2 m)

does not seem to be realistic. It should be noted however, that the empirical relative density equation (Gibbs and Holtz (1957) uses an overburden corrected SPT-N value, i.e.,  $N_{1,60}$  (in-situ effective vertical stresses are normalized by 100 kPa). This may result in a misinterpretation of the in-situ density state of the soil for the shallow sea bottom sands. The weighted average relative density of the mixture layers at both sites is 29% (it is 28% for site 1 and 40% for site 2) and the mixture of marine soils of this region can be classified as loose to medium dense.

### Method of analyses

During standard penetration testing, disturbed and undisturbed soil samples are obtained from the field at each

SPT depth. In this study, the soils at each borehole (e.g., Figure 7) are first classified according to the Unified Soil Classification System using laboratory tests (sieve analyses, hydrometer test, and Atterberg limits) and observations from the field as reported in borehole logs interpreted. The layers are then grouped into two: (1) mixture layers and (2) clay layers to calculate the corresponding vertical SCL based on SPT-N data. The soil layers classified as sandy gravel, gravelly sand or fill are eliminated because of SPT-N refusal data within these layers. The same type of soil layers at the same boreholes are not lumped together to obtain a single vertical SCL. Instead, they are considered separately and each layer has its own vertical SCL. In this study, the vertical SCL is not calculated for layers that are less than 7.5 m thickness. Twenty seven boreholes at site 1 (average borehole depth of 31 m from seabed) and 14 boreholes at site 2 (average borehole depth of 25 m from seabed) are investigated and vertical SCL's based on SPT-N blowcounts are reported. In the evaluation of SCL, the SPT-N data is not corrected for energy efficiency, borehole diameter, rod length etc. It is known that SPT is prone to measurement errors (equipment-related and operator effects, etc.), however, this has not been considered in the current work. Therefore, evaluated spatial correlation lengths



**Figure 4.** Classification of soils at both sites, (a) sieve analyses, (b) Atterberg test results.

**Table 4.** Variability of SPT-N data for two sites.

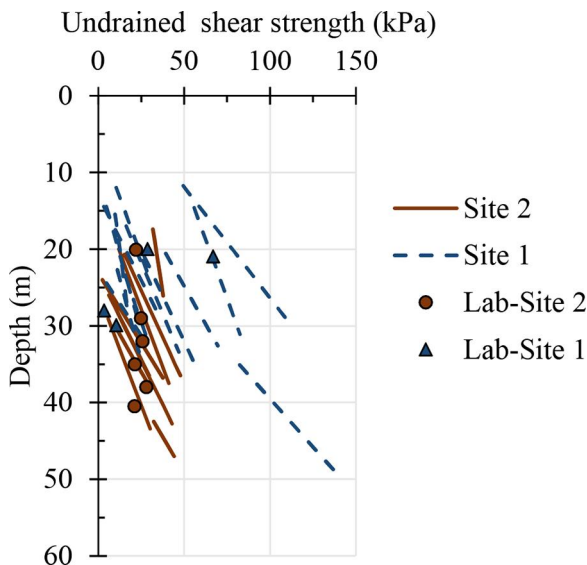
	Sublayer identification	Number of data	Mean SPT-N	COV (%)
Site 1	Mixture Soil (clayey, silty, sandy)	330	17	71
	Clay	100	10	80
Site 2	Mixture soil (clayey, silty, sandy)	89	22	77
	Clay	73	8	88

based on SPT-N data represent not only the inherent variability of soils but also the effect of measurement errors.

Exponential and squared-exponential autocorrelation functions are widely used to calculate SCL in the literature (e.g., DeGroot 1996; Akkaya and Vanmarcke 2003; Zhang and Chen 2012; Huber 2013; Fenton 1999; Firouzianbandpey et al. 2014; Zhang et al. 2016; Peng et al. 2017). In this study, four autocorrelation functions proposed by Vanmarcke (1977) (Table 1) have been utilized to see the effects of the autocorrelation functions on SCL and their corresponding coefficient of determination ( $R$ -squared) values. The evaluation of the SPT-N data and calculation of vertical SCL values are conducted by a MATLAB code developed in this study, which analyzes the data, and fits the autocorrelation functions and reports the SCL.

The SPT-N data can be treated as stationary (i.e., “constant mean with depth”) or nonstationary (i.e., “depth-dependent mean (trend)” approach). In these approaches, the fluctuations about the mean are considered and the corresponding SCL is evaluated. Vanmarcke (1977) states that the initial steps of the spatial variability analysis should be the determination of the existence of a trend (i.e., stationarity or nonstationarity) and standardizing the data. This check can be done by calculation of mean first order increment (Eq. 1) of the data in the vertical direction (Chiasson et al. 1995). If there is an increase or decrease in values with increasing depth, the data should be treated nonstationary (“trend approach”).

$$\bar{d}(\tau) = \frac{1}{n} [N(z_i + \tau) - N(z_i)] \quad [1]$$

**Figure 5.** Undrained shear strength profile at sites 1 and 2 by utilizing empirical equation of Stroud (1974).**Table 5.** Rate of increase of undrained shear strength with depth.

Reference	Rate of increase of $c_u$ with depth (kPa/m)	Remarks
This study	2.1 (range: 0.6–4.0, std. dev.: 1.0) at site 1	Clays nearshore Turkey
	2.2 (range: 1.7–2.8, std. dev.: 0.4) at site 2	
Basack and Purkayastha (2009)	2.5	–
Cao and Wang (2014)	1.6	Marine clays
Hossain et al. (2014)	1.02–2.55	Clays at 14 sites, Gulf of Mexico
Wei, Pant, and Tumay (2010), Kamei and Iwasaki (1995), Li-Zhong et al. (2008), Terzaghi, Peck, and Mesri (1996)	0.8–3.5 for $(c_u/\sigma'_v) = 0.12 - 0.35^*$	–

\*Using buoyant unit weight of 7 to 10 kN/m<sup>3</sup>.

where  $\bar{d}(\tau)$  is the mean first order increment,  $N(z_i)$  is the SPT-N blowcount at depth  $z_i$ ,  $\tau$  is the spacing and  $n$  is the number of data points separated by  $\tau$  (lag distance).

In this study, the calculated SCL's for both “constant” and “trend” approaches are compared. Treating the data as having constant mean and depth-dependent mean are illustrated in Figure 8. It is clear that these two approaches will result in different SCL's and it is reported that removing the trend (detrending) eliminates the longer fluctuations (Akkaya and Vanmarcke 2003). If there exists a trend, the means should be subtracted from the measurements and then this deviation should be divided by standard deviation at each depth (Eq. 2).

$$N_C(z) = \frac{N(z) - \bar{N}(z)}{\sigma_{N(z)}} \quad [2]$$

where the  $N(z)$  and  $\bar{N}(z)$  are the real measurement and trend (mean) at depth  $z$ , respectively,  $N_C(z)$  is standardized data, and  $\sigma_{N(z)}$  is the standard deviation of the measurement. By doing this, the data can be treated as statistically homogeneous which means that the mean ( $\mu = 0$ ) and standard deviation ( $\sigma = 1$ ) are constant with depth. The similar procedure is called as “detrending” in the literature (DeGroot and Baecher 1993; Phoon and Kulhawy 1999a, 1999b; Firouzianbandpey et al. 2014). Detrending of the data also provides stationarity (constant mean and variation). The only difference between standardizing and detrending is that standardizing provides unit standard deviation. In this study, in the “constant mean” approach, the average of the measurements is taken as the mean of the data and kept constant with depth. In the “trend approach”, a linear function is used as the trend of the measurements, and the fluctuations about that trend is evaluated. It is found out that, a linear trend in SPT-N data is generally sufficient to represent the depth-dependency (the fit for the linear trend-line has an average of 0.60 coefficient of determination).

The developed MATLAB code calculates the autocorrelation coefficients (Eq. 3) of the data and plots these coefficient versus lag distance which is the distance between two points of concern. It should be noted that the data sampling interval between observation points has to be constant (Vanmarcke 1977; Fenton and Griffiths 2008; Liu and Chen 2010;

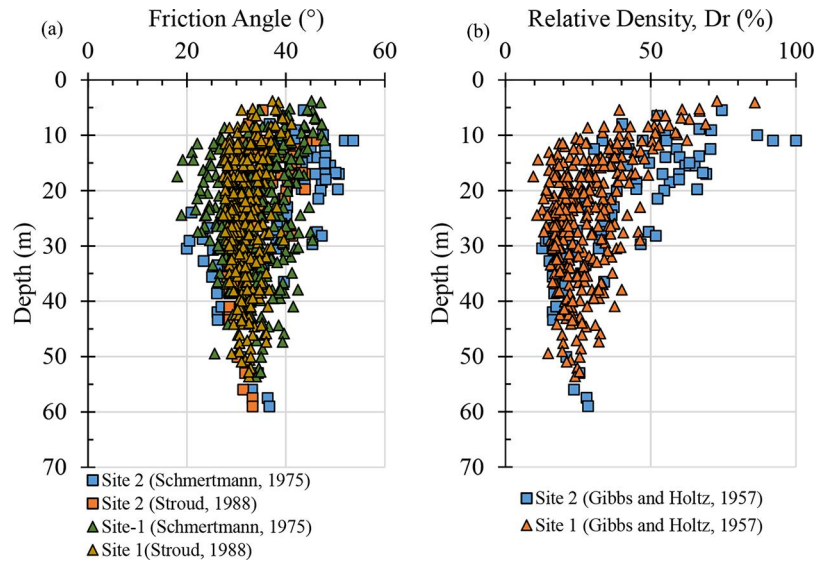


Figure 6. Estimated (a) effective friction angle, (b) relative density, with depth.

Firouzianbandpey et al. 2014; Lloret-Cabot et al. 2014; Yan and Guo 2015; Zhang and Chen 2012). The SCL is calculated by utilizing autocorrelation functions provided by Vanmarcke (1977) (Table 1).

Autocorrelation coefficient is defined as:

$$\rho_k = \frac{\sum_{i=1}^{n-k} (N_i - \bar{N}_i)(N_{i+k} - \bar{N}_{i+k})}{\sum_{i=1}^{n-k} (N_i - \bar{N}_i)^2} \quad k = 0, 1, 2, \dots, (n-1) \quad [3]$$

where the  $N_i$  and  $\bar{N}_i$  are the real measurement and trend (mean) at depth  $i$ , respectively, and  $N_{i+k}$  is the measurement at depth  $i+k$ . The autocorrelation coefficient is constrained by  $[-1.0, 1.0]$ . If the coefficient is positive, both variables tend to be higher and lower together. However, if the coefficient is negative, high value of one variable tends to be associated with a low value of the other variable (Kottogoda and Rosso 2008). In the literature, the same autocorrelation function is also defined in terms of autocovariance function. The autocovariance of the SPT-N blowcounts may be calculated by the method of moments (Eq. 4) and the autocorrelation coefficients may be calculated by normalizing with the data variation (Eq. 5). It is seen that combining Eqs. 4 and 5 results in Eq. 3.

Autocovariance function:

$$c_k = \text{Cov}(X_i, X_{i+k}) = E[(X_i - \bar{X})(X_{i+k} - \bar{X})] \quad [4]$$

where  $k$  is the lag distance,  $X_i$  is the value of parameter  $X$  at the location of  $i$  and  $E$  is the expectation operator.

Autocorrelation function:

$$\rho_k = \frac{c_k}{c_0} \quad [5]$$

where  $\rho_k$  and  $c_k$  are the autocorrelation coefficient and the autocovariance at lag  $k$ , and  $c_0$  is the autocovariance at lag 0.

Estimation of SCL remains as a significant challenge due to a lack of high-resolution measurement data in geotechnical practice. Among the in-situ tests, the cone penetration test data has the highest resolution (typically on the order of a few cm's). However, other properties of soil such as water content, unit weight, undrained shear strength (from laboratory tests), and SPT-N data from the field can also be used to calculate the SCL, even though they have larger spacing between observation points (lower resolution) (Table 2). Using the conventional statistical method described in the manuscript, vertical SCL values found based on SPT in the literature are 2.4 m (Vanmarcke 1977), 0–4 m (Alonso and Krizek 1975 reported in Huber 2013), 0.3 m (Lumb 1975 reported in Huber 2013), and 1.36–1.63 m (Zhang and Chen 2012).

It is also seen that if the mean of the measurement,  $\bar{N}$ , is taken as the trend value in Eq. 3, there is no need to standardize or detrend the measurements. That is, the computed SCL for normal data, detrended data (zero mean) and standardized data (zero mean and unit standard deviation) become the same in case of taking mean as trend value.

Figure 9 shows an example plot of autocorrelation coefficients versus lag distance for a sample borehole at site 1. The four autocorrelation functions in Table 1 are utilized to

Table 6. Friction angle and relative density obtained through SPT-N correlations.

	Friction angle (°)				Dr (%)	
	Schmertmann (1975)		Stroud (1988)		Gibbs and Holtz (1957)	
	Site 1	Site 2	Site 1	Site 2	Site 1	Site 2
Average	33.6	37.6	31.7	34.2	28.3	39.6
Range	18.0–47.7	20.0–53.6	27.3–41.1	27.8–46.3	9.6–85.9	12.1–100
Stan. Dev.	6.1	8.5	2.9	4.9	11.7	20.1
COV (%)	18.0	22.7	9.0	14.4	41.2	50.7



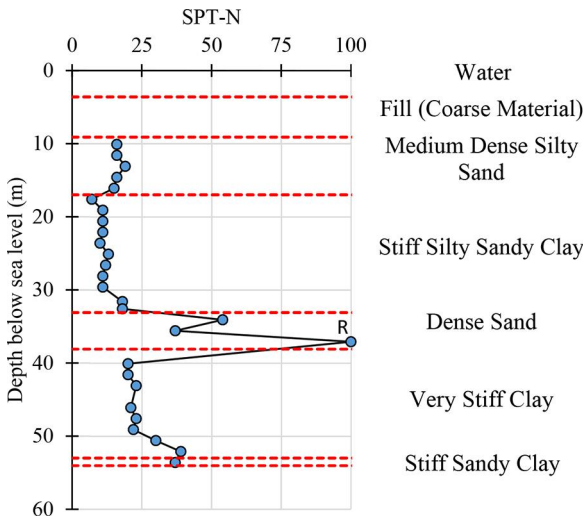


Figure 7. YDSK-1 borehole at site 1.

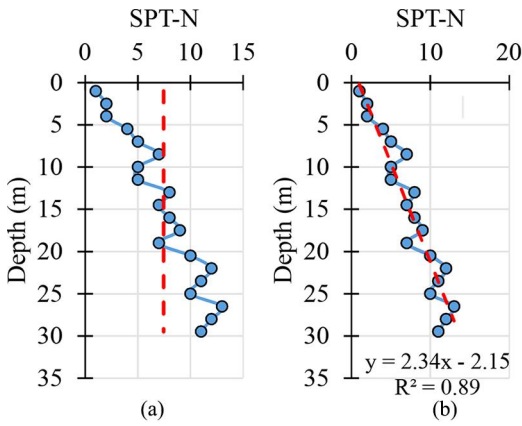


Figure 8. (a) Constant mean and (b) depth-dependent mean approaches.

fit the data and corresponding SCL's and coefficient of determinations of fit are provided in Table 7. The results indicate that although the coefficient of determination does not

Table 7. The spatial correlation lengths, SCL (both “constant” and “trend” approaches) for four autocorrelation functions for borehole YDSK-16.

Correlation Function	“Constant mean with depth” approach		“Trend” approach	
	SCLm)	R <sup>2</sup>	SCLm)	R <sup>2</sup>
Exponential	6.05	0.75	2.30	0.71
Squared-Exponential	6.49	0.79	2.93	0.76
Cosine-Exponential	5.77	0.84	2.23	0.81
2nd Order Autoregressive	6.43	0.78	2.75	0.74

change significantly for “trend” and “constant” approaches, evaluated SCL values in the vertical direction do.

Results

The SPT-N data at both sites are statistically evaluated. Autocorrelation coefficients and the vertical SCL's are calculated by utilizing four different autocorrelation functions (Table 1). The mean values, ranges and the standard deviations of the SCL's with “trend approach” are tabulated in Table 8, Figure 10a,b. In Table 8, the results are reported as the mean vertical SCL obtained by exponential autocovariance function and by all four autocorrelation functions for all boreholes. Figure 10a shows the SCL of mixture soils and Figure 10b shows that of clay layers, with four autocorrelation functions, for both sites 1 and 2.

The mean vertical SCL based on SPT-N data of both sites using all four autocovariance functions and using the “trend approach”, is 1.72 m (±0.91 m standard deviation) for clay layers, whereas it is 2.03 m (±1.29 m standard deviation) for mixture layers. The vertical SCL values are within typical ranges reported in the literature for similar soil groups, both onshore and offshore (Table 2).

In the analyses, two different approaches, constant and trend, have been used and, as stated in the study of Akkaya and Vanmarcke (2003), the “trend approach”, where the fluctuations about the trend are considered, results in shorter fluctuation (shorter SCL). While the average of SCL's with the “trend approach” for clays and mixtures are 1.72 and 2.03 m,

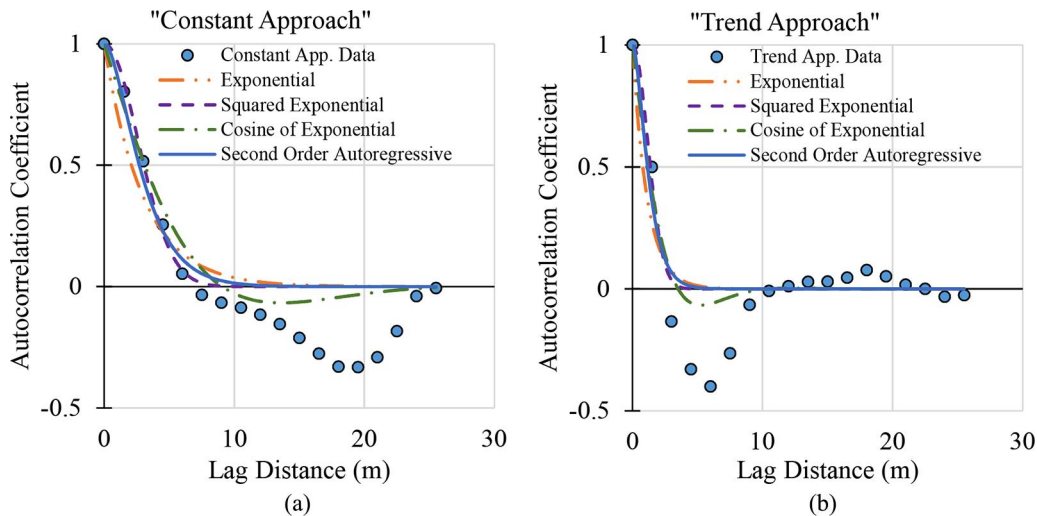


Figure 9. Autocorrelation coefficient vs lag distance for borehole YDSK-16 and utilized autocorrelation functions (a) “constant approach” (b) “trend approach”.

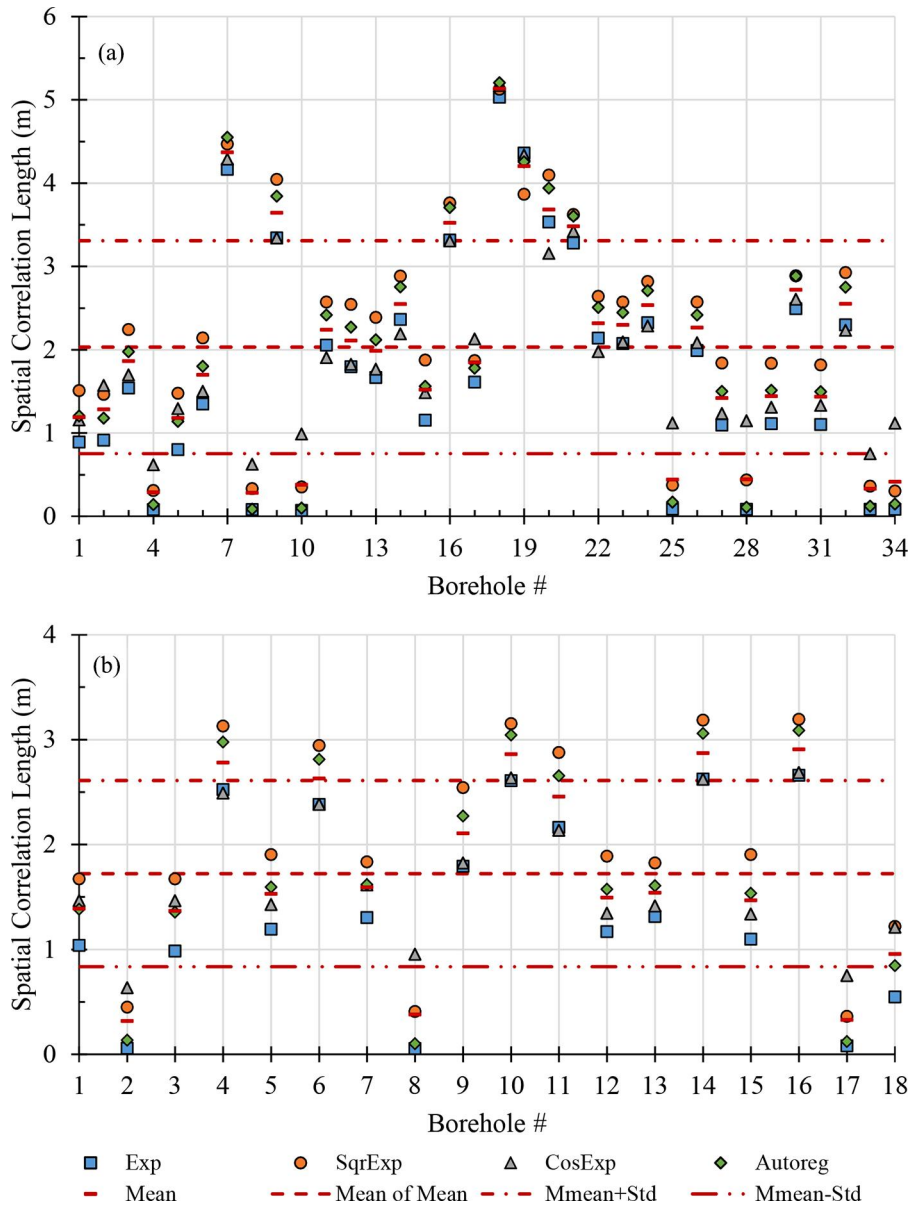
**Table 8.** The mean, and standard deviation of SCL (with “trend approach”) for clays and mixtures.

		Site 1		Site 2	
		Mixtures	Clays	Mixtures	Clays
Four Functions	Mean (m)	2.19	1.75	1.52	1.67
	Range	0.07–5.20	0.06–3.19	0.08–4.55	0.06–3.13
	Standard Deviation (m)	1.27	0.93	1.25	0.89
Exponential Function only	Mean (m)	1.94	1.45	1.23	1.36
	Range	0.07–5.03	0.06–2.66	0.08–4.17	0.06–2.53
	Standard Deviation (m)	1.34	0.94	1.30	0.93

respectively, the SCL’s with “constant-mean with depth” approach are 3.85 and 4.20 m, for clays and mixtures, respectively. This indicates that the SCL with “constant-mean” approach is larger than the SCL with “trend approach”. Figure 8 also illustrates the difference in two approaches. The fluctuation about depth-dependent mean (trend line) is more frequent than the fluctuation about constant mean, which means that constant mean approach results in larger

SCL. In addition to these, it should be noted that, the order of the trend function is important, as the order of the polynomial increases, the SCL decreases (Phoon 2008).

When all four utilized autocorrelation functions are compared, the results show that squared exponential (Gaussian) autocorrelation function gives the highest SCL (in 72% of all evaluations) compared to the others; while the exponential (Markov) autocorrelation function results in the lowest SCL

**Figure 10.** Spatial correlation length of (a) all mixture soils, (b) all clay layers, for both sites 1 and 2, using “trend approach.”

**Table 9.** Goodness of fit, represented by  $R^2$  values, for four different autocorrelation functions (mean value and range in parenthesis).

	Exponential	Squared Exponential	Cosine Exponential	Second Order Autoregressive
Mixtures	0.68 (0.55–0.83)	0.70 (0.56–0.83)	0.74 (0.59–0.85)	0.69 (0.56–0.83)
Clays	0.68 (0.56–0.83)	0.70 (0.56–0.84)	0.74 (0.59–0.86)	0.69 (0.56–0.84)

(in 78% of all evaluations). In addition, the autocorrelation functions, exponential and cosine-exponential mostly gives closer SCL's to each other (in 43% of all evaluations).

Cao and Wang (2014) state that selection of the most suitable correlation function is an important issue and the goodness of fit can be used to help select the most suitable functions. Table 9 shows the goodness of fit in terms of  $R^2$  values for all the SPT-N data reported in the manuscript with their means and ranges. All  $R^2$  values are in the range of 0.55 and 0.86 with an average of 0.70. Results with a coefficient of determination smaller than 0.50 are not considered. Considering all boreholes data for the mixture soils and for the clay soils, the Cosine Exponential Autocorrelation Function gives the highest  $R^2$  values (greater than 0.64 with an average of 0.74), i.e., seems like the best fit among the four types of autocovariance functions.

### Concluding remarks

In this study, the vertical spatial correlation length is determined using site investigation data from two sites on the southern coast of Turkey, based on SPT-N values at 1.5 m depth intervals, from 41 boreholes (depths of 14 to 51 m from seabed) at average water depths of 8.9 and 16.1 m for sites 1 and 2, respectively. At both sites, marine deposits exist where the soil profile generally consists of mixture layers (clayey, silty and sandy materials with different proportions) and low plasticity clay layers. The clay layers at both sites are generally in a soft to medium stiff state (0–50 kPa of  $c_u$ ), but a few clay layers are observed with an undrained shear strength of 50–100 kPa. For “mixture” layers at both sites, the mean friction angle is 34° and it is seen that the mixture soils are mostly in loose to medium-dense state with a mean relative density of about 29%.

Vertical spatial correlation length based on SPT-N data is calculated using four autocovariance functions; namely, exponential (Markov), squared-exponential (Gaussian), cosine exponential and second-order autoregressive. The results indicate that clays and mixtures have mean SPT-N based SCL of 1.72 and 2.03 m, respectively. The results of this study contribute to the SCL studies in the literature and serve as an example from Turkish offshore/nearshore sea bottom soils, and also can be useful in future studies on reliability-based design of offshore/nearshore structures.

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